CENTRO DE ESTUDIOS MONETARIOS Y FINANCIEROS **T. J. KEHOE**
MACROECONOMICS I **WINTER 2011**

PROBLEM SET #6

1. This question requires you to apply the Hodrick-Prescott filter to the time series for macroeconomic variables for the country that you studied in problem sets #3 and #4.

a) First consider a set of data the includes the components of real GDP — that is, consumption, investment, government expenditures, exports, and imports — and of labor inputs — that is, hours (if these data are available) and employment. Construct a table of business cycle facts like that on pp. 29–33 of Cooley and Prescott. Do this both for the country that you are studying and for the United States. If quarterly data are available for the country that you are studying, use them and use quarterly data for the United States. Otherwise use annual data. Take the natural logarithm of all variables. (If the trade balance is one of your variables, calculate the trade balance as a fraction of GDP. You cannot take the logarithm of a negative number!) If you have quarterly data, use the smoothing parameter $\lambda = 1600$. If you only have annual data, use the smoothing parameter $\lambda = 100$. Calculate the standard deviations and autocorrelations of the variables and the correlations of the real GDP with the leads and lags of the other variables. If you have quarterly data, follow Cooley and Prescott in calculating correlations with 5 leads and lags, as well as the contemporary correlation. If you only have annual data, calculate correlations with 3 leads and lags, as well as the contemporary correlation. Discuss your results, stressing the comparison of the business cycle facts for the country that you are studying with those for the United States. Also compare the filter for the country that you are studying with that for the United States.

Note: If you have quarterly data, make sure that they are seasonally adjusted. If they are not seasonally adjusted, and you cannot find seasonally adjusted data, then you have three choices: (1) Use annual data. (2) Use the raw quarterly data. In this case, the fourth leads and lags will display jumps, so only compute 3 leads and lags. (3) Seasonally adjust the data yourself. Be explicit about what you are doing.

b) Now focus on a subset of variables for which you will compare the times series data with data generated by your model in question 3: real GDP, real investment (investment in current prices deflated by the GDP deflator as in question 2 on problem set #3), consumption (GDP in current prices minus investment in current prices deflated by the GDP deflator), capital, and labor input (hour worked or, if these data are not available, employment). Construct a series for the variable that corresponds to Also construct a series for total factor productivity. Construct a table for the country that you are studying and one for the United States. Discuss your results, stressing the comparison of the business cycle facts for the country that you are studying with those for the United States.

2. Consider an economy like that in question 1 on problem set #4 in which the equilibrium allocation is the solution to the optimal growth problem

$$
\begin{gathered}
\max E\sum_{t=0}^{\infty}\beta^t\left[\theta\log C_t+(1-\theta)\log(N_t\bar{h}-L_t)\right] \\
\text{s.t.}\quad C_t+K_{t+1}-(1-\delta)K_t\le e^{z_t}(\gamma^{1-\alpha})^t A_0K_t^{\alpha}L_t^{1-\alpha} \\
C_t,\, K_t\ge 0 \\
K_0=\bar{K}_0 \\
N_t=\eta^t N_0\,.
\end{gathered}
$$

Here z_t is a random variable that takes on two values $\bar{z}_1=-\zeta$, $\bar{z}_2=\zeta$, and whose evolution is governed by the stationary, first order Markov chain with transition matrix

$$
\Pi=\begin{bmatrix}1-\pi & \pi\\ \pi & 1-\pi\end{bmatrix}.
$$

Assume, for the sake of specificity, that, at $t=0$, $z_0=\bar{z}_1=-\zeta$.

a) Define an Arrow-Debreu equilibrium for this economy.

b) Define a sequential markets equilibrium for this economy.

