INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
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EXAMINATION

Please answer **two** of the three questions:

1. Consider an economy in which the consumption space is the set of functions $c : R_+ \times R_+ \to R_+$. In $c(x,t)$ the index *x* denotes the type of good and the index *t* denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c(x,t)+1) dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t) / a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to *t* and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no storage.

a) Provide a motivation for the both the utility function and the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two-country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. There is no borrowing or lending across countries. Define an equilibrium for this economy.

d) Suppose that $z^1(t) > z^2(t)$. Explain carefully and illustrate two of the five qualitatively different possible equilibrium configurations for production, consumption, and trade at time t. (To make things easy, assume that $z^1(t)$ and $z^2(t)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Briefly describe the dynamics of this model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$.

2. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b} .$$

The investment good is produced according to

$$k_{t+1} = d(a_1 x_{1t}^b + a_2 x_{2t}^b)^{1/b} .$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t} .$$

where

$$k_{1t} + k_{2t} = k_t$$
$$\ell_{1t} + \ell_{2t} = \ell_t .$$

The initial value of k_t is $\bar{k}_0$. ℓ_t is normalized to 1.

a) Define an equilibrium for this economy.

b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in k_t and c_t and a transversality condition. Here $c_t = d(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$ is aggregate consumption. (You do not need to go through all of the algebra, but you need to explain all of the logical steps carefully.)

c) Suppose now that there is a world made up of two different countries, each with the same technologies and preferences, but with different constant populations, $L_t^j = \bar{L}^j$, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_{1t} + p_{2t} = r_t k_t + w_t = d(a_1 k_t^b + a_2)^{1/b}$ is income per capita. Transform the two difference equation in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

where $y_t^i = p_{1t}k_t^i + p_{2t} = r_t k_t^i + w_t = d(a_1 k_t^{i\,b} + a_2)^{1/b}$ is income per capita in country i. Explain the economic significance of this result.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha)\log c_0 + (\alpha/\rho)\log\int_0^\mu c(v)^\rho\, dv$$
$$\text{s.t. } p_0 c_0 + \int_0^\mu p(v)c(v)dv = w\bar{\ell} + \pi$$
$$c(v) \geq 0 .$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good v takes the price function $p(v)$ as given. Suppose too that this producer has the production function

$$y(v) = \max\left[z(v)\left(\ell(v) - f\right), 0\right].$$

Solve the consumer's profit maximization problem to derive and optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $z \geq 1$ according to the Pareto distribution with distribution function

$$F(z) = 1 - z^{-\gamma} .$$

Define an equilibrium for this economy.

c) Suppose that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Find an expression for the cutoff productivity level $\bar{z}$ such that firms with productivity $\bar{z}$ earn zero profits. Find an expression for profits π.

d) Suppose now that there are two countries that engage in free trade. Each country i, $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(z) = 1 - z^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_x where $f_x > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this economy.

e) Suppose now that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Explain now why there are two relevant cutoff levels of firm productivity, $\bar{z}_d$ and $\bar{z}_x$. Find expressions for these cutoff productivity levels. Find an expression for profits π.