

PROBLEM SET #4

1. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1-\alpha)\log c_0 + (\alpha/\rho)\log \int_0^m c(z)^\rho dz \\ \text{s.t.} \quad & p_0 c_0 + \int_0^\mu p(z)c(z)dz = w\bar{\ell} + \pi \\ & c(z) \geq 0. \end{aligned}$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max[x(z)(\ell(z) - f), 0].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma}.$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first, μ is so small that all firms make nonnegative profits. In the second, μ is large enough so that there is a cutoff level of productivity $\bar{x} > 1$ such that all firms with productivity $x(z) \geq \bar{x}$ produce and earn nonnegative profits but no firm with productivity $x(z) < \bar{x}$ finds it profitable to produce. Find the relation between m and μ and \bar{x} .

d) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{\ell} = 40$, $\mu = 10$, $\gamma = 4$, and $f = 2$. Calculate the equilibrium of this economy.

2. Consider a world with two countries like that in question 1 that engage in free trade. Each country i , $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i . Firms' productivities are again distributed according to the Pareto distribution,

$F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j , $j \neq i$, of f_e where $f_e > f_d = f$ and an iceberg transportation cost of $\tau_i^j - 1 = \tau - 1 \geq 0$, $j \neq i$.

- a) Define an equilibrium for this economy.
- b) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that $\bar{l}_1 = \bar{l}_2 = \bar{l}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels, \bar{x}_d and \bar{x}_e , where $\bar{x}_e > \bar{x}_d > 1$. Firms with $x(z) \geq \bar{x}_e$ produce for both the domestic and the export market; firms with $\bar{x}_e > x(z) \geq \bar{x}_d$ produce only for the domestic market, and firms with $\bar{x}_d > x(z)$ do not produce.
- c) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{l}_1 = \bar{l}_2 = \bar{l} = 40$, $\mu_1 = \mu_2 = \mu = 10$, $\gamma = 4$, $f_d = 2$, $f_e = 3$, and $\tau_2^1 = \tau_1^2 = \tau = 1.2$. Calculate the equilibrium of this economy.
- d) Suppose now that a free trade agreement sets $\tau_2^1 = \tau_1^2 = \tau = 1$. Recalculate the equilibrium in part c.
- e) Suppose now that a different reform sets $f_e = 2$. Again recalculate the equilibrium in part b. Contrast the results with those in part d. What sort of reform can lower f_e ?