Gambling for Redemption and Self-Fulfilling Debt Crises

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Motivation

Sovereign debt crisis have recently occurred in Greece, Ireland, and Portugal.

Crises threaten Spain and Italy.

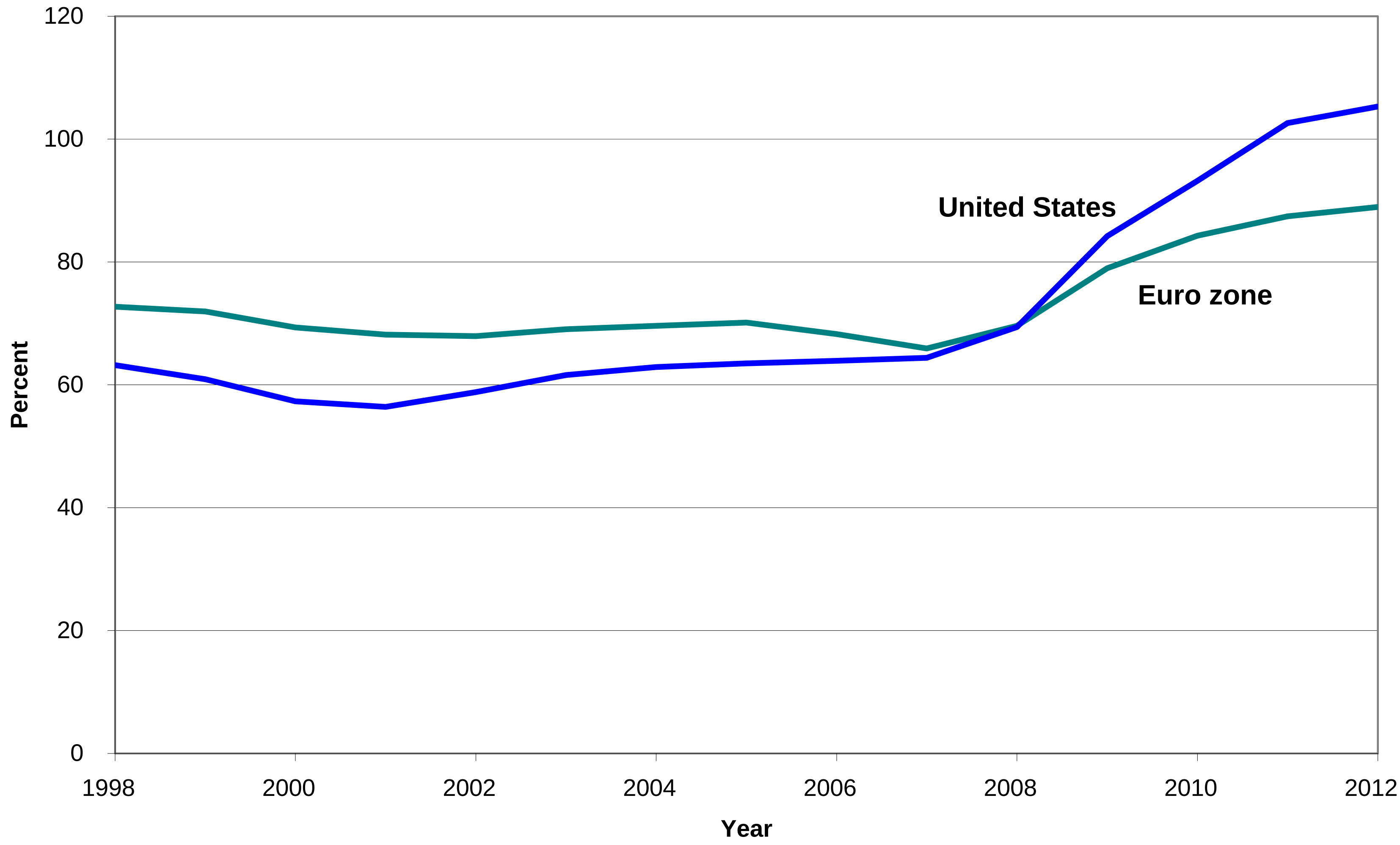
PIIGS — Portugal, Ireland, Italy, Greece, and Spain.

The timing of the crises and its relation to fundamentals seems arbitrary.

