Liquidity Constrained Markets versus Debt Constrained Markets

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Evidence favors incomplete asset markets over complete, frictionless Arrow-Debreu markets:

- There is not much correlation between individual and aggregate consumption

  Hayashi (1985), Zeldes (1989)

- The correlation between the trade balance and GDP is negative

  Backus, Kehoe, and Kydland (1992)

How can we model incomplete asset markets?

Two different models with a common environment:

- Models with liquidity constraints
  Bewley (1980)
  Townsend (1980)
  Scheinkman and Weiss (1986)
  Kehoe, Levine, and Woodford (1990)

- Models with debt constraints
  Schechtman and Escudero (1977)
  Manuelli (1986)
  Marcet and Marimon (1992)
  Kehoe and Levine (1993)
  Kocherlakota (1996)

another possibility would be

- Models with private information
  Prescott and Townsend (1984)
  Kehoe, Levine, and Prescott (2002)

Environment

continuum of consumers of two symmetric types

$$U(x_0^i, x_1^i \ldots) = (1-\delta)\sum_{t=0}^{\infty} \delta^t u(x_t^i)$$

$$(x_0^i, x_1^i \ldots) \in \ell_\infty^{++}$$

$$Du(x) > 0, \quad Du(x) \to \infty \text{ as } x \to 0$$
$$D^2u(x) < 0$$

two factors of production

- human capital (labor)
- physical capital (tree)

two levels of human capital endowment

$$w_t^i = \begin{cases} \omega^g \\ \omega^b \end{cases} \qquad \omega^b < \omega^g$$

in cyclic economy - alternate

in stochastic economy - π is the probability of reversal

one unit of physical capital in the economy - yields $r \geq 0$

θ_t^i the ownership share of physical capital

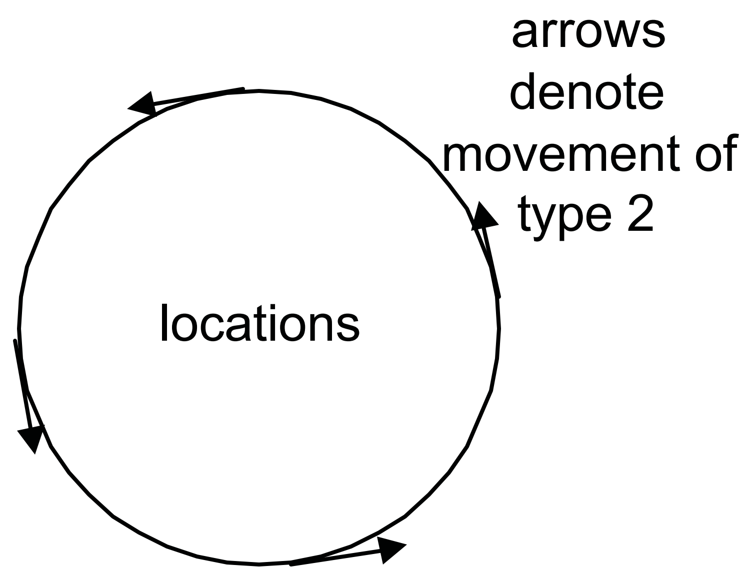
Goods market clearing

$x_t^1 + x_t^2 \leq \omega^g + \omega^b + r \ \ (= \omega)$

Capital market clearing

$\theta_t^1 + \theta_t^2 \leq 1$

Locational story (Townsend)



consumers of different types can only meet once

there is common knowledge

Two alternative economies

1. Economy with liquidity constraints

- max $(1-\delta)\sum_{t=0}^{\infty}\delta^t u(x_t^i)$
  subject to
$$x_t^i + v_t\theta_{t+1}^i \le w_t^i + (v_t + r)\theta_t^i$$
$$\theta_t^i \ge 0, \quad \theta_0^i \text{ given}$$
$$t = 0, 1, \ldots$$

- goods market clearing

- capital market clearing

equilibrium is $x,\ \theta,\ v$.

