PROBLEM SET #3

1. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha)\log c_0 + (\alpha/\rho)\log \int_0^m c(z)^\rho dz$$

$$\text{s.t.} \ p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$

$$c(z) \geq 0.$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\big(\ell(z) - f\big), 0 \right].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma}.$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first, μ is so small that all firms make nonnegative profits. In the second, μ is large enough so that there is a cutoff level of productivity $\bar{x} > 1$ such that all firms with productivity $x(z) \geq \bar{x}$ produce and earn nonnegative profits but no firm with productivity $x(z) < \bar{x}$ finds it profitable to produce.

d) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{\ell} = 40$, $\mu = 10$, $\gamma = 4$, and $f = 2$. Calculate the equilibrium of this economy.

e) Suppose now that there are two countries that engage in free trade. Each country i, $i = 1,2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$ and an iceberg transportation cost of $\tau_i^j - 1 \geq 0$. Define an equilibrium for this economy.

f) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels, $\bar{x}_d$ and $\bar{x}_d$, where $\bar{x}_e > \bar{x}_d > 1$. Firms with $x(z) \geq \bar{x}_e$ produce for both the domestic and the export market; firms with $\bar{x}_e > x(z) \geq \bar{x}_d$ produce only for the domestic market, and firms with $\bar{x}_d > x(z)$ do not produce.

g) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$, $\mu_1 = \mu_2 = \mu = 10$, $\gamma = 4$, $f_d = 2$, $f_e = 3$, and $\tau_2^1 = \tau_1^2 = \tau = 1.2$. Calculate the equilibrium of this economy.

h) Suppose now that a free trade agreement sets $\tau_2^1 = \tau_1^2 = \tau = 1$. Recalculate the equilibrium in part g.

i) Suppose now that a different reform sets $f_e = 2$. Again recalculate the equilibrium in part g. Contrast the results with those in part h. What sort of reform can lower f_e?

2. Suppose now that the measure of firms in question 1 is determined by costly entry, where the entry cost is ϕ. Assume that $\phi = 1$. Redo question 1.

3. Choose two countries that are important (to each other) trading partners, one with higher output per worker than the other. Use Summers-Heston data to try to answer Lucas's question: To what extent can differences in output per worker be explained by differences in capital per worker? Use both Summers-Heston data on capital per worker and IMF real interest rate data to try to make inferences about differences in capital per worker across countries. Discuss what you have learned from this exercise and what more you think would be relevant in explaining differences in output per worker across countries.

4. Find annual time series data on real output, real investment, employment, working age population, and — if you can — hours worked for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate δ and a capital share α. Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1}/K_{T_0} = (K_{T_0+10}/K_{T_0})^{1/10}.$$

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0}/Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t/Y_t\right)/10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.

5. Consider a model with an infinitely-lived, representative consumer. The production function is $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. The consumer solves the problem

$$\max \sum_{t=T_0}^{\infty} \beta^t [\gamma \log C_t + (1-\gamma)\log(N_t \bar{h} - L_t)]$$
$$\text{s.t. } C_t + K_{t+1} - K_t = w_t L_t + (r_t - \delta)K_t.$$
$$K_{t_0} = \bar{K}_{t_0}$$

a) Define an equilibrium of this economy.

b) Use the results from question 4 and the first order conditions from the consumer's problem to estimate the values of the parameters β and γ.

c) Using the MATLAB programs found at http://www.greatdepressionsbook.com/, calculate the equilibrium of this model. If you have enough data, you should calibrate the parameters β and γ for a period in which you are not very interested and then calibrate the equilibrium for a more interesting period. Briefly explain your computational methodology. Discuss your results.