CENTRO DE ESTUDIOS MONETARIOS Y FINANCIEROS **T. J. KEHOE**
MACROECONOMICS I **WINTER 2011**

PROBLEM SET #7

1. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

2. Consider an economy with a continuum [0,1] of consumers of two symmetric types who live forever. Consumers have utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

where $0 < \beta < 1$ and u is strictly concave, increasing, and continuously differentiable. Consumers of type i have an endowment stream of the single good in each period

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (\omega^g, \omega^b, \omega^g, \omega^b, ...),$$

while consumers of type 2 have

$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (\omega^b, \omega^g, \omega^b, \omega^g, ...)$$

where $\omega^g > \omega^b$. There is one unit of trees that produce r units of the good every period. Each consumer of type i owns $\bar{\theta}_0^i$ of such trees in period 0, $\bar{\theta}_0^1 + \bar{\theta}_0^2 = 1$.

a) Define an Arrow-Debreu equilibrium for this economy. Define the corresponding sequential markets equilibrium. Find initial asset holdings $\bar{\theta}_0^1$ and $\bar{\theta}_0^2$ such that, in equilibrium, $\hat{c}_t^1 = \hat{c}_t^2$, $t = 0,1,...$. Find conditions under which $\bar{\theta}_0^i \geq 0$, $i = 1,2$.

b) Suppose now that contracts can be enforced only by the threat of exclusion from future trades and the seizure of any tree holdings. That is, at any date, consumers of either type can opt to renege on any debts and to subsist in autarky where $c_t^i = w_t^i$ forever after. Define an Arrow-Debreu equilibrium for this economy with enforcement constraints. Define the corresponding sequential markets equilibrium.

c) Define a symmetric steady state in which consumption depends only on the value of w_t^i, but not on i or t. (You can base this definition either on the Arrow-Debreu equilibrium or the sequential markets equilibrium.) Consider the function

$$f^D(c) = u(c) - u(\omega^g) + \beta\left(u(\omega^g + \omega^b + r - c) - u(\omega^b)\right).$$

Let $\hat{c}$ be the symmetric consumption in the equilibrium in part a. Explain carefully why, if $f^D(\hat{c}) \geq 0$, then the symmetric steady state consumption allocation in the economy with enforcement constraints is the same as in the economy without enforcement constraints. Explain why, if $f^D(\hat{c}) < 0$, however, the symmetric steady state consumption allocation must give higher consumption to the consumer when $w_t^i = \omega^g$ than when $w_t^i = \omega^b$.

d) Suppose that $f^D(\hat{c}) < 0$ in part c. Explain why there exists a unique c^g, $\omega^g > c^g > \hat{c}$ such that

$$f^D(c^g) = u(c^g) - u(\omega^g) + \beta\left(u(\omega^g + \omega^b + r - c^g) - u(\omega^b)\right) = 0.$$

Use this c^g to calculate the values of all of the variables in the definition of a symmetric steady state in this case.