

APPLIED GENERAL EQUILIBRIUM MODELS

Data

The data source is an input-output matrix.

	Agr.	Man.	Con.	Inv.	Exp.	Total
Agriculture	2	3	8	2	5	20
Manufacturing	4	7	12	4	3	30
Imports	4	4				8
Tariff Revenue	2	1				3
Labor Compensation	4	10				14
Returns to Capital	4	5				9
Total	20	30	20	6	8	

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This matrix reports the value of all real transactions made in an economy in one year. The columns report expenditures made by a sector and the rows report the receipts received by a sector.

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Suppose that the units are hundreds of billions of 1990 pesos. Then manufacturing firms spent 300 billion pesos on intermediate inputs of agricultural goods, for example, and 1000 billion pesos on labor compensation.

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Domestic consumers spent 1200 billion pesos on manufactured goods, while foreigners spent 300 billion on exports of manufactured goods.

Here is how the data in this sort of matrix relate to national income and product accounts:

Product		Income	
Consumption	20	Labor Income	14
Investment	6	Capital Income	9
Exports	8	Tariffs	3
-Imports	-8		
GDP	26	GDP	26

Calibration

We calibrate an applied general equilibrium model so that, in equilibrium, the agents in the model make the same transactions as do their counterparts in the world according to the input-output matrix.

- Armington aggregators for agriculture and manufactured goods:

$$y_j = \gamma_j y_{j,d}^{\delta_j} y_{j,f}^{1-\delta_j}, \quad j = agr, man.$$

- Tariff rates:

$$\tau_{agr}, \tau_{man}.$$

- Utility function for the representative domestic consumer:

$$\theta_{agr} \log c_{agr} + \theta_{man} \log c_{man} + \theta_{inv} \log c_{inv}.$$

- Endowments for the representative domestic consumer:

$$\bar{\ell}, \bar{k}.$$

- Production functions for agriculture and manufactured goods:

$$y_{j,d} = \min [x_{agr,j} / a_{agr,j}, x_{man,j} / a_{man,j}, \beta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}], \quad j = agr, man.$$

- Production function for the investment good:

$$y_{inv} = \min [x_{agr,inv} / a_{agr,inv}, x_{man,inv} / a_{man,inv}]$$

- Utility function for the representative consumer in the rest of the world:

$$\theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{f,f} \log x_{f,f}.$$

A level of income I_f and tariff rates $\tau_{agr,f}$, $\tau_{man,f}$ in the rest of the world need to be exogenously specified. We need additional data for this.

Equilibrium of AGE Model

An equilibrium is

prices for the final goods $\hat{p}_{agr}, \hat{p}_{man}, \hat{p}_{inv}$,

prices for the domestic goods $\hat{p}_{agr,d}, \hat{p}_{man,d}$,

a real exchange rate \hat{e} ,

factor prices \hat{r}, \hat{w} ,

total supplies of final goods $\hat{y}_{agr}, \hat{y}_{man}, \hat{y}_{inv}$,

consumption levels $\hat{c}_{agr}, \hat{c}_{man}, \hat{c}_{inv}$,

imports $\hat{y}_{agr,f}, \hat{y}_{man,f}$,

exports $\hat{x}_{agr,f}, \hat{x}_{man,f}$,

production plans for agriculture and manufacturing

$\left(\hat{y}_{agr,d}, \hat{x}_{agr,agr}, \hat{x}_{man,agr}, \hat{\ell}_{agr}, \hat{k}_{agr} \right), \left(\hat{y}_{man,d}, \hat{x}_{agr,man}, \hat{x}_{man,man}, \hat{\ell}_{man}, \hat{k}_{man} \right)$,

a production plan for investment $\left(\hat{y}_{inv}, \hat{x}_{agr,inv}, \hat{x}_{agr,inv} \right)$,

a lump sum transfer \hat{T} ,

and consumption of the foreign good in the rest of the world $\hat{x}_{f,f}$

such that

- $\hat{c}_{agr}, \hat{c}_{man}, \hat{c}_{inv}$ solve

$$\begin{aligned} & \max \theta_{agr} \log c_{agr} + \theta_{man} \log c_{man} + \theta_{inv} \log c_{inv} \\ \text{s.t. } & \hat{p}_{agr} c_{agr} + \hat{p}_{man} c_{man} + \hat{p}_{inv} c_{inv} = \hat{r} \bar{k} + \hat{w} \bar{\ell} + \hat{T} \\ & c_{agr}, c_{man}, c_{inv} \geq 0. \end{aligned}$$