2. Economy with debt constraints

- max $(1-\delta)\sum_{t=0}^{\infty}\delta^t u(x_t^i)$
  subject to
  $$\sum_{t=0}^{\infty} p_t x_t^i \leq \sum_{t=0}^{\infty} p_t (w_t^i + r\theta_0^i)$$
  (Arrow-Debreu budget constraint)
  $$(1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(x_\tau^i) \geq (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}u(w_\tau^i)$$
  $$t = 0, 1, \ldots$$
  (individual rationality constraints)

- goods market clearing

equilibrium is x, p.

alternatively, we could impose sequential market budget constraints

$$x_t^i + v_t \theta_{t+1}^i \leq w_t^i + (v_t + r)\theta_t^i$$

$$\theta_t^i \geq -\Theta, \quad \theta_0^i \text{ given}$$

$$t = 0, 1, \ldots$$

Symmetric steady state

$$x_t^i = \begin{cases} x^g & \text{if } w_t^i = \omega^g \\ x^b & \text{if } w_t^i = \omega^b \end{cases}$$

v_t constant in liquidity constrained economy

p_t / p_{t-1} constant in debt constrained economy

$$f^D(x^g) = u(x^g) - u(\omega^g) + \delta\left(u(\omega - x^g) - u(\omega^b)\right)$$
$$f^L(x^g) = Du(x^g)(x^g - \omega^g) + \delta Du(\omega - x^g)(\omega - x^g - \omega^b)$$

Proposition: There exists a unique symmetric steady state x^g of the liquidity constrained economy. It is characterized by

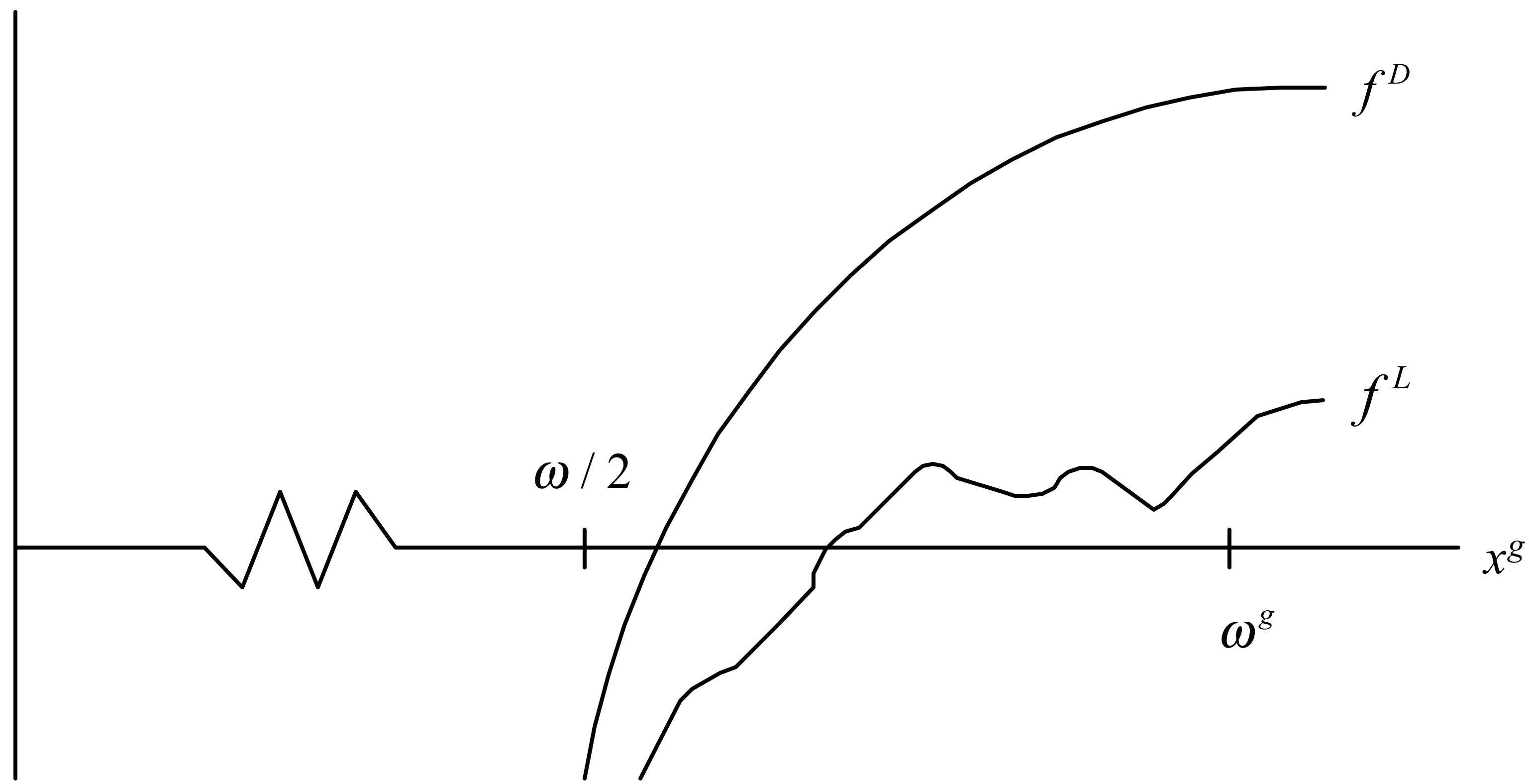
$$f^L(\omega/2) \geq 0 \text{ and } x^g = \omega/2 \text{ or}$$

