ECON 8401 **T. J. HOLMES AND T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2010**

PROBLEM SET #1

1. Consider an economy in which the representative consumer has the utility function

$$u(c_1, c_2) = a_1 \log c_1 + a_2 \log c_2 \,,\ i = 1,2 \,.$$

where $a_i > 0$ and $a_1 + a_2 = 1$. This consumer is endowed with capital and labor in the amounts $(\bar{k}, \bar{\ell})$. There are two goods, which are produced with the production technologies

$$y_j = \theta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j} \,,\ j = 1,2 \,,$$

where $1 > \alpha_1 > \alpha_2 > 0$.

a) Define an autarky equilibrium for this economy. Calculate this equilibrium.

b) Now suppose the country opens to trade. Suppose that the country is so small that it does not affect the world prices p_1, p_2. Define an equilibrium for this small open economy.

c) Find conditions on p_1, p_2, $\bar{k}$, and $\bar{\ell}$ such that the country produces positive amounts of both goods in the trade equilibrium.

d) Use the answer to part c to calculate a cone of diversification that depends on (p_1, p_2) — a set of $(\bar{k}, \bar{\ell})$ — such that small open economies with their endowments in this cone produce both goods.

e) State and prove a version of the Stolper-Samuelson theorem. [Hint: The proofs of this theorem and those in part e and in question 2, parts c and d are simple for this particular model. Please do not copy a general proof out of a book. Do the proof for this specific model. You should provide careful statements of the theorems. In particular, you should be careful about the restrictions on parameters needed for the theorem to hold.]

f) State and prove a version of the Rybczynski theorem.

2. Now consider a two-country version of question 1. The representative consumer in each country has the utility function

$$u(c_1^i, c_2^i) = a_1 \log c_1^i + a_2 \log c_2^i\,,\ i = 1,2\,.$$

This consumer is endowed with capital and labor in the amounts $(\bar{k}^i, \bar{\ell}^i)$ where $\bar{k}^1 / \bar{\ell}^1 > \bar{k}^2 / \bar{\ell}^2$. The production technologies in the two countries are identical.

$$y_j^i = \theta_j (k_j^i)^{\alpha_j} (\ell_j^i)^{1-\alpha_j}\,,\ i, j = 1,2\,.$$

a) Define a free trade equilibrium. Explain the four possible patterns of specialization that are possible. [Hint: You need to distinguish among three conceptually different cases: one in which both countries produce both goods, another in which one country produces both goods and the other produces only one good, and the last in which each country produces only one good.]

b) Under what conditions on $(\bar{k}^1, \bar{\ell}^1)$ and $(\bar{k}^2, \bar{\ell}^2)$ do both countries produce both goods? Calculate a cone of diversification that depends on $(\bar{k}^1 + \bar{k}^2, \bar{\ell}^1 + \bar{\ell}^2)$ for the world economy such that, if both countries have endowments of labor and capital in the cone, they both produce positive amounts of both goods. [Be careful: This concept of the cone of diversification differs from that in question 1.]

c) Calculate the equilibrium in the case where both country produce both goods and for the case where country 1 specializes in the production of good 1 but country 2 produces both goods.

d) State and prove a version of the factor price equalization theorem for this particular world economy.

e) State and prove a version of the Heckscher-Ohlin theorem. Does this theorem hold when the endowments of one or both countries are outside the cone of diversification?

3. Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z)\,.$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}\,.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz .$$

This consumer is endowed with $\bar{\ell}_j$ units of labor where $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.

c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.

d) For the model in part c, calculate gross domestic product, exports, and the real income index

$$v_j = \exp \int_0^1 \log c_j(z)\, dz$$

as functions of τ . Suppose that in the base period $\tau = 0$ and calculate real GDP — that is, GDP in base period prices — as well as GDP in current prices.

4. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index *x* denotes the type of good and the index *t* denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c(x,t)+1) dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t) / a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^{\infty} b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Describe the dynamics of the model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$.