- $(\hat{y}_{j,d}, \hat{x}_{agr,j}, \hat{x}_{man,j}, \hat{\ell}_j, \hat{k}_j)$, $j = agr, man$, satisfy

$$\begin{aligned} \hat{y}_{j,d} &= \min [\hat{x}_{agr,j} / a_{agr,j}, \hat{x}_{man,j} / a_{man,j}, \beta_j \hat{k}_j^{\alpha_j} \hat{\ell}_j^{1-\alpha_j}] \\ \hat{p}_{j,d} \hat{y}_{j,d} - \hat{p}_{agr} \hat{x}_{agr,j} - \hat{p}_{man} \hat{x}_{man,j} - \hat{r} \hat{k}_j - \hat{w} \hat{\ell}_j &= 0 \end{aligned}$$

where $\hat{k}_j, \hat{\ell}_j$ solve

$$\begin{aligned}
& \min \hat{r}k_j + \hat{w}\ell_j \\
& \text{s.t. } \beta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j} = \hat{y}_{j,d} \\
& \quad k_j, \ell_j \geq 0.
\end{aligned}$$

- $(\hat{y}_{inv}, \hat{x}_{agr,inv}, \hat{x}_{man,inv})$ satisfy

$$\begin{aligned}
\hat{y}_{inv} &= \min [\hat{x}_{agr,inv} / a_{agr,inv}, \hat{x}_{man,inv} / a_{man,inv}] \\
\hat{p}_{inv} \hat{y}_{inv} - \hat{p}_{agr} \hat{x}_{agr,inv} - \hat{p}_{man} \hat{x}_{man,inv} &= 0.
\end{aligned}$$

- $\hat{y}_j, \hat{y}_{j,d}, \hat{y}_{j,f}$ $j = agr, man$, satisfy

$$\hat{p}_j \hat{y}_j - \hat{p}_{j,d} \hat{y}_{j,d} - (1 + \tau_j) \hat{e}\bar{p}_{j,f} \hat{y}_{j,f} = 0$$

where $\hat{y}_{j,d}, \hat{y}_{j,f}$ solve

$$\begin{aligned} \min \quad & \hat{p}_{j,d} y_{j,d} + (1 + \tau_j) \hat{e} \bar{p}_{j,f} y_{j,f} \\ \text{s.t.} \quad & \gamma_j y_{j,d}^{\delta_j} y_{j,f}^{1-\delta_j} = \hat{y}_j \\ & y_{j,d}, y_{j,f} \geq 0. \end{aligned}$$

- The prices in the rest of the world are exogenously fixed, $\bar{p}_{agr,f}$, $\bar{p}_{man,f}$.
- $\hat{c}_j + \hat{x}_{j,agr} + \hat{x}_{j,man} + \hat{x}_{j,inv} + \hat{x}_{j,f} = \hat{y}_j$, $j = agr, man$.
- $\hat{c}_{inv} = \hat{y}_{inv}$.
- $\hat{k}_{agr} + \hat{k}_{man} = \bar{k}$.
- $\hat{l}_{agr} + \hat{l}_{man} = \bar{l}$.
- $\hat{p}_{agr,f} \tau_{agr} \hat{y}_{agr,f} + \hat{p}_{man,f} \tau_{man} \hat{y}_{man,f} = \hat{T}$.