$$\omega^g > \omega/2, f^L(x^g) = 0 \text{ and } x^g \in [\omega/2, \omega^g].$$

Proposition: There exists a symmetric steady state x^g of the debt constrained economy. It is characterized by

$$f^D(\omega/2) \geq 0 \text{ and } x^g = \omega/2 \text{ or}$$

$$\omega^g > \omega/2, f^D(x^g) = 0 \text{ and } x^g \in [\omega/2, \omega^g].$$



$$f^D(x^g) = u(x^g) - u(\omega^g) + \delta\left(u(\omega - x^g) - u(\omega^b)\right)$$

$$f^L(x^g) = Du(x^g)(x^g - \omega^g) + \delta Du(\omega - x^g)(\omega - x^g - \omega^b)$$

Other results

- An equilibrium in the debt constrained economy is efficient among allocations that satisfy the individual rationality constraints.

- If $r > 0$, then as $\delta \to 1$ the only symmetric steady state is $x^g = \omega / 2$ in both the liquidity constrained economy and in the debt constrained economy.

- If the liquidity constraints bind in the liquidity constrained economy of if the individual rationality constraints bind in the debt constrained economy, then the real interest rate is lower than in the corresponding complete markets economy.

$$i = \frac{Du(x^g)}{\delta Du(x^b)} - 1$$

Numerical Example

$$u(x) = \log x$$

$$\omega^g = 24, \quad \omega^b = 9, \quad r = 1, \quad \delta = 1/2$$

liquidity constrained economy:

$$f^L(x^g) = (x^g - 24)/x^g + \frac{1}{2}(25 - x^g)/(34 - x^g) = 0$$

$$x^g = 20.63, \quad x^b = 13.37.$$

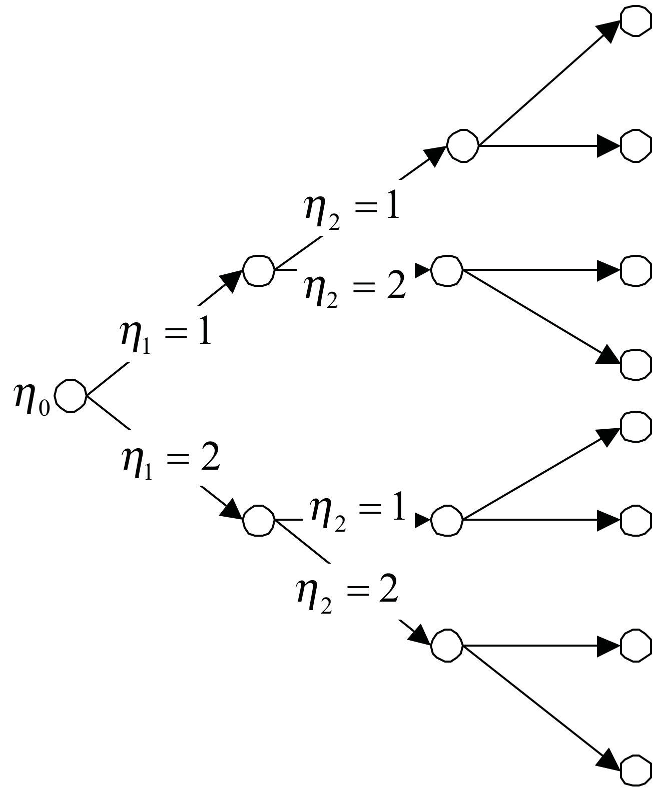
debt constrained economy:

$$f^D(x^g) = \log x^g - \log 24 + \frac{1}{2}\left(\log(34 - x^g) - \log 9\right) = 0$$

