ECON 8401   **T. J. HOLMES AND T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY   FALL 2011
                              EXAMINATION

This exam has two parts. Each part has two questions. Please answer **one** of the two
questions in each part for a total of two answers. Please keep your answers for Part I and
Part II separate.

You can consult class notes, working papers, and articles while you are working on the
exam, but you are asked not to discuss the exam with anyone until the exam period is
finished for everyone.

Part I

1. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

The investment good is produced according to

$$k_{t+1} = d x_{1t}^{a_1} x_{2t}^{a_2}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t},$$

where

$$k_{1t} + k_{2t} = k_t$$
$$\ell_{1t} + \ell_{2t} = 1.$$

The initial value of k_t is $\bar{k}_0$. All of the variables specified above are in per capita terms. There is a measure L of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner's problem for this economy. Explain how solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t. } c_t + k_{t+1} = dk_t^{a_1}$$
$$c_t, k_t \geq 0$$
$$k_0 = \bar{k}_0.$$

[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner's problem in part b. [Recall that the policy function for investment has the form $k_{t+1}(k_t) = Adk_t^{a_1}$ where A is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of n different countries, all with the same technologies and preferences, but with different constant populations, L^i, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_t + p_{2t} = dk_t^{a_1}$ is world GDP per capita. Transform the first-order conditions for the one-sector social planner's problem in part b into two difference equations in k_t and s_t. Use the first-order conditions for the consumer's problem of the equilibrium in part d to show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

g) Use the solution to the one-sector social planner's problem in part c to solve for s_t. Discuss the economic significance of the result that you obtain.

2. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha)\log c_0 + \frac{\alpha}{\rho}\log \int_0^m c(z)^\rho dz$$

$$\text{s.t. } p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$

$$c(z) \geq 0.$$

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Furthermore, $m > 0$ is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0 \right].$$

where $x(z) > 0$ is the firm's productivity level and $f > 0$. Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$ Suppose that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where $\gamma > 2$ and $\gamma > \rho/(1-\rho)$. Also suppose that the measure of potential firms is fixed at μ. Define an equilibrium for this economy.

c) Suppose that, in equilibrium not all potential firms actually produce. Find an expression for the productivity of the least productive firm that produces. That is, find a productivity $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce. Relate the measure of firms that produce m to the measure of potential firms μ and the cutoff $\bar{x}$.

d) Suppose now that there are two countries that engage in trade. Each country i, $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$. Each country also imposes an *ad valorem* tariff τ on imports of differentiated goods from the other country. The revenue from these tariffs is redistributed in lump-sum form to the consumer in that country. Define an equilibrium for this world economy.

e) Suppose that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to calculate any cutoff values, but you to not actually need to calculate it.] Compare this value, or these values, with that in part c. Draw a graph depicting what happens when a closed economy opens to trade.

f) Discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.

International Trade and Payments Theory

T.J. Holmes and T.J. Kehoe

Final Exam: Part II

Question 1

Consider a model with two countries. Suppose labor is the only factor of production and assume that the number of workers N is the same in each country. Each worker is endowed with one unit of time.

There are two sectors, manufacturing and services. In the service sector, one unit of labor produces one unit of services. (Note service productivity is the same in both countries.)

The manufacturing sector follows Eaton and Kortum (2002). There are a continuum of differentiated manufacturing goods, $j \in [0, 1]$. Let T_i be the productivity of country i (we will make this endogenous below). This governs the distribution of productivity draws, so that the c.d.f. of productivity z in country i is

$$F_i(z) = e^{-T_i z^{-\theta}}.$$

Suppose there is a CES aggregator of the differentiated manufacturing goods j,

$$Q = \left[\int_0^1 q(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma < 1 + \theta$, and Q is a level of the manufacturing composite, while $q(j)$ is a quantity of differentiated good.

Finally, utility of the manufactured good composite and services is Cobb-Douglas,

$$U = Q^\mu S^{1-\mu},$$

where S is the quantity of services.

Let $\tau \geq 1$ be the iceberg cost of shipping between the two locations.

Let N_i^S and N_i^M be the quantity of labor working in each sector at country i. Assume that there is an exogenous external effect in productivity, so that $T_i = g(N_i^M)$, where $g(\cdot)$ an nondecreasing function, and $g(0) > 0$.