- $\hat{x}_{agr,f}$, $\hat{x}_{man,f}$ solve

$$\begin{aligned} & \max \theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{f,f} \log x_{f,f} \\ \text{s.t. } & \hat{p}_{agr} (1 + \tau_{agr,f}) x_{agr,f} + \hat{p}_{man} (1 + \tau_{man,f}) x_{man,f} + \hat{e} \hat{x}_f = \hat{e} I_f \\ & x_{agr,f}, x_{man,f}, x_f \geq 0. \end{aligned}$$

- Trade is balanced,

$$\hat{p}_{agr} \hat{x}_{agr,f} + \hat{p}_{man} \hat{x}_{man,f} = \hat{e} \bar{p}_{agr,f} \hat{y}_{agr,f} + \hat{e} \bar{p}_{man,f} \hat{y}_{man,f}.$$

Calibration of Parameters

- We know that $\hat{c}_{agr}, \hat{c}_{man}, \hat{c}_{inv}$ solve

$$\begin{aligned} & \max \theta_{agr} \log c_{agr} + \theta_{man} \log c_{man} + \theta_{inv} \log c_{inv} \\ \text{s.t. } & \hat{p}_{agr} c_{agr} + \hat{p}_{man} c_{man} + \hat{p}_{inv} c_{inv} = \hat{r} \bar{k} + \hat{w} \bar{\ell} + \hat{T} \\ & c_{agr}, c_{man}, c_{inv} \geq 0. \end{aligned}$$

Normalizing units of all goods to be base period values, so that $\hat{p}_{agr} = 1$, $\hat{p}_{man} = 1$, $\hat{p}_{inv} = 1$, $\hat{r} = 1$, and $\hat{w} = 1$, we calibrate

$$\bar{k} = 9, \bar{\ell} = 14.$$

Solving the consumer's problem, we obtain, for example,

$$\hat{c}_{agr} = \theta_{agr} \frac{\hat{r}\bar{k} + \hat{w}\bar{\ell} + \hat{T}}{\hat{p}_{agr}}.$$

We calibrate

$$8 = \theta_{agr} \frac{9 + 14 + 3}{1}$$

$$\theta_{agr} = \frac{8}{26} = 0.3077.$$

Similarly, $\theta_{man} = 12/26 = 0.4615$ and $\theta_{inv} = 6/26 = 0.2308$.

- We know that $\hat{p}_{agr}, \hat{p}_{man}, \hat{p}_{agr,d}, \hat{p}_{man,d}, \hat{r}, \hat{w},$
 $\left(\hat{y}_{agr,d}, \hat{x}_{agr,agr}, \hat{x}_{man,agr}, \hat{\ell}_{agr}, \hat{k}_{agr} \right), \left(\hat{y}_{man,d}, \hat{x}_{agr,man}, \hat{x}_{man,man}, \hat{\ell}_{man}, \hat{k}_{man} \right)$ satisfy

$$\hat{y}_{j,d} = \min [\hat{x}_{agr,j} / a_{agr,j}, \hat{x}_{man,j} / a_{man,j}, \beta_j \hat{k}_j^{\alpha_j} \hat{\ell}_j^{1-\alpha_j}] \quad j = agr, man$$

Therefore, for example,

$$\begin{aligned} \hat{y}_{agr,d} &= \frac{\hat{x}_{agr,agr}}{a_{agr,agr}} \\ 14 &= \frac{2}{a_{agr,agr}} \\ a_{agr,agr} &= \frac{2}{14} = 0.1426. \end{aligned}$$

Similarly, $a_{man,agr} = 4/14 = 0.2857$, $a_{agr,man} = 3/25 = 0.1200$,
 $a_{man,man} = 7/25 = 0.2800$.

We know that $\hat{k}_j, \hat{\ell}_j, j = agr, man$, solve

$$\begin{aligned}
& \min \hat{r}k_j + \hat{w}l_j \\
& \text{s.t. } \beta_j k_j^{\alpha_j} l_j^{1-\alpha_j} = \hat{y}_{j,d} \\
& k_j, l_j \geq 0.
\end{aligned}$$

