CENTRO DE ESTUDIOS MONETARIOS Y FINANCIEROS MACROECONOMICS I

## EXAMINATION

Answer three of the four questions. Only the first three questions attempted will be graded.

1. Consider the social planner's problem of choosing sequences of $c_{t}, \ell_{t}$, and $k_{t}$ to solve

$$
\begin{gathered}
\max \sum_{t=0}^{\infty} \beta^{t}\left[\gamma \log c_{t}+(1-\gamma) \log \left(1-\ell_{t}\right)\right] \\
\text { s.t. } \quad c_{t}+k_{t+1} \leq \theta k_{t}^{\alpha} \ell_{t}^{1-\alpha} \\
c_{t}, k_{t}, \ell_{t},\left(1-\ell_{t}\right) \geq 0 \\
k_{0} \leq \bar{k}_{0} .
\end{gathered}
$$

Here $1>\beta>0,1>\gamma>0,1>\alpha>0$, and $\theta>0$.
a) Write down the Euler conditions and the transversality condition for this problem.
b) Formulate this social planner's problem as a dynamic programming problem by writing down the relevant Bellman's equation. Guessing that the value function takes the form

$$
V(k)=a_{0}+a_{1} \log k,
$$

solve for the value function $V(k)$ and the policy functions $c=c(k), \ell=\ell(k)$, and $k^{\prime}=k^{\prime}(k)$. [Hint: The optimal value of $\ell$ does not vary with $k$.]
c) Verify that the solution to the social planner's generated by the policy functions in part b satisfies the Euler conditions and transversality condition in part a.
d) Specify an economic environment for which the solution to this social planning problem is a Pareto efficient allocation. Define a sequential markets equilibrium for this economy. Explain how you can use the policy functions from part b to calculate this equilibrium.
e) Define an Arrow-Debreu equilibrium for the economy in part d. Explain how you can use the policy functions from part b to calculate this equilibrium.
2. Consider an economy in which the equilibrium allocation solves the optimal growth problem

$$
\begin{array}{lc} 
& \max E \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}, 1-\ell_{t}\right) \\
\text { s.t. } & c_{t}+k_{t+1}-(1-\delta) k_{t} \leq z_{t} f\left(k_{t}, \ell_{t}\right) \\
& c_{t}, k_{t} \geq 0 \\
& k_{0}=\bar{k}_{0},
\end{array}
$$

where $1>\beta>0,1 \geq \delta \geq 0, u$ is continuously differentiable, strictly concave, and monotonically increasing, and $f$ is continuously differentiable, monotonically increasing, concave, and homogeneous of degree 1. Here $z_{t}$ is a random variable that takes on two values $\bar{z}_{1}$ and $\bar{z}_{2}, \bar{z}_{2}>\bar{Z}_{1}$ and whose evolution is governed by the stationary, first order Markov chain with transition matrix

$$
\Pi=\left[\begin{array}{ll}
\pi_{11} & \pi_{12} \\
\pi_{21} & \pi_{22}
\end{array}\right]
$$

Assume, for the sake of specificity, that, at $t=0, z_{0}=\bar{z}_{1}$.
a) Define an Arrow-Debreu equilibrium for this economy.
b) Define a sequential markets equilibrium for this economy.
c) Write down Bellman's equation for solving the social planner's problem as a dynamic programming problem.
d) Suppose that you have solved this dynamic programming problem and have found the policy functions $k^{\prime}=k^{\prime}(k, z), c=c(k, z)$, and $\ell=\ell(k, z)$. Explain how you can use these policy functions to calculate the Arrow-Debreu equilibrium. Explain how you can use these policy functions to calculate the sequential markets equilibrium.
e) Suppose now that there are two types of consumers of equal measure, measure 1. Consumers of type $i, i=1,2$, have the utility function

$$
E \sum_{t=0}^{\infty} \beta^{t} u_{i}\left(c_{t}^{i}, 1-\ell_{t}^{i}\right)
$$

and endowment of capital $\bar{k}_{0}^{i}$ in period 0 . Define a sequential markets equilibrium for this economy. Does the equilibrium for this economy with heterogeneous consumers solve a social planner's problem? If it does, write down the associated Bellman's equation. If it does not, explain why it does not.
3. Consider an overlapping generations economy in which the representative consumer in generation $t, t=1,2, \ldots$, has preferences over the consumption of the single good in each of the two periods of her life given by the utility function

$$
u\left(c_{t}^{t}, c_{t+1}^{t}\right)=\log c_{t}^{t}+\beta \log c_{t+1}^{t},
$$

where $\beta>0$. This consumer is endowed with quantities of labor $\left(\ell_{t}^{t}, \ell_{t+1}^{t}\right)=\left(\ell_{1}, \ell_{2}\right)$. In addition, there is a generation 0 whose representative consumer lives only in period 1 and has the utility function

$$
u^{0}\left(c_{1}^{0}\right)=\log c_{1}^{0},
$$

and the endowment of $\ell_{1}^{0}=\ell_{2}$ units of labor and $k_{1}^{0}=\bar{k}_{1}$ units of capital. This consumer also has an endowment of fiat money $m$, which can be positive, negative, or zero. The production function is

$$
f\left(k_{t}, \ell_{t}\right)=\theta k_{t}^{\alpha} \ell_{t}^{1-\alpha},
$$

where $\theta>0$ and $1>\alpha>0$. Capital depreciates at the rate $\delta$ per period, $0 \leq \delta \leq 1$.
a) Define a sequential market equilibrium for this economy.
b) Define an Arrow-Debreu equilibrium for this economy.
c) State and prove two theorems that establish the essential equivalence between a sequential market equilibrium and an Arrow-Debreu equilibrium.
d) Define a steady state for this economy. Write down an equation or equations that characterize a steady state.
e) Suppose now that consumers born in period 1 and afterwards live for three, rather than two, periods. Suppose too that there are three generations alive in period 1 generation 1 , which lives in periods 1,2 , and 3 , generation 0 , which lives in periods 1 and 2 , and generation -1 , which lives in period 1 . Specify a stationary environment analogous to that above by specifying utility functions, endowments, and a production technology. Define a sequential markets equilibrium for this economy. Define a steady state for this economy. Write down an equation or equations that characterize a steady state.
4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage $w$ drawn independently from the time invariant probability distribution $F(v)=\operatorname{prob}(w \leq v)$, $v \in[0, B], B>0$. After receiving the wage offer $w$ the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit $b$, and search again next period. That is,

$$
y_{t}=\left\{\begin{array}{ll}
w & \text { if job offer has been accepted } \\
b & \text { if searching }
\end{array} .\right.
$$

The worker solves

$$
\max E \sum_{t=0}^{\infty} \beta^{t} y_{t}
$$

where $1>\beta>0$. Once a job offer has been accepted, there are no fires or quits.
a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.
b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w<\bar{w}$.
c) Consider two economies with different unemployment benefits $b_{1}$ and $b_{2}$, but otherwise identical. Let $\bar{w}_{1}$ and $\bar{w}_{2}$ be the reservation wages in these two economies. Suppose that that $b_{2}>b_{1}$. Prove that $\bar{w}_{2}>\bar{w}_{1}$. Provide some intuition for this result.
d) Consider two economies with different wage distributions $F_{1}$ and $F_{2}$, but otherwise identical. Let $\bar{w}_{1}$ and $\bar{w}_{2}$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that $F_{2}$ is a mean preserving spread of $F_{1}$. Prove that $\bar{w}_{2}>\bar{w}_{1}$. Provide some intuition for this result.

