CENTRO DE ESTUDIOS MONETARIOS Y FINANCIEROS MACROECONOMICS I PROBLEM SET #3 T. J. KEHOE WINTER 2011

1. Consider an economy with an infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

This consumer is endowed with $\ell_t = 1$ in every period and $\overline{k_0}$ period 0. Feasible consumption/production plans satisfy

$$c_t + k_{t+1} \le \theta k_t^{\alpha}$$
$$c_t, k_t \ge 0.$$

Here $1 > \alpha > 0$, $1 > \beta > 0$, $\theta > 0$. Notice that the rate of depreciation, δ , is implicitly equal to 1.

a) Define a sequential markets equilibrium of this economy.

b) Define a steady state of this economy. Calculate the steady state values of c and k.

c) Write down an optimal growth problem for this economy. Write down the Euler conditions and the transversality condition for this problem. Suppose that in the solution to the optimal growth problem

$$k_{t+1} = b\theta k_t^{\alpha}$$

for some constant b. Using the Euler conditions, solve for b. Verify that the resulting solution to the Euler conditions satisfies the transversality condition.

d) Use the solution to part c to calculate the sequential markets equilibrium for this economy.

e) Define an Arrow-Debreu equilibrium for this economy. Use the solution to part d to calculate this Arrow-Debreu equilibrium.

Questions 2 and 3 require you to find annual time series data on constant price GDP, current price GDP, current price investment, hours worked, and working age population for the some country not the United States. Instead of constant price GDP, you may need to use a chained weighted quantity index. If you cannot find data on hours worked, use data on employment and assume that all workers worked a fixed amount, like 40 hours per week.

2. Construct a real investment series by calculating

$$I_t = \frac{\tilde{I}_t}{P_t},$$

where \tilde{I}_t is nominal (current price) investment and P_t is the GDP deflator

$$P_t = \frac{\tilde{Y}_t}{Y_t},$$

where \tilde{Y}_t is nominal (current price) GDP and Y_t is real (constant price) GDP or the chain weighted quantity index for GDP.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$\begin{split} K_{t+1} &= (1-\delta)K_t + I_t \\ K_{T_0} &= \overline{K}_{T_0} \;. \end{split}$$

where T_0 is the first year for which you have data on output and investment. Choose \overline{K}_{T_0} so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}$$

If you have sufficient data, calibrate the depreciation rate δ . Otherwise, use $\delta = 0.05$.

b) Repeat part a, but choose \overline{K}_{T_0} so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10.$$

c) Compare the two series constructed in parts a and b.

3. Suppose that the aggregate production function for the country that you are studying takes the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} .$$

If you have sufficient data, calibrate the capital share α . Otherwise, use $\alpha = 0.30$.

a) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}$$

b) Discuss what happens during different time periods.