Therefore, for example,

$$\begin{aligned}
\frac{\hat{w}}{\hat{r}} &= \frac{(1-\alpha_{agr})\beta_{agr}\hat{k}_{agr}^{\alpha_{agr}}\hat{l}_{agr}^{-\alpha_{agr}}}{\alpha_{agr}\beta_{agr}\hat{k}_{agr}^{\alpha_{agr}-1}\hat{l}_{agr}^{1-\alpha_{agr}}} = \frac{(1-\alpha_{agr})\hat{k}_{agr}}{\alpha_{agr}\hat{l}_{agr}} \\
\frac{\hat{w}\hat{l}_{agr}}{\hat{r}\hat{k}_{agr}} &= \frac{(1-\alpha_{agr})}{\alpha_{agr}} \\
\frac{4}{4} &= \frac{(1-\alpha_{agr})}{\alpha_{agr}} \\
\alpha_{agr} &= \frac{1}{2} = 0.5000.
\end{aligned}$$

Since $\hat{y}_{agr,d} = \beta_{agr} \hat{k}_{agr}^{\alpha_{agr}} \hat{\ell}_{agr}^{1-\alpha_{agr}}$, this implies that

$$14 = \beta_{agr} 4^{1/2} 4^{1/2}$$
$$\beta_{agr} = \frac{14}{4} = 3.5000$$

Similarly, $\alpha_{man} = 1/3 = 0.3333$ and $\beta_{man} = 25 / (5^{1/3} 10^{2/3}) = 3.1498$.

- We know that $(\hat{y}_{inv}, \hat{x}_{agr,inv}, \hat{x}_{man,inv})$ satisfy

$$\hat{y}_{inv} = \min [\hat{x}_{agr,inv} / a_{agr,inv}, \hat{x}_{man,inv} / a_{man,inv}].$$

Therefore, for example,

$$\hat{y}_{inv} = \frac{\hat{x}_{agr,inv}}{a_{agr,inv}}$$

$$6 = \frac{2}{a_{agr,inv}}$$

$$a_{agr,inv} = \frac{2}{6} = 0.3333.$$

Similarly, $a_{man,inv} = 4/6 = 0.6667$.

- We know that $(1 + \tau_{agr}) \hat{e}\bar{p}_{agr,f} y_{agr,f} = 6$ while $\hat{e}\bar{p}_{agr,f} y_{agr,f} = 4$.

Therefore

$$(1 + \tau_{agr}) = \frac{6}{4}$$

$$\tau_{agr} = \frac{2}{4} = 0.5000$$

Similarly, $\tau_{man} = 1/4 = 0.2500$.

- We know that $\hat{y}_{j,d}, \hat{y}_{j,f}$, $j = agr, man$, solve

$$\begin{aligned} & \min \hat{p}_{j,d} y_{j,d} + (1 + \tau_j) \hat{e}\bar{p}_{j,f} y_{j,f} \\ \text{s.t. } & \gamma_j y_{j,d}^{\delta_j} y_{j,f}^{1-\delta_j} = \hat{y}_j \\ & y_{j,d}, y_{j,f} \geq 0. \end{aligned}$$

Therefore, for example,

$$\begin{aligned} \frac{(1 + \tau_{agr}) \hat{e}\bar{p}_{agr,f}}{\hat{p}_{agr,d}} &= \frac{(1 - \delta_{agr}) \gamma_{agr} \hat{y}_{agr,d}^{\delta_{agr}} \hat{y}_{agr,f}^{-\delta_{agr}}}{\delta_{agr} \gamma_{agr} \hat{y}_{agr,d}^{\delta_{agr}-1} \hat{y}_{agr,f}^{1-\delta_{agr}}} = \frac{(1 - \delta_{agr}) \hat{y}_{agr,d}}{\delta_{agr} \hat{y}_{agr,f}} \\ &= \frac{(1 + \tau_{agr}) \hat{e}\bar{p}_{agr,f} \hat{y}_{agr,f}}{\hat{p}_{agr,d} \hat{y}_{agr,d}} = \frac{(1 - \delta_{agr})}{\delta_{agr}} \end{aligned}$$

$$\frac{6}{14} = \frac{(1 - \delta_{agr})}{\delta_{agr}}$$

$$\delta_{agr} = \frac{14}{20} = 0.7000.$$

Since $\hat{y}_{agr} = \gamma_{agr} \hat{y}_{agr,d}^{\delta_{agr}} \hat{y}_{agr,f}^{1-\delta_{agr}}$, this implies that

$$20 = \gamma_{agr} 14^{7/10} 4^{3/10}$$

$$\gamma_{agr} = \frac{20}{14^{7/10} 4^{3/10}} = 2.0803.$$

Similarly, $\delta_{man} = 25/30 = 0.8333$ and $\gamma_{man} = 30/(25^{5/6} 4^{1/6}) = 1.6287$.

