TOPICS IN INTERNAIONAL TRADE **T. J. KEHOE**
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EXERCISES: MONOPOLISTIC COMPETITION

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho},$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 2$, $f = 4$, $\rho = 1/2$, and $\bar{\ell} = 36$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 180$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 36$ and the second with $\bar{\ell}^2 = 144$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

2. Repeat the analysis of question 1 for two variants of the model. Compare the gains from trade in these two alternative models with those in the model in question 1.

a) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.

b) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

c) Compare the gains in real income in parts a and b with each other and with those in question 1, part d.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha)\log c_0 + (\alpha/\rho)\log \int_0^m c(z)^\rho \, dz$$

$$\text{s.t. } p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$

$$c(z) \geq 0 .$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0\right].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma} .$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first, μ is so small that all firms make nonnegative profits. In the second, μ is large enough so that there is a cutoff level of productivity $\bar{x} > 1$ such that all firms with productivity $x(z) \geq \bar{x}$ produce and earn nonnegative profits but no firm with productivity $x(z) < \bar{x}$ finds it profitable to produce. Find the relation between m and μ and $\bar{x}$.

d) Suppose that $\alpha = 0.5$, $\rho = 0.5$, $\bar{\ell} = 40$, $\mu = 10$, $\gamma = 4$, and $f = 2$. Calculate the equilibrium of this economy.

4. Consider a world with two countries like that in question 3 that engage in free trade. Each country i, $i = 1,2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$ and an iceberg transportation cost of $\tau_i^j - 1 = \tau - 1 \geq 0$, $j \neq i$.

a) Define an equilibrium for this economy.

b) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels, $\bar{x}_d$ and $\bar{x}_d$, where $\bar{x}_e > \bar{x}_d > 1$. Firms with $x(z) \geq \bar{x}_e$ produce for both the domestic and the export market; firms with $\bar{x}_e > x(z) \geq \bar{x}_d$ produce only for the domestic market, and firms with $\bar{x}_d > x(z)$ do not produce.

c) Suppose that $\mu = 0.5$, $\rho = 0.5$, $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$, $\mu_1 = \mu_2 = \mu = 10$, $\gamma = 4$, $f_d = 2$, $f_e = 3$, and $\tau_2^1 = \tau_1^2 = \tau = 1.2$. Calculate the equilibrium of this economy.

d) Suppose now that a free trade agreement sets $\tau_2^1 = \tau_1^2 = \tau = 1$. Recalculate the equilibrium in part c.

e) Suppose now that a different reform sets $f_e = 2$. Again recalculate the equilibrium in part b. Contrast the results with those in part d. What sort of reform can lower f_e?