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DYNAMIC MODELS OF INTERNATIONAL TRADE   **SPRING 2009**

PROBLEM SET

1. Find annual time series data on real output, real investment, employment, working age population, and — if you can — hours worked for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate δ and a capital share α. Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t \right)/10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.

2. Consider a model with an infinitely-lived, representative consumer. The production function is $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. The consumer solves the problem

$$\max \sum_{t=T_0}^{\infty} \beta^t [\gamma \log C_t + (1-\gamma)\log(N_t \bar{h} - L_t)]$$
$$\text{s.t. } C_t + K_{t+1} - K_t = w_t L_t + (r_t - \delta)K_t.$$
$$K_{t_0} = \bar{K}_{t_0}$$

a) Define an equilibrium of this economy.

b) Use the results from question 1 and the first order conditions from the consumer's problem to estimate the values of the parameters β and γ.

c) Using the MATLAB programs found at http://www.greatdepressionsbook.com/, calculate the equilibrium of this model. If you have enough data, you should calibrate the parameters β and γ for a period in which you are not very interested and then calibrate the equilibrium for a more interesting period. Briefly explain your computational methodology. Discuss your results.

3. Choose two countries that are important (to each other) trading partners, one with higher output per worker than the other. Use Summers-Heston data or World Development Indicators data to try to answer Lucas's question: To what extent can differences in output per worker be explained by differences in capital per worker?

a) Use IMF real interest rate data to try to make inferences about differences in capital per worker across countries. How much of the difference in output per worker be explained by differences in capital per worker? If capital flows equalized the rental rates on capital, what would happened to output per worker in the different countries?

b) Use data on investment and the accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t,$$

where $\delta = 0.05$ to determine capital per worker in some year. Answer the same questions as in part a and compare.

c) Discuss what you have learned from this exercise and what more you think would be relevant in explaining differences in output per worker across countries.

4. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_t = rer_t^T + rer_t^N,$$

where rer_t is the natural logarithm of the bilateral real exchange rate and rer_t^T is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate ratio of the standard deviations of rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of rer_t in terms of rer_t^T and rer_t^N in levels, in 1 year differences, and in 4 year differences.