- Let $I_f = 1000$ be the income in the rest of the world. Let $\tau_{agr,f} = 0.20$ and $\tau_{man,f} = 0.10$ be the tariff rates in the rest of the world. We know that $\hat{x}_{agr,f}$, $\hat{x}_{man,f}$ solve

$$\begin{aligned} & \max \theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{f,f} \log x_{f,f} \\ \text{s.t. } & \hat{p}_{agr} (1 + \tau_{agr,f}) x_{agr,f} + (1 + \tau_{man,f}) \hat{p}_{man} x_{man,f} + \hat{e} \hat{x}_f = \hat{e} I_f \\ & x_{agr,f}, x_{man,f}, x_f \geq 0. \end{aligned}$$

Solving this problem, we obtain, for example,

$$\begin{aligned} \hat{x}_{agr,f} &= \theta_{agr,f} \frac{\hat{e} I_f}{\hat{p}_{agr} (1 + \tau_{agr,f})} \\ 5 &= \theta_{agr,f} \frac{1000}{1.2} \\ \theta_{agr,f} &= \frac{6}{1000} = 0.0060. \end{aligned}$$

Similarly, $\theta_{man,f} = 3.3/1000 = 0.0033$ and $\theta_{f,f} = 990.7/1000 = 0.9907$.

Armington Elasticities

Suppose now that the Armington elasticity for imports is $\sigma_{imp} = 1/(1 - \rho_{imp}) = 5$, or $\rho_{imp} = 0.8$. Then we can recalibrate the Armington aggregators

$$y_j = \gamma_j \left[\delta_j y_{j,d}^{\rho_{imp}} + (1 - \delta_j) y_{j,f}^{\rho_{imp}} \right]^{\frac{1}{\rho_{imp}}}, \quad j = agr, man.$$

With more information, we could impose a different Armington elasticity for each good $\sigma_{j,imp}$, $j = agr, man$.

Suppose too that the Armington elasticity for exports is $1/(1 - \rho_{exp}) = 10$, or $\rho_{exp} = 0.9$. Then we can recalibrate the foreign utility function

$$\left(\theta_{agr,f} x_{agr,f}^{\rho_{exp}} + \theta_{man,f} x_{man,f}^{\rho_{exp}} + \theta_{f,f} x_{f,f}^{\rho_{exp}} - 1 \right) / \rho_{exp}.$$

- We know that $\hat{y}_{j,d}, \hat{y}_{j,f}$, $j = agr, man$, solve

$$\begin{aligned} & \min \hat{p}_{j,d} y_{j,d} + (1 + \tau_j) \hat{e}\bar{p}_{j,f} y_{j,f} \\ \text{s.t. } & \gamma_j \left[\delta_j y_{j,d}^{\rho_{imp}} + (1 - \delta_j) y_{j,f}^{\rho_{imp}} \right]^{\frac{1}{\rho_{imp}}} = \hat{y}_j \\ & y_{j,d}, y_{j,f} \geq 0. \end{aligned}$$

Therefore

$$\begin{aligned} \frac{(1 + \tau_{agr}) \hat{e}\bar{p}_{agr,f}}{\hat{p}_{agr,d}} &= \frac{(1 - \delta_{agr}) \hat{y}_{agr,f}^{\rho_{imp}^{-1}}}{\delta_{agr} \hat{y}_{agr,d}^{\rho_{imp}^{-1}}} \\ \frac{(1 - \delta_{agr})}{\delta_{agr}} &= \frac{(1 + \tau_{agr}) \hat{e}\bar{p}_{agr,f} \hat{y}_{agr,f}^{1 - \rho_{imp}}}{\hat{p}_{agr,d} \hat{y}_{agr,d}^{1 - \rho_{imp}}} = \frac{1.5 \times 4^{0.2}}{14^{0.2}} \end{aligned}$$