Let w_i be the wage at location i, let P_i^M be the price index for the manufactured good composite at location i. To simplify calculations, we review some of the results of EK that you can take as given. Define Φ_1 and Φ_2 by

$$\Phi_1 = T_1 w_1^{-\theta} + T_2 w_2^{-\theta} \tau^{-\theta},$$
$$\Phi_2 = T_1 w_1^{-\theta} \tau^{-\theta} + T_2 w_2^{-\theta}.$$

Then P_i^M equals

$$P_i^M = \gamma \Phi_i^{-1/\theta},$$

for a constant γ. Also, let $n \neq i$. Then the probability that country i is the lowest cost provider to country n equals

$$\pi_{ni} = \frac{T_i w_i^{-\theta} \tau^{-\theta}}{T_n w_n^{-\theta} + T_i w_i^{-\theta} \tau^{-\theta}},$$

if $n \neq i$, and the probability that i is the lowest cost provider to itself is

$$\pi_{ii} = \frac{T_i w_i^{-\theta}}{T_n w_n^{-\theta} \tau^{-\theta} + T_i w_i^{-\theta}}.$$

(1) Suppose $\tau = \infty$, so that each country is in autarky. Solve for the equilibrium, and determine the price level of composite manufactured goods.

(2) Suppose $\tau = 1$, so there is free trade. Show there exists a symmetric equilibrium where $N_1^M = N_2^M$. What is the price level of composite manufactured goods?

(3) Suppose $\tau = 1$. Suppose that $2\mu < 1$. (This ensures that both locations produce service goods.) Define an asymmetric equilibrium in which country 1 has a higher manufacturing share than country 2, $N_1^M > N_2^M$, and derive a condition

that depends upon N_1^M that characterizes an asymmetric equilibrium. Show that $N_2^M = 0$ is not possible in equilibrium. Show that if

$$g(\mu N) - g'(\mu N)\mu N < 0,$$

there exists an asymmetric equilibrium. (Hint, take your equilibrium condition and look what happens locally around the symmetric equilibrium.)

Question 2

Suppose there are a unit measure of goods indexed by j, and utility is Cobb-Douglas with equals weight on each good,

$$U = \int_0^1 \log q(j)dj,$$

where $q(j)$ is consumption of j.

There are two countries. Let τ be the iceberg transportation cost, so that to deliver one unit from one country to the other, τ units must be shipped.

There are two kinds of labor, high skill and low skill. A low skill worker is endowed with 1 efficiency unit of labor. A high skill worker is endowed with $a > 1$ efficiency units of labor. Let N_H and N_L be the measure of high and low skill workers in each country.

At each location there exists a freely available "backyard" technology in which it is possible to produce one unit of each kind of good with one efficiency unit of labor.

In addition, in country 1, for good $j < \frac{1}{2}$, there exists an entrepreneur who has a monopoly on a special production technology that is more efficient than the backyard technology. In particular, the entrepreneur with a special technology for producing j can produce $\theta > 1$ units of j, for every one efficiency unit of labor. In country 1, for $j \geq \frac{1}{2}$, the only technology for producing j is the backyard technology.

Country 2 is the mirror image of country 1 in that for $j > \frac{1}{2}$ there is an an

entrepreneur who can produce with the special production technology, while for $j \leq \frac{1}{2}$ the only technology for producing in country 2 is the backyard technology.

There is one final constraint. Let $n_H(j)$ and $n_L(j)$ be the employment of an entrepreneur with a special technology for producing j. There is a management constraint that

$$n_H(j) + n_L(j) \leq \bar{n}.$$

Assume firms engage in Bertrand pricing. Focus on symmetric equilibria in which the wage as a function of skill, w_L and w_H, is the same in both countries.

(a) Consider first the autarky case where $\tau = \infty$. Define an equilibrium. In equilibrium, the skill premium w_H/w_L will either equal a (the ratio of the efficiency units) or will be strictly greater than a. Derive a condition on the underlying model parameters under which the skill premium must necessarily be greater than a. (Hint: To understand out how this model works, it is useful to start by assuming that the constraint parameter $\bar{n}$ is so large that is is irrelevant and determine the equilibrium in this case. Then reduce $\bar{n}$ to where it is binding. It is also distinguish between the case where λ is small relative to N_H and where λ is large relative to N_H.)

(b) Now consider what happens if the economy is opened up to free trade, $\tau = 1$. In particular: (i) Show that if the skill premium equals a with free trade then it also equals a under autarky. (ii) Derive a condition under which the skill premium equals a under autarky but must be strictly greater than a under free trade.

4