

EXAMINATION

Please answer **two** of the three questions:

1. Consider an economy in which there are two types of goods, primary goods and manufactured goods. Primary goods are homogeneous and are produced using capital services and labor services subject to the production function

$$y_0 = k_0^{\alpha_p} \ell_0^{1-\alpha_p}.$$

Manufactured goods are differentiated by firm. There are  $n$  firms and the production function for firm  $j$  is

$$y_j = \max[\beta k_j^{\alpha_m} \ell_j^{1-\alpha_m} - f, 0].$$

where  $f$  is the fixed cost. Suppose that  $1 > \alpha_p > \alpha_m > 0$ . Suppose that there is a representative consumer with preferences given by the utility function

$$\log x_0 + (1/\rho) \log \sum_{j=1}^n x_j^\rho$$

where  $1 \geq \rho > 0$ . There is an endowment of  $\bar{k}$  units of capital and  $\bar{\ell}$  units of labor.

- a) Suppose that the number of manufacturing firms is variable, that these firms are Cournot competitors, and that there is free entry and exit in manufacturing. Define an (autarkic) equilibrium. Explain carefully how you would calculate this equilibrium (You do not need to calculate it.)
- b) Suppose now that there are two such countries, one with endowments  $(\bar{k}^1, \bar{\ell}^1)$  and the other with endowments  $(\bar{k}^2, \bar{\ell}^2)$ , but otherwise identical. Define a trade equilibrium.
- c) Suppose that  $\bar{k}^1 / \bar{\ell}^1 > \bar{k}^2 / \bar{\ell}^2$ . Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.
- d) Suppose now that the manufacturing firms are Bertrand competitors, redefine the trade equilibrium in part b.

2. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods  $X = [0, 1]$  given by the utility function

$$\int_X \log c(x) dx.$$

In each country there is a single factor, labor. Endowments are  $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ . Production functions are linear but differ across countries:

$$y_j(x) = \ell_j(x) / a_j(x).$$

Here  $y_j(x)$  is the amount of good  $x$  produced in country  $j$ ;  $\ell_j(x)$  is the amount of labor used in this production; and  $a_j(x)$  is the unit labor required, given by

$$\begin{aligned} a_1(x) &= e^x \\ a_2(x) &= e^{1-x}. \end{aligned}$$

- a) Define a competitive equilibrium of the world economy.
- b) Characterize as much as possible the patterns of specialization and trade in the unique competitive equilibrium
- c) Suppose now that there are tariffs, so that the price paid by a consumer in county  $i$  for a good  $x$  imported from country  $j$ ,  $i \neq j$ , is

$$p^i(x) = (1 + \tau) a_j(x) w_j$$

where  $\tau$  is the *ad valorem* tariff rate and  $w_j$  is the wage rate in country  $j$ . Tariff revenues are redistributed to consumers in a lump-sum form. Explain how your definition of equilibrium is altered and characterize as much as possible how the equilibrium in the world with tariffs differs from the equilibrium in the world without tariffs.

- d) For the world economy in part c explain how to calculate the value of trade as a fraction of GDP. Calculate this fraction as much as possible.

3. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}.$$

The investment is produced according to

$$k_{t+1} - (1 - \delta)k_t = d(a_1 x_{1t}^b + a_2 x_{2t}^b)^{1/b}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}. \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= \ell_t. \end{aligned}$$

The initial value of  $k_t$  is  $\bar{k}_0$  per capita. In per capita terms  $\ell_t$  is fixed at 1. There is a continuum of measure  $\bar{\ell}$  of consumers.

- Carefully define a competitive equilibrium for this economy.
- Reduce the equilibrium conditions to two difference equations in  $k_t$  and  $c_t$  and a transversality condition. Here  $c_t = d(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$  is aggregate consumption per capita.
- Suppose now that there is a world made up of  $m$  different countries, all with the same technologies and preferences. The countries have different sizes of population,  $\bar{\ell}^j$ , however, and different initial endowments of capital per capita,  $\bar{k}_0^j$ . Suppose that there is no international borrowing or lending and there are no international capital flows. Define an equilibrium for the world economy. Prove that in this equilibrium the variables  $c_{it} = \sum_{j=1}^m \bar{\ell}^j c_{it}^j / \sum_{j=1}^m \bar{\ell}^j$ ,  $k_t = \sum_{j=1}^m \bar{\ell}^j k_t^j / \sum_{j=1}^m \bar{\ell}^j$ ,  $p_{it}$ ,  $r_t$ , and  $w_t$  satisfy the equilibrium conditions for the equilibrium in part a where  $\bar{k}_0 = \sum_{j=1}^m \bar{\ell}^j \bar{k}_0^j / \sum_{j=1}^m \bar{\ell}^j$ .
- Consider the case where  $\delta = 1$ . Let  $s_t = c_t / y_t$  where  $y_t = p_{1t} k_t + p_{2t} = d(a_1 k_t^b + a_2)^{1/b}$ . Transform the two difference equation in part b into two difference equations in  $k_t$  and  $s_t$ . Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left( \frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left( \frac{y_0^i - y_0}{y_0} \right)$$

where  $y_t^i = p_{1t}k_t^i + p_{2t}$ . Explain carefully the significance of this result.