$$\delta_{agr} = 0.4613.$$

Since $y_{agr} = \gamma_{agr} \left[\delta_{agr} y_{agr,d}^{\rho_{imp}} + (1 - \delta_{agr}) y_{agr,f}^{\rho_{imp}} \right]^{\frac{1}{\rho_{imp}}}$, this implies that

$$20 = \gamma_{agr} \left[0.4613 \times 14^{0.8} + 0.5387 \times 4^{0.8} \right]^{1.25}$$

$$\gamma_{agr} = \frac{20}{\left[0.4613 \times 14^{0.8} + 0.5387 \times 4^{0.8} \right]^{1.25}} = 2.4057.$$

Similarly, $\delta_{man} = 0.5358$ and

$$\gamma_{agr} = 30 / \left[0.5358 \times 25^{0.9} + 0.4642 \times 4^{0.9} \right]^{1.1111} = 2.0843.$$

- Let $I_f = 1000$ and $\tau_{agr,f} = 0.20$ and $\tau_{man,f} = 0.10$. We know that $\hat{x}_{agr,f}$, $\hat{x}_{man,f}$ solve

$$\begin{aligned} & \max \left(\theta_{agr,f} x_{agr,f}^{\rho_{exp}} + \theta_{man,f} x_{man,f}^{\rho_{exp}} + \theta_{f,f} x_{f,f}^{\rho_{exp}} - 1 \right) / \rho_{exp} \\ \text{s.t. } & \hat{p}_{agr} (1 + \tau_{agr,f}) x_{agr,f} + (1 + \tau_{man,f}) \hat{p}_{man} x_{man,f} + \hat{e} \hat{x}_f = \hat{e} I_f \\ & x_{agr,f}, x_{man,f}, x_f \geq 0. \end{aligned}$$

Solving this problem, we obtain, for example,

$$\hat{x}_{agr,f} = \frac{\theta_{agr,f}^{1-\rho_{exp}} \hat{e} I_f}{\left((1 + \tau_{agr,f}) \hat{p}_{agr} \right)^{\frac{1}{1-\rho_{exp}}} \Delta}$$

where

$$\Delta = \theta_{agr,f}^{\frac{1}{1-\rho_{exp}}} \left((1 + \tau_{agr,f}) \hat{p}_{agr} \right)^{\frac{-\rho_{exp}}{1-\rho_{exp}}} + \theta_{man,f}^{\frac{1}{1-\rho_{exp}}} \left((1 + \tau_{man,f}) \hat{p}_{man} \right)^{\frac{-\rho_{exp}}{1-\rho_{exp}}} + \theta_{f,f}^{\frac{1}{1-\rho_{exp}}} \hat{e}^{\frac{-\rho_{exp}}{1-\rho_{exp}}}.$$

$$\theta_{agr,f} = \frac{(1.2)5^{0.1}}{(1.2)5^{0.1} + (1.1)3^{0.1} + 990.7^{0.1}} = 0.3044.$$

Similarly, $\theta_{man,f} = 0.2651$ and $\theta_{f,f} = 0.4305$.

Numerical Experiments I: Unilateral Reform

Tariffs

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
τ_{agr}	0.5000	0.2000	0.2000	0.0000	0.0000
τ_{man}	0.2500	0.2000	0.2000	0.0000	0.0000
$\tau_{agr,f}$	0.2000	0.2000	0.2000	0.2000	0.2000
$\tau_{man,f}$	0.1000	0.1000	0.1000	0.1000	0.1000

Prices

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{p}_{agr}	1.0000	0.9749	0.9527	0.9758	0.9221
\hat{p}_{man}	1.0000	1.0167	1.0315	1.0161	1.0519
\hat{p}_{inv}	1.0000	1.0028	1.0053	1.0027	1.0087
$\hat{p}_{agr,d}$	1.0000	1.0002	1.0332	0.9990	1.0432
$\hat{p}_{man,d}$	1.0000	1.0005	1.0363	0.9986	1.0470
\hat{e}	1.0000	1.1479	1.0512	1.3858	1.1404
\hat{r}	1.0000	0.9988	1.0508	0.9999	1.1888
\hat{w}	1.0000	0.9977	1.0575	0.9924	1.1347

