INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
ECON 8401 **FALL 2004**

EXAMINATION

Please answer **two** of the three questions:

1. Consider an economy in which there are two types of goods, primary goods and manufactured goods. Primary goods are homogeneous and are produced using land services and services and land subject to the production function

$$y_0 = t_0^{1/2} \ell_0^{1/2}$$

Manufactured goods are differentiated by firm using capital services and labor services. There are *n* firms and the production function for firm *j* is

$$y_j = \max[\theta k_j^{1/2} \ell_j^{1/2} - f,\ 0].$$

where *f* is the fixed cost. Suppose that there is a representative consumer with preferences given by the utility function

$$\log\ c_0 + (1/\rho)\log \sum\nolimits_{j=1}^{n} c_j^{\rho}$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{t}$ units of land, $\bar{\ell}$ units of labor, and $\bar{k}$ units of capital.

a) Suppose that the number of manufacturing firms is variable, that these firms are Cournot competitors, and that there is free entry and exit in manufacturing. Define an (autarkic) equilibrium. Explain carefully how you would calculate this equilibrium (You do not need to calculate it.)

b) Suppose now that there are two such countries, one with endowments $(\bar{t}^1, \bar{\ell}^1, \bar{k}^1)$ and and the other with endowments $(\bar{t}^2, \bar{\ell}^2, \bar{k}^2)$, but otherwise identical. Define a trade equilibrium.

c) Suppose that $\bar{t}^1 / \bar{k}^1 > \bar{t}^2 / \bar{k}^2$. Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

d) Suppose now that the manufacturing firms are Bertrand competitors, redefine the trade equilibrium in part b.

2. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0, 1]$ given by the utility function

$$\int_X \log c(x) dx.$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$. Production functions are linear but differ across countries:

$$y_j(x) = \ell_j(x) / a_j(x).$$

Here $y_j(x)$ is the amount of good *x* produced in country *j*; $\ell_j(x)$ is the amount of labor used in this production; and $a_j(x)$ is the unit labor required, given by

$$a_1(x) = e^x$$
$$a_2(x) = e^{1-x}.$$

a) Define a competitive equilibrium of the world economy.

b) Characterize as much as possible the patterns of specialization and trade in the unique competitive equilibrium.

c) Suppose now that there are iceberg transportation costs, so that the unit labor requirement for producing good z in country j for consumption in country i is

$$a_1^1(x) = e^x, \ a_1^2(x) = \left(1+\tau\right)e^x$$
$$a_2^1(x) = \left(1+\tau\right)e^{1-x}, \ a_2^2(x) = e^{1-x}.$$

where $\tau > 0$ is the transportation cost. Explain how your definition of equilibrium is altered and characterize as much as possible how the equilibrium in the world with transportation costs differs from the equilibrium in the world without transportation costs.

d) For the world economy in part c explain how to calculate the value of trade as a fraction of GDP. What is this fraction as a function of $\tau > 0$?

3. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}.$$

The investment good is produced according to

$$k_{t+1} = d(a_1 x_{1t}^b + a_2 x_{2t}^b)^{1/b} .$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t} . \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= \ell_t . \end{aligned}$$

The initial value of k_t is $\bar{k}_0$. ℓ_t is normalized to 1.

a) Define an equilibrium for this economy.

b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in k_t and c_t and a transversality condition. Here $c_t = d(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$ is aggregate consumption.

c) Suppose now that there is a world made up of m different countries, all with the same technologies and preferences, but with different constant populations, $\ell_t^j = \bar{\ell}^j$, and with different initial capital-labor ratios $\bar{k}_0^j$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Let $s_t = c_t / y_t$ where $y_t = p_{1t} k_t + p_{2t} = d(a_1 k_t^b + a_2)^{1/b}$ is per-capita income. Transform the two difference equation in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right).$$

Explain the significance of this result.