## ECON 8401 T. J. HOLMES AND T. J. KEHOE INTERNATIONAL TRADE AND PAYMENTS THEORY FALL 2010 EXAMINATION

This exam has two parts. Each part has two questions. Please answer **one** of the two questions in each part for a total of two answers. Please keep your answers for Part I and Part II separate.

You can consult class notes, working papers, and articles while you are working on the exam, but you are asked not to discuss the exam with anyone until the exam period is finished for everyone.

## Part I

1. Consider an economy in which there are two types of goods, primary goods and manufactured goods. Primary goods are homogeneous and are produced using capital and labor services subject to the production function

$$y_0 = k_0^{\alpha_p} \ell_0^{1-\alpha_p}.$$

Manufactured goods are differentiated by firm. There are n firms and the production function for firm j is

$$y_{i} = \max[\beta k_{i}^{\alpha_{m}} \ell_{i}^{1-\alpha_{m}} - f, 0].$$

Here *f* is the fixed cost. Suppose that  $1 > \alpha_p > \alpha_m > 0$ . Suppose that there is a representative consumer with preferences given by the utility function

$$\log c_0 + (1/\rho) \log \sum_{j=1}^n c_j^{\rho}$$
,

where  $1 \ge \rho > 0$ . There is an endowment of  $\overline{k}$  units of capital and  $\overline{\ell}$  units of labor.

a) Suppose that the number of manufacturing firms, n, is variable, that these firms are Cournot competitors, and that there is free entry and exit in manufacturing. Define an (autarkic) equilibrium. Explain carefully how you would calculate this equilibrium. (You do not need to calculate it.)

b) Suppose now that there are two such countries, one with endowments  $(\bar{k}^1, \bar{\ell}^1)$  and and the other with endowments  $(\bar{k}^2, \bar{\ell}^2)$ , but otherwise identical. Define a trade equilibrium.

c) Suppose that  $\overline{k}^1 / \overline{\ell}^1 > \overline{k}^2 / \overline{\ell}^2$ . Explain what changes you would expect to see in

prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

d) Suppose now that country *i*, *i* = 1,2, imposes *ad valorem* tariffs of  $\tau_p^i$  and  $\tau_m^i$  on imports of primary and manufactured goods respectively. Tariff revenues are redistributed to the representative consumer in lump-sum form. Define a trade equilibrium.

2. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

The investment good is produced according to

$$k_{t+1} = dx_{1t}^{a_1} x_{2t}^{a_2} \,.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
  
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t},$$

where

$$k_{1t} + k_{2t} = k_t$$
  
$$\ell_{1t} + \ell_{2t} = 1.$$

The initial value of  $k_t$  is  $\overline{k}_0$ . All of the variables specified above are in per capita terms. There is a measure L of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner's problem for this economy. Explain how solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\sum_{t=0}^{\infty} \beta^{t} \log c_{t}$$
  
s.t.  $c_{t} + k_{t+1} = dk_{t}^{a_{t}}$   
 $c_{t}, k_{t} \ge 0$   
 $k_{0} = \overline{k_{0}}$ .

[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner's problem in part b. [Recall that the policy function for investment has the form  $k_{t+1}(k_t) = Adk^{a_1}$  where A is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of *n* different countries, all with the same technologies and preferences, but with different constant populations,  $L^i$ , and with different initial capital-labor ratios  $\overline{k}_0^i$ . Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let  $s_t = c_t / y_t$  where  $y_t = p_{1t}k_t + p_{2t} = dk_t^{a_1}$  is world GDP per capita. Transform the first-order conditions for the one-sector social planner's problem in part b into two difference equations in  $k_t$  and  $s_t$ . Use the first-order conditions for the consumer's problem of the equilibrium in part d to show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left( \frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left( \frac{y_0^i - y_0}{y_0} \right).$$

g) Use the solution to the one-sector social planner's problem in part c to solve for  $s_t$ . Discuss the economic significance of the result that you obtain.