Normalize prices

$$\frac{\theta_{agr}}{\theta_{agr} + \theta_{man}} \hat{p}_{agr} + \frac{\theta_{man}}{\theta_{agr} + \theta_{man}} \hat{p}_{man} = 1 \text{ (CPI).}$$

Domestic production

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
$\hat{y}_{agr,d}$	14.0000	14.0661	13.6456	14.4193	16.6160
$\hat{x}_{agr,agr}$	2.0000	2.0094	1.9494	2.0599	2.3737
$\hat{x}_{man,agr}$	4.0000	4.0189	3.8987	4.1198	4.7474
$\hat{\ell}_{agr}$	4.0000	4.0213	3.8864	4.1353	4.8595
\hat{k}_{agr}	4.0000	4.0165	3.9111	4.1044	4.6380
$\hat{y}_{man,d}$	25.0000	24.9370	25.3373	24.6004	22.4988
$\hat{x}_{agr,man}$	3.0000	2.9924	3.0405	2.9520	2.6999
$\hat{x}_{man,man}$	7.0000	6.9824	7.0945	6.8881	6.2997
$\hat{\ell}_{man}$	10.0000	9.9787	10.1136	9.8647	9.1405
\hat{k}_{man}	5.0000	4.9835	5.0889	4.8956	4.3620

Investment

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{y}_{inv}	6.0000	5.7056	6.3199	5.2689	6.0823
$\hat{x}_{agr,inv}$	2.0000	1.9019	2.1066	1.7563	2.0274
$\hat{x}_{man,inv}$	4.0000	3.8038	4.2133	3.5126	4.0549

Total supply and international trade

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{y}_{agr}	20.0000	20.6162	29.2483	21.0877	56.2916
\hat{y}_{man}	30.0000	29.4466	31.1158	29.0105	33.2486
$\hat{y}_{agr,d}$	14.0000	14.0661	13.6456	14.4193	16.6160
$\hat{y}_{man,d}$	25.0000	24.9370	25.3373	24.6004	22.4988
$\hat{y}_{agr,f}$	4.0000	4.3774	10.9121	4.4547	29.5479
$\hat{y}_{man,f}$	4.0000	3.6226	4.6291	3.5453	9.0329
\hat{T}	3.0000	1.8366	3.2675	0.0000	0.0000
$\hat{x}_{agr,f}$	5.0000	5.8872	13.2602	7.1007	40.3199
$\hat{x}_{man,f}$	3.0000	3.3869	3.5916	4.0914	6.4824

Consumption, savings, and welfare

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{C}_{agr}	8.0000	7.8253	8.8916	7.2187	8.8707
\hat{C}_{man}	12.0000	11.2547	12.3177	10.3985	11.6642
\hat{C}_{inv}	6.0000	5.7056	6.3199	5.2689	6.0823
real income	1.0000	0.9531	1.0582	0.8801	1.0221

Real income index:

$$\hat{Y} = \frac{\hat{C}_{agr}^{\theta_{agr}} \hat{C}_{man}^{\theta_{man}} \hat{C}_{inv}^{\theta_{inv}}}{8^{\theta_{agr}} 12^{\theta_{man}} 6^{\theta_{inv}}}.$$

Numerical Experiments II: Free Trade Agreement

Tariffs

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
τ_{agr}	0.5000	0.2000	0.2000	0.0000	0.0000
τ_{man}	0.2500	0.2000	0.2000	0.0000	0.0000
$\tau_{agr,f}$	0.2000	0.0500	0.0500	0.0000	0.0000
$\tau_{man,f}$	0.1000	0.0500	0.0500	0.0000	0.0000

Prices

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{p}_{agr}	1.0000	0.9666	0.9307	0.9635	0.8865
\hat{p}_{man}	1.0000	1.0223	1.0462	1.0243	1.0757
\hat{p}_{inv}	1.0000	1.0037	1.0077	1.0041	1.0126
$\hat{p}_{agr,d}$	1.0000	1.0215	1.0770	1.0307	1.2098
$\hat{p}_{man,d}$	1.0000	1.0228	1.0781	1.0318	1.2008
\hat{e}	1.0000	1.0622	0.9743	1.2347	1.0412
\hat{r}	1.0000	1.0366	1.1491	1.0561	1.4702
\hat{w}	1.0000	1.0331	1.1093	1.0454	1.2538