Redefine variables C_t and K_t by dividing by the number of effective working age persons $\tilde{N}_t=\gamma^t N_t=(\gamma\eta)^t N_0$. Divide L_t by N_t:

$$
\begin{gathered}
c_t=C_t/\tilde{N}_t=\gamma^{-t}(C_t/N_t) \\
k_t=K_t/\tilde{N}_t=\gamma^{-t}(K_t/N_t) \\
\ell_t=L_t/N_t
\end{gathered}
$$

Consider the social planner's problem

$$
\begin{gathered}
\max\; E\sum_{t=0}^{\infty}\beta^t\left[\theta\log\; c_t+(1-\theta)\log(\bar{h}-\ell_t)\right] \\
\text{s. t.}\; c_t+\gamma\eta k_{t+1}-(1-\delta)k_t\le e^{z_t}A_0k_t^{\alpha}\ell_t^{1-\alpha} \\
c_t,\; k_t\ge 0,\; \bar{h}\ge\ell_t\ge 0 \\
k_0=\bar{K}_0/N_0,
\end{gathered}
$$

with the associated Bellman's equation

$$
\begin{aligned}
V(k,z) = \max\ & \theta \log\ c + (1-\theta)\log(\bar{h}-\ell) + \beta E V(k',z') \\
\text{s. t. }\ & c + \gamma\eta k' - (1-\delta)k \le e^z A_0 k^\alpha \ell^{1-\alpha} \\
& c,\ k' \ge 0,\ \bar{h} \ge \ell \ge 0 \\
& k,\ z \text{ given.}
\end{aligned}
$$

c) Suppose that you have solved this dynamic programming problem and have found the policy functions $k' = g(k,z)$, $c = c(k,z)$, and $\ell = \ell(k,z)$. Explain how you can use these policy functions to calculate the Arrow-Debreu equilibrium. Explain how you can use these policy functions to calculate the sequential markets equilibrium.

3. (Optional) This question requires you to use the parameters — $\beta, \theta, \gamma, A_0$, and η and, if you have sufficient data, α and δ — that you calibrated in question 1 in problem set #4, the computer program that you developed in question 3 in problem set #4, and the H-P filtered data from question 1 to build a DSGE (dynamic, stochastic general equilibrium) model of the economy that you are studying.

Note: If you are using quarterly data, you should modify the parameters $\beta, \gamma, \eta, \delta$:

$$
\begin{aligned}
\tilde{\beta} &= \beta^{1/4} \\
\tilde{\gamma} &= \gamma^{1/4} \\
\tilde{\eta} &= \eta^{1/4} \\
\tilde{\delta} &= 1-(1-\delta)^{1/4}.
\end{aligned}
$$

a) Find the sample standard deviation and autocorrelation for the H-P filtered logarithm of the total factor productivity that you calculated in part b of question 1. Use these statistics to calibrate the parameters ζ and π for the Markov chain for the productivity shock z_t. [Hint: There are analytical expressions for the standard deviation and autocorrelation of z_t in its invariant distribution.]

b) Find the steady state capital stocks for two deterministic versions of your economy: $\hat{k}_2$ when z_t is constant at $z_t = \bar{z}_2 = \zeta$ and $\hat{k}_1$ when z_t is constant at $z_t = \bar{z}_1 = -\zeta$. This requires you to solve

$$
\beta\left(\alpha e^z A_0 k^{\alpha-1}\ell^{1-\alpha} + 1 - \delta\right) = \gamma\eta
$$

$$
\frac{1-\theta}{\bar{h}-\ell} = \frac{\theta(1-\alpha)e^z A_0 k^\alpha \ell^{-\alpha}}{e^z A_0 k^\alpha \ell^{1-\alpha} - \gamma\eta k + (1-\delta)k}
$$

for k and ℓ.

c) Rewrite the Bellman's equation in question 2 as the system of two functional equations

$$
\begin{aligned}
V(k,\bar{z}_1) = \max \ & \theta \log\ c + (1-\theta)\log(\bar{h}-\ell) + \beta\big((1-\pi)V(k',\bar{z}_1) + \pi V(k',\bar{z}_2)\big) \\
\text{s. t. } & c + \gamma\eta k' - (1-\delta)k \le e^{\bar{z}_1} A_0 k^{\alpha} \ell^{1-\alpha} \\
& c,\ k' \ge 0,\ \bar{h} \ge \ell \ge 0 \\
& k \text{ given.}
\end{aligned}
$$

$$
\begin{aligned}
V(k,\bar{z}_2) = \max \ & \theta \log\ c + (1-\theta)\log(\bar{h}-\ell) + \beta\big(\pi V(k',\bar{z}_1) + (1-\pi)V(k',\bar{z}_2)\big) \\
\text{s. t. } & c + \gamma\eta k' - (1-\delta)k \le e^{\bar{z}_2} A_0 k^{\alpha} \ell^{1-\alpha} \\
& c,\ k' \ge 0,\ \bar{h} \ge \ell \ge 0 \\
& k \text{ given.}
\end{aligned}
$$

