PROBLEM SET #1

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log x_0 + (1/\rho)\log\sum_{j=1}^{n} x_j^\rho$$

where $\rho > 0$. There is an endowment of $\bar{\ell}$ units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 1$, $f = 3$, $\rho = 1/2$, and $\bar{\ell} = 50$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 200$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell} = 50$ and the second with $\bar{\ell} = 150$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility? Illustrate the efficiency gains using an average cost curve diagram.

e) Repeat the analysis of parts a-d in a model in which consumers have the utility function

$$\log x_0 + (1/\rho)\log\int_0^n x(j)^\rho \, dj.$$

Here there is a continuum $[0,n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

2. Consider a world with two countries. The representative consumer in each country has the utility function

$$u(x_1^i, x_2^i) = \log x_1^i + \log x_2^i, \quad i = 1, 2.$$

This consumer is endowed with capital and labor in the amounts $(\bar{k}^i, \bar{\ell}^i)$. The production technologies in the two countries are identical.

$$y_j^i = \min[k_j^i / a_{Kj}, \ell_j^i / a_{Lj}], \quad i, j = 1, 2.$$

To make things simple assume that $a_{K1} = a_{L2} = 1$ and that $a_{K2} = a_{L1} = 2$.

a) Define an autarky equilibrium for country i. Under what conditions on $(\bar{k}^i, \bar{\ell}^i)$ are both of the equilibrium factor prices positive?

b) Define a free trade equilibrium. Under what conditions on $(\bar{k}^1, \bar{\ell}^1)$ and $(\bar{k}^2, \bar{\ell}^2)$ do both countries produce both goods?

c) State and prove a version of the factor price equalization theorem for this world.

d) State and prove a version of the Stolper-Samuelson theorem.

e) State and prove a version of the Rybczynski theorem.

f) State and prove a version of the Heckscher-Ohlin theorem.

3. Consider a two sector growth model in which both consumption and investment are formed from two intermediate goods using the same aggregator function,

$$c + \dot{k} + \delta k = (x_1^b + x_2^b)^{1/b}.$$

The two goods are produced from capital and labor using the production functions

$$x_1 = \ell$$
$$x_2 = k.$$

The initial value of $k(t)$ is $\bar{k}_0$. $\ell(t)$ is fixed at $\bar{\ell}$. The representative consumer has a utility function of the form

$$\int_0^\infty e^{-\rho t} \log(c) dt.$$

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two differential equations in k and $z = c/k$ and a transversality condition. Draw phase diagrams illustrating the different possible equilibrium paths for any $\bar{k}_0 > 0$.

c) Suppose now that there is a world made up of m different countries all with the same technologies and preferences, but different endowments, $\bar{\ell}^j$ and $\bar{k}_0^j$. Suppose that there is no international borrowing or lending and no international capital flows. Define an equilibrium for the world economy. Prove that in this equilibrium the variables $c(t) = \sum_{j=1}^m c^j(t)$, $k(t) = \sum_{j=1}^m k^j(t)$, $p_1(t)$, $p_2(t)$, $r(t)$, and $w(t)$ satisfy the equilibrium conditions for the equilibrium in part a where $\bar{k}_0 = \sum_{j=1}^m \bar{k}_0^j$, $\bar{\ell} = \sum_{j=1}^m \bar{\ell}^j$.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this world economy.

e) Derive equations that govern the relationship between $k^j(t)$ and $k(t)$ and that between $y^j(t) = p_1(0)\bar{\ell}^j + p_2(0)k^j(t)$ and $y(t)$.

f) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis?

4. Repeat parts a and b of question 3 for an economy where the production technologies take the form

$$x_1 = \theta_1 \ell_1^{1-\alpha_1} k_1^{\alpha_1}$$
$$x_2 = \theta_2 \ell_2^{1-\alpha_2} k_2^{\alpha_2}.$$

Where does the integrated equilibrium approach of part c break down? What impact would assuming international borrowing and lending or capital flows have on your analysis?