Domestic production

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
$\hat{y}_{agr,d}$	14.0000	14.1916	15.9768	14.5680	22.9709
$\hat{x}_{agr,agr}$	2.0000	2.0274	2.2824	2.0811	3.2816
$\hat{x}_{man,agr}$	4.0000	4.0547	4.5648	4.1623	6.5631
$\hat{\ell}_{agr}$	4.0000	4.0617	4.6460	4.1834	7.1070
\hat{k}_{agr}	4.0000	4.0478	4.4850	4.1412	6.0608
$\hat{y}_{man,d}$	25.0000	24.8175	23.1118	24.4586	16.3417
$\hat{x}_{agr,man}$	3.0000	2.9781	2.7734	2.9350	1.9610
$\hat{x}_{man,man}$	7.0000	6.9489	6.4713	6.8484	4.5757
$\hat{\ell}_{man}$	10.0000	9.9383	9.3540	9.8166	6.8930
\hat{k}_{man}	5.0000	4.9522	4.5150	4.8588	2.9392

Investment

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{y}_{inv}	6.0000	5.9029	7.2865	5.5485	7.0158
$\hat{x}_{agr,inv}$	2.0000	1.9676	2.4288	1.8495	2.3386
$\hat{x}_{man,inv}$	4.0000	3.9352	4.8577	3.6990	4.6772

Total supply and international trade

	benchmark	partial liberalization		free trade	
variable		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{y}_{agr}	20.0000	21.4249	47.3734	22.2641	155.3193
\hat{y}_{man}	30.0000	29.7960	32.2208	29.5646	34.0054
$\hat{y}_{agr,d}$	14.0000	14.1916	15.9768	14.5680	22.9709
$\hat{y}_{man,d}$	25.0000	24.8175	23.1118	24.4586	16.3417
$\hat{y}_{agr,f}$	4.0000	4.8743	22.9916	5.2121	105.5540
$\hat{y}_{man,f}$	4.0000	3.9828	7.5212	4.0879	16.2835
\hat{T}	3.0000	1.8816	5.9459	0.0000	0.0000
$\hat{x}_{agr,f}$	5.0000	6.2792	29.3693	7.6890	137.0533
$\hat{x}_{man,f}$	3.0000	3.2656	2.2905	3.9777	4.9804

Consumption, savings, and welfare

variable	benchmark	partial liberalization		free trade	
		$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$	$\sigma_{imp} = \sigma_{exp} = 1$	$\sigma_{imp} = 5, \sigma_{exp} = 10$
\hat{c}_{agr}	8.0000	8.1726	10.5194	7.7095	10.6849
\hat{c}_{man}	12.0000	11.5915	14.0365	10.8772	13.2091
\hat{c}_{inv}	6.0000	5.9029	7.2865	5.5485	7.0158
real income	1.0000	0.9869	1.2231	0.9280	1.1846

To calculate a post-reform input-output matrix that balances, we need to use post-reform prices. That is, the elements need to be quantities multiplied by prices. Here, for example, is the input-output matrix for the economy in the numerical experiment where

$$\tau_{agr} = \tau_{man} = \tau_{agr,f} = \tau_{man,f} = 0, \sigma_{imp} = 5, \sigma_{exp} = 10.$$

	Agr.	Man.	Con.	Inv.	Exp.	Total
Agriculture	2.9091	1.7385	9.4723	2.0732	121.4995	137.6925
Manufacturing	7.0597	4.9219	14.2084	5.0310	5.3572	36.5782
Imports	109.9023	16.9543				126.8566
Tariff Revenue	0.0000	0.0000				0.0000
Labor Compensation	8.9107	8.6424				17.5531
Returns to Capital	8.9107	4.3212				13.2319
Total	137.6925	36.5782	23.6807	7.1042	126.8566	