

PROBLEM SET #3

1. Consider an economy similar to that in question 2, part b in problem set 2. Again, consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Also the production function for the differentiated good remains

$$y(j) = (1/b) \max[\ell(j) - f, 0].$$

Suppose that there are  $m$  symmetric countries, each with an endowment of  $\bar{\ell}$  units of labor.

- a) Suppose that there are iceberg transportation costs of  $\tau$  per good. Define and calculate the equilibrium.
- b) Consider a situation where  $\tau > 0$  in the base period. Calculate an expression for real GDP and for an ideal real income index. How do these change with a multilateral free trade agreement that reduces  $\tau$  to 0 in all of the countries?
- c) Repeat parts a and b for the case where  $\tau$  is an *ad valorem* tariff.

2. Consider an economy similar to that in question 1. Again, consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Also the production function for the differentiated good remains

$$y(j) = (1/b) \max[\ell(j) - f, 0].$$

Suppose now, however, that production of the agricultural good is governed by the function

$$y_0 = \ell_0^\alpha t_0^{1-\alpha}.$$

Here  $t_0$  denotes inputs of land. Suppose now that there are two such countries, one with endowments  $(\bar{\ell}^1, \bar{t}^1)$  and the other with endowments  $(\bar{\ell}^2, \bar{t}^2)$ , but otherwise identical.

- a) Define an autarky equilibrium. Calculate this autarky equilibrium.

b) Define a trade equilibrium.

c) Suppose that  $\bar{\ell}^1 / \bar{t}^1 > \bar{\ell}^2 / \bar{t}^2$ . Derive conditions on  $(\bar{\ell}^1, \bar{t}^1)$  and  $(\bar{\ell}^2, \bar{t}^2)$  for which factor price equalization holds. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

d) Continuing to suppose that  $\bar{\ell}^1 / \bar{t}^1 > \bar{\ell}^2 / \bar{t}^2$ , explain what changes you would expect to see in prices, factor prices, number of firms, average output levels, and utility levels as these two countries, initially in autarky, open to trade.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1-\alpha) \log c_0 + (\alpha / \rho) \log \int_0^m c(z)^\rho dz \\ \text{s.t.} \quad & p_0 c_0 + \int_0^m p(z) c(z) dz = w \bar{\ell} + \pi \\ & c(z) \geq 0. \end{aligned}$$

Here  $\pi$  are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good  $z$  takes the prices  $p(z')$ , for  $z' \neq z$ , as given. Suppose too that this producer has the production function

$$y(z) = \max [x(z)(\ell(z) - f), 0].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure  $\mu$  of potential firms. Firm productivities are distributed on the interval  $x \geq 1$  according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma}.$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first,  $\mu$  is so small that all firms make nonnegative profits. In the second,  $\mu$  is large enough so that there is a cutoff level of productivity  $\bar{x} > 1$  such that all firms with productivity  $x(z) \geq \bar{x}$  produce and earn nonnegative profits but no firm with productivity  $x(z) < \bar{x}$  finds it profitable to produce. Find the relation between  $m$  and  $\mu$  and  $\bar{x}$ .

d) Suppose that  $\alpha = 0.5$ ,  $\rho = 0.5$ ,  $\bar{\ell} = 40$ ,  $\mu = 10$ ,  $\gamma = 4$ , and  $f = 2$ . Calculate the equilibrium of this economy.

4. Consider a world with two countries like that in question 3 that engage in free trade. Each country  $i$ ,  $i=1,2$ , has a population of  $\bar{\ell}_i$  and a measure of potential firms of  $\mu_i$ . Firms' productivities are again distributed according to the Pareto distribution,  $F(x)=1-x^{-\gamma}$ . A firm in country  $i$  faces a fixed cost of exporting to country  $j$ ,  $j \neq i$ , of  $f_e$  where  $f_e > f_d = f$  and an iceberg transportation cost of  $\tau_i^j - 1 = \tau - 1 \geq 0$ ,  $j \neq i$ . Define an equilibrium for this economy.

a) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that  $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$  and  $\mu_1 = \mu_2 = \mu$ . Suppose too that  $\mu$  is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels,  $\bar{x}_d$  and  $\bar{x}_e$ , where  $\bar{x}_e > \bar{x}_d > 1$ . Firms with  $x(z) \geq \bar{x}_e$  produce for both the domestic and the export market; firms with  $\bar{x}_e > x(z) \geq \bar{x}_d$  produce only for the domestic market, and firms with  $\bar{x}_d > x(z)$  do not produce.

b) Suppose that  $\mu = 0.5$ ,  $\rho = 0.5$ ,  $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$ ,  $\mu_1 = \mu_2 = \mu = 10$ ,  $\gamma = 4$ ,  $f_d = 2$ ,  $f_e = 3$ , and  $\tau_2^1 = \tau_1^2 = \tau = 1.2$ . Calculate the equilibrium of this economy.

c) Suppose now that a free trade agreement sets  $\tau_2^1 = \tau_1^2 = \tau = 1$ . Recalculate the equilibrium in part b.

d) Suppose now that a different reform sets  $f_e = 2$ . Again recalculate the equilibrium in part b. Contrast the results with those in part c. What sort of reform can lower  $f_e$ ?