ECON 8401 **T. J. HOLMES AND T. J. KEHOE**
INTERNATIONAL TRADE AND PAYMENTS THEORY **FALL 2010**

PROBLEM SET #3

1. Download data on bilateral trade by sector at the 4 digit SITC level from the OECD web site, http://oberon.sourceoecd.org. Follow the methodology in Kehoe and Ruhl, "How Important is the New Goods Margin in International Trade?" to create a set of least traded goods and carry out one of the two following exercises:

a) Consider trade between two countries over time. Construct diagrams with fractions of trade at the end of the period by deciles of sets of goods at the beginning of the period. Graph of the fraction of trade accounted for by the least traded decile over time. Do imports and exports separately.

b) Consider exports of one country to a number of trading partners during one year. Compare the sets of least traded goods. Do you see any patterns?

2. Consider an economy with two goods that enter both consumption and investment. The utility function of the representative consumer is

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment goods are produced according to

$$k_{t+1} - (1-\delta)k_t = d x_{1t}^{a_1} x_{2t}^{a_2}.$$

Feasible consumption/investment plans satisfy the feasibility conditions

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$

$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}$$

where

$$k_{1t} + k_{2t} = k_t$$

$$\ell_{1t} + \ell_{2t} = \ell_t.$$

The initial endowment of k_t is $\bar{k}_0$. ℓ_t is equal to 1. (In other words, all variables are expressed in per capita terms.)

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two difference equations in k_t and c_t and a transversality condition. Here $c_t = dc_{1t}^{a_1} c_{2t}^{a_2}$ is aggregate consumption. [Here is one possible approach: Prove a version of the first and second welfare theorems for this economy. Show that the two-sector social planer's problem is equivalent to a one-sector social planner's problem. Derive the difference equations and transversality conditions from the one-sector social planner's problem.]

c) Suppose now that there is a world composed of n different countries, all with the same preferences and technologies, but with different initial endowments of capital per worker, $\bar{k}_0^i$ The countries also have different population sizes, L^i, which are constant over time. (In other words, there is a continuum of identical consumers/workers of measure L^i in country i.) Suppose that there is no international borrowing or lending and no and no international capital flows. Define an equilibrium for this world economy. Prove that in this equilibrium the variables $c_{jt} = \sum_{i=1}^{n} L^i c_{jt}^i \Big/ \sum_{j=1}^{n} L^i$, $k_t = \sum_{i=1}^{n} L^i k_t^i \Big/ \sum_{i=1}^{n} L^i$, p_{it}, r_t, and w_t satisfy the equilibrium conditions of the economy in part a when $\bar{k}_0 = \sum_{i=1}^{n} L^i \bar{k}_0^i \Big/ \sum_{i=1}^{n} L^i$.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis? Try to be precise about which variables are uniquely determined in the equilibrium with borrowing and lending and/or capital flows are which would not be uniquely determined.

f) Consider the case where $\delta = 1$. Set $z_0 = c_0 / \left(\beta r_0 k_0\right)$ and $z_t = c_{t-1} / k_t$, $t = 1, 2, \ldots$. Transform the two difference equations in part b into two difference equations in k_t and z_t. Prove that

$$\frac{k_t^i - k_t}{k_t} = \frac{z_t}{z_{t-1}} \left(\frac{k_{t-1}^i - k_{t-1}}{k_{t-1}} \right) = \frac{z_t}{z_0} \left(\frac{\bar{k}_0^i - \bar{k}_0}{\bar{k}_0} \right).$$

g) Consider again the case where $\delta = 1$. Let $s_t = c_t / y_t$ where

$$y_t = p_{1t} k_t + p_{2t} = dk_t^{a_1} = r_t k_t + w_t .$$

Transform the two difference equations in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right)$$

where $y_t^i = p_{1t}y_{1t}^i + p_{2t}y_{2t}^i = r_t k_t^i + w_t$. Calculate an expression for s_t and discuss the significance of this result.

3. Find annual time series data on real output, real investment, employment, working age population, and — if you can — hours worked for some country. If you have sufficient data for other variables, calibrate an annual deprecation rate δ and a capital share α. Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t\right) / 10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.