$$x^g = 18, \quad x^b = 16.$$

Stochastic model

Markov process $\pi = pr(\eta_t \neq i | \eta_{t-1} = i)$



state history

$$s = (\eta_0, \eta_1, \ldots, \eta_t)$$

$t(s) = t$ (length of vector s plus 1)

$$\pi_s = pr(\eta_{t(s)} | \eta_{t(s)-1}) pr(\eta_{t(s)-1} | \eta_{t(s)-2}) \cdots pr(\eta_1 | \eta_0)$$

$$U(x^i) = (1-\delta) \sum\nolimits_{s \in S} \delta^{t(s)} \pi_s u(x_s^i)$$

individual rationality constraint

$$(1-\delta) \sum\nolimits_{\sigma \geq s} \delta^{t(\sigma)} \pi_\sigma u(x_\sigma^i) \geq (1-\delta) \sum\nolimits_{\sigma \geq s} \delta^{t(\sigma)} \pi_\sigma u(w^i(\eta_\sigma)$$

symmetric stochastic steady state

$$x_s^i = \begin{cases} x^g & \text{if } w^i(\eta_s) = \omega^g \\ x^b & \text{if } w^i(\eta_s) = \omega^b \end{cases}$$

budget constraints in the liquidity constrained economy

$$x_s^i + v_s \theta_{(s,\eta)}^i \le w_s^i + (v_s + r)\theta_s^i$$

$$\theta_s^i \ge 0, \quad \theta_0^i \text{ fixed}$$

Arrow-Debreu budget constraint in the debt constrained economy

$$\sum_{s \in S} p_s x_s^i \le \sum_{s \in S} p_s (w_s^i + \theta_0^i r)$$

sequential markets budget constraints in the debt constrained economy

$$x_s^i + q_{(s,1)} \theta_{(s,1)}^i + q_{(s,2)} \theta_{(s,2)}^i \le w_s^i + (v_s + r)\theta_s^i$$

$$\theta_s^i \ge -\Theta, \quad \theta_0^i \text{ fixed}$$

$$f^D(x^g) = \left(1-\delta(1-\pi)\right)\left(u(x^g)-u(\omega^g)\right)+\delta\pi\left(u(\omega-x^g)-u(\omega^b)\right).$$

Proposition: There exists a unique symmetric stochastic steady state x^g of the debt constrained economy. It is characterized by

$$f^D(\omega/2)\geq 0 \text{ and } x^g = \omega/2 \text{ or}$$

$$\omega^g > \omega/2, f^D(x^g)=0 \text{ and } x^g \in[\omega/2,\omega^g].$$

Proposition: If $0<\pi<1$ there is no symmetric stochastic steady state with liquidity constraints.

In the stochastic economy with liquidity constraints consumption needs to depend on the holdings of physical capital θ_s^i as well as on the event η_s. In general, it is very difficult to compute equilibria.

Numerical Example

$u(x) = \log x$,

$\omega^g = 24, \quad \omega^b = 9, \quad r = 1, \quad \delta = 1/2$

$\pi = 1/2$

debt constrained economy

$$f^D(x^g) = \frac{3}{4}(\log x^g - \log 24) + \frac{1}{4}\left(\log(34 - x^g) - \log 9\right) = 0$$

$x^g = 21.52, \quad x^b = 12.48$

Dynamic analysis of debt constrained economy

binding case: individual rationality constraint binds at the symmetric first best

non-binding case: individual rationality constraint does not bind at the symmetric first best

initial phase: until the two consumer types exchange roles

Proposition: There is a unique dynamic equilibrium of the debt constrained model. During the initial phase, equilibrium consumption is constant. In the binding case, following the initial phase, consumption follows the symmetric steady state. In the non-binding case, following the initial phase, consumption is constant, although possibly different than during the initial phase.

Applications of debt constrained models

Alvarez and Jermann (2000) show that a consumption/asset accumulation model with debt constraints can solve the equity premium puzzle. A model with liquidity constraints cannot.

Kehoe and Perri (1998) show that an international real business cycle model with debt constraints can solve the quantity anomaly. A model with liquidity constraints cannot.

Krueger and Perri (2000) show that a model with debt constraints can explain why the inequality in the distribution of consumption has fallen or stayed constant even as income inequality has risen in the United States over the past two decades. A model with liquidity constraints cannot.