Choose a maximal capital stock $\bar{k}$ such that $\bar{k} > \hat{k}_2$. Choose a grid of capital stocks in the interval $(0,\bar{k}]$. Modify the computer program that you developed to solve question 3 in problem set #4 to solve for $V(k,\bar{z}_1)$ and $V(k,\bar{z}_2)$. Also calculate the policy functions $g(k,\bar{z}_1)$ and $g(k,\bar{z}_2)$.

Note: It is efficient to calculate the return function

$$
\begin{aligned}
v(k,k',z) = \max \ & \theta \log\ c + (1-\theta)\log(\bar{h}-\ell) \\
\text{s. t. } & c + \gamma\eta k' - (1-\delta)k \le e^{\bar{z}_2} A_0 k^{\alpha} \ell^{1-\alpha} \\
& c,\ k' \ge 0,\ \bar{h} \ge \ell \ge 0 \\
& k,\ k' \text{ given}
\end{aligned}
$$

at the beginning of the program, before you do any value function iterations. It is also useful to calculate the optimizing values of $c(k,k',z)$ and $\ell(k,k',z)$. Notice that, when you have obtained the policy function for the next period capital stock $k' = g(k,z)$, the policy functions for consumption and labor supply are $c = c(k,g(k,z),z)$ and $\ell = \ell(k,g(k,z),z)$.

d) Graph the two policy functions $g(k,\bar{z}_1)$ and $g(k,\bar{z}_2)$ in (k,k') space along with the 45° line. Explain how you could use these two functions along with a random number generator that generates uniform random numbers on the interval $[0,1]$ to produce realizations of an equilibrium path of capital stocks for this economy.

e) Write a computer program to simulate the DSGE model that you have built. Choose a value of k_0 such that $\hat{k}_2 > k_0 > \hat{k}_1$. Set $z_0 = \bar{z}_1 = -\zeta$. Simulate the economy for $n+20$

periods. Here n is the number of periods for which you have data in question 1. Discard the first 20 periods, $t = 0, 1, ..., 19$ of results. (This is to eliminate any sensitivity of your results to the choice of (k_0, z_0).) For every period t, $t = 20, 21, ..., 19 + n$, calculate the logarithms of real GDP, real investment, capital, labor input, and total factor productivity. Apply the Hodrick-Prescott filter to these data. Calculate the same statistics for these data as you did in the table that you constructed in part b of question 1. Do this sort of simulation 5 times and take averages of these statistics. Produce a table like that in part b of question 1 and compare the results from your model with the data for the country that you are studying.

4. (Optional) This question guides you in producing a more professional DSGE model than that in question 3, but it requires you to write a computer program to apply the Hodrick-Prescott filter and to seasonally adjust the data if they are not seasonally adjusted to start with.

a) If you are working with quarterly data, make sure that it is seasonally adjusted. That is, if the data are not seasonally adjusted, seasonally adjust them. [Hint: You can find programs for doing this in Matlab on the internet.]

b) Modify your program for solving the dynamic programming problem in part b of question 3 to solve for models with a Markov chain with 3 states $(\bar{z}_1, \bar{z}_2, \bar{z}_3) = (-\zeta, 0, \zeta)$ and transition matrix

$$\Pi = \begin{bmatrix} 1-2\pi & \pi & \pi \\ \pi & 1-2\pi & \pi \\ \pi & \pi & 1-2\pi \end{bmatrix}.$$

c) Calibrate the parameters ζ and π for the Markov chain for the productivity to match the sample standard deviation and autocorrelation for series for the H-P filtered logarithm of the total factor productivity that you calculated in part b of question 1. [Hint: It is here that you need to include the program for the Hodrick-Prescott filter. You can find such programs in Matlab on the internet.]

d) Simulate the model 100 times, for $n + 20$ periods each simulation. Calculate the same statistics as in the tables in part c of question 3 and part b of question 1. Compare these tables and discuss.