INTERNATIONAL TRADE AND PAYMENTS THEORY

## EXAMINATION

Please answer two of the three questions:

1. Consider an economy in which there are two types of goods, primary goods and manufactured goods. Primary goods are homogeneous and are produced using capital and labor services subject to the production function

$$
y_{0}=\theta_{p} k_{0}^{\alpha_{p}} \ell_{0}^{1-\alpha_{p}} .
$$

Manufactured goods are differentiated by firm. There are $n$ firms and the production function for firm $j$ is

$$
y_{j}=\max \left[\theta_{m} k_{j}^{\alpha_{m}} \ell_{j}^{1-\alpha_{m}}-f, 0\right] .
$$

where $f$ is the fixed cost. Suppose that $1>\alpha_{p}>\alpha_{m}>0$. Suppose that there is a representative consumer with preferences given by the utility function

$$
\log c_{0}+(1 / \rho) \log \sum_{j=1}^{n} c_{j}^{\rho}
$$

where $1 \geq \rho>0$. There is an endowment of $\bar{k}$ units of capital and $\bar{\ell}$ units of labor.
(a) Suppose that the number of manufacturing firms is variable, that these firms are Cournot competitors, and that there is free entry and exit in manufacturing. Define an (autarkic) equilibrium. Explain carefully how you would calculate this equilibrium (You do not need to calculate it.)
(b) Suppose now that there are two such countries, one with endowments ( $\bar{k}^{1}, \bar{\ell}^{1}$ ) and the other with endowments $\left(\bar{k}^{2}, \bar{\ell}^{2}\right)$, but otherwise identical. Define a trade equilibrium.
(c) Suppose that $\bar{k}^{1} / \bar{\ell}^{1}>\bar{k}^{2} / \bar{\ell}^{2}$. Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.
(d) Repeat parts b and c supposing that, rather than having free entry and exit, the number of manufacturing firms in each country, $n^{i}>0, i=1,2$, is fixed.
2. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $\mathrm{X}=[0,1]$ given by the utility function

$$
\int_{X} \log c(x) d x
$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_{1}=\bar{\ell}_{2}=\bar{\ell}$. Production functions are linear but differ across countries:

$$
\begin{aligned}
& y_{j}^{i}(x)=\ell_{j}^{i}(x) / a_{j}^{i}(x) \\
& a_{1}^{1}(x)=a_{1}^{2}(x)=\alpha+\beta x \\
& a_{2}^{1}(x)=a_{2}^{2}(x)=\alpha+\beta-\beta x .
\end{aligned}
$$

Here, for example, $y_{j}^{i}(x)$ is the amount of good $x$ produced in country $j$ for consumption in country $i$. Initially, there are no transportation costs or tariffs.
(a) Define an equilibrium for this model.
(b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.
(c) Suppose now that there are transportation costs,

$$
a_{j}^{i}(x)=(1+\tau(x)) a_{j}^{j}(x) \text { when } i \neq j
$$

where $\tau(x)=|0.5-x|$. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in parts a and b.
(d) Suppose now that the countries engage in a trade war in which each imposes an ad valorem tariff of $\tau(x)=|0.5-x|$ on imports of good $x$ from the country. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in part c.
3. Consider a two-sector growth model in which the representative consumer has the utility function

$$
\sum_{t=0}^{\infty} \beta^{t} \log \left(a_{1} c_{1 t}^{b}+a_{2} c_{2 t}^{b}\right)^{1 / b} .
$$

The investment good is produced according to

$$
k_{t+1}=d\left(a_{1} x_{1 t}^{b}+a_{2} x_{2 t}^{b}\right)^{1 / b} .
$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$
\begin{gathered}
c_{1 t}+x_{1 t}=\phi_{1}\left(k_{1 t}, \ell_{1 t}\right)=k_{1 t} \\
c_{2 t}+x_{2 t}=\phi_{2}\left(k_{2 t}, \ell_{2 t}\right)=\ell_{2 t} .
\end{gathered}
$$

where

$$
\begin{aligned}
& k_{1 t}+k_{2 t}=k_{t} \\
& \ell_{1 t}+\ell_{2 t}=\ell_{t} .
\end{aligned}
$$

The initial value of $k_{t}$ is $\bar{k}_{0} . \ell_{t}$ is normalized to 1 .
(a) Define an equilibrium for this economy.
(b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in $k_{t}$ and $c_{t}$ and a transversality condition. Here $c_{t}=d\left(a_{1} c_{1 t}^{b}+a_{2} c_{2 t}^{b}\right)^{1 / b}$ is aggregate consumption. (You do not need to go through all of the algebra, but you need to explain all of the logical steps carefully.)
(c) Suppose now that there is a world made up of $m$ different countries, all with the same technologies and preferences, but with different constant populations, $L_{t}^{j}=\bar{L}^{j}$, and with different initial capital-labor ratios $\bar{k}_{0}^{j}$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.
(d) State and prove versions of the factor price equalization theorem, the StolperSamuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.
(e) Let $s_{t}=c_{t} / y_{t}$ where $y_{t}=p_{1 t} k_{1 t}+p_{2 t}=r_{t} k_{t}+w_{t}=d\left(a_{1} k_{t}^{b}+a_{2}\right)^{1 / b}$ is income per capita. Transform the two difference equation in part b into two difference equations in $k_{t}$ and $s_{t}$. Prove that

$$
\frac{y_{t}^{i}-y_{t}}{y_{t}}=\frac{s_{t}}{s_{t-1}}\left(\frac{y_{t-1}^{i}-y_{t-1}}{y_{t-1}}\right)=\frac{s_{t}}{s_{0}}\left(\frac{y_{0}^{i}-y_{0}}{y_{0}}\right) .
$$

where $y_{t}^{i}=p_{1 t} k_{t}^{i}+p_{2 t}=r_{t} k_{t}^{i}+w_{t}=d\left(a_{1} k_{t}^{i b}+a_{2}\right)^{1 / b}$ is income per capita in country $i$. Explain the significance of this result.