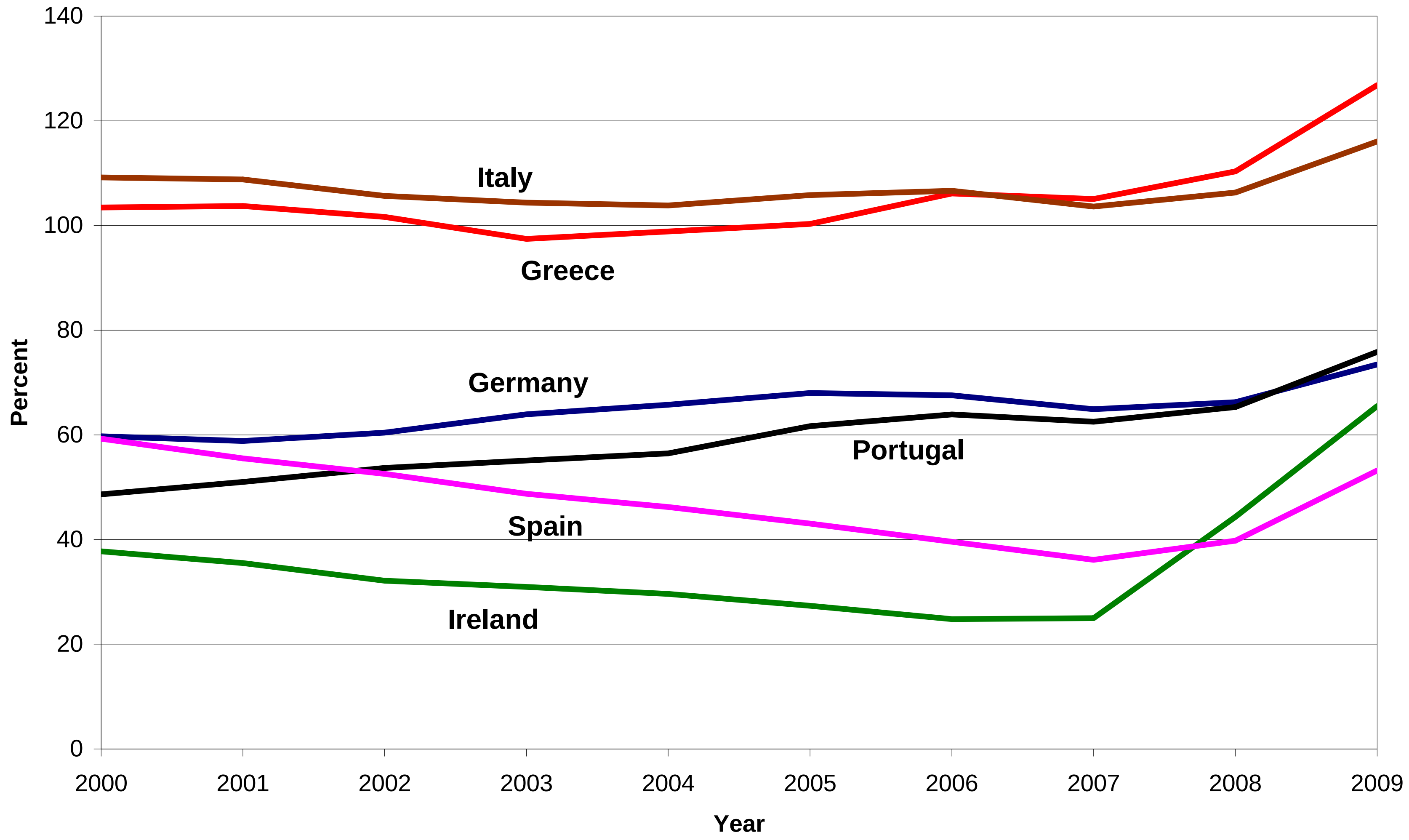
Can we model these crises as self-fulfilling in the sense that some researchers modeled the Mexican crises of 1994–95?

Cole and Kehoe (1996, 2000)

Debt to GDP ratio in the United States and the Euro zone



Debt to GDP ratio in select European countries



Question

Why did the European Union's loan package for Greece not end the crisis there in the same way that U.S. President Bill Clinton's loan package ended the 1994–95 Mexican crisis?

Answer in Chamley-Pinto (2011): The Greek crisis is a solvency crisis, while the Mexican crisis was a liquidity crisis.

Our answer: The Clinton loan package motivated the Mexican government to run down its debt. The EU loan package has motivated the Greek government to increase its debt. The Greek government is trying to smooth government expenditures as it awaits a recovery of fiscal revenues.

Gambling for redemption — a martingale gambling strategy.

Additional question

We also provide a more satisfactory answer to a question raised by the analysis of Cole and Kehoe (1996, 2000):

Why would a government rationally increase its debt even though this increased debt would make it susceptible to a self-fulfilling crisis?

Answer in Cole-Kehoe (1996): The government is less patient than other agents.

Our answer: The government is rationally gambling for redemption.

Crucial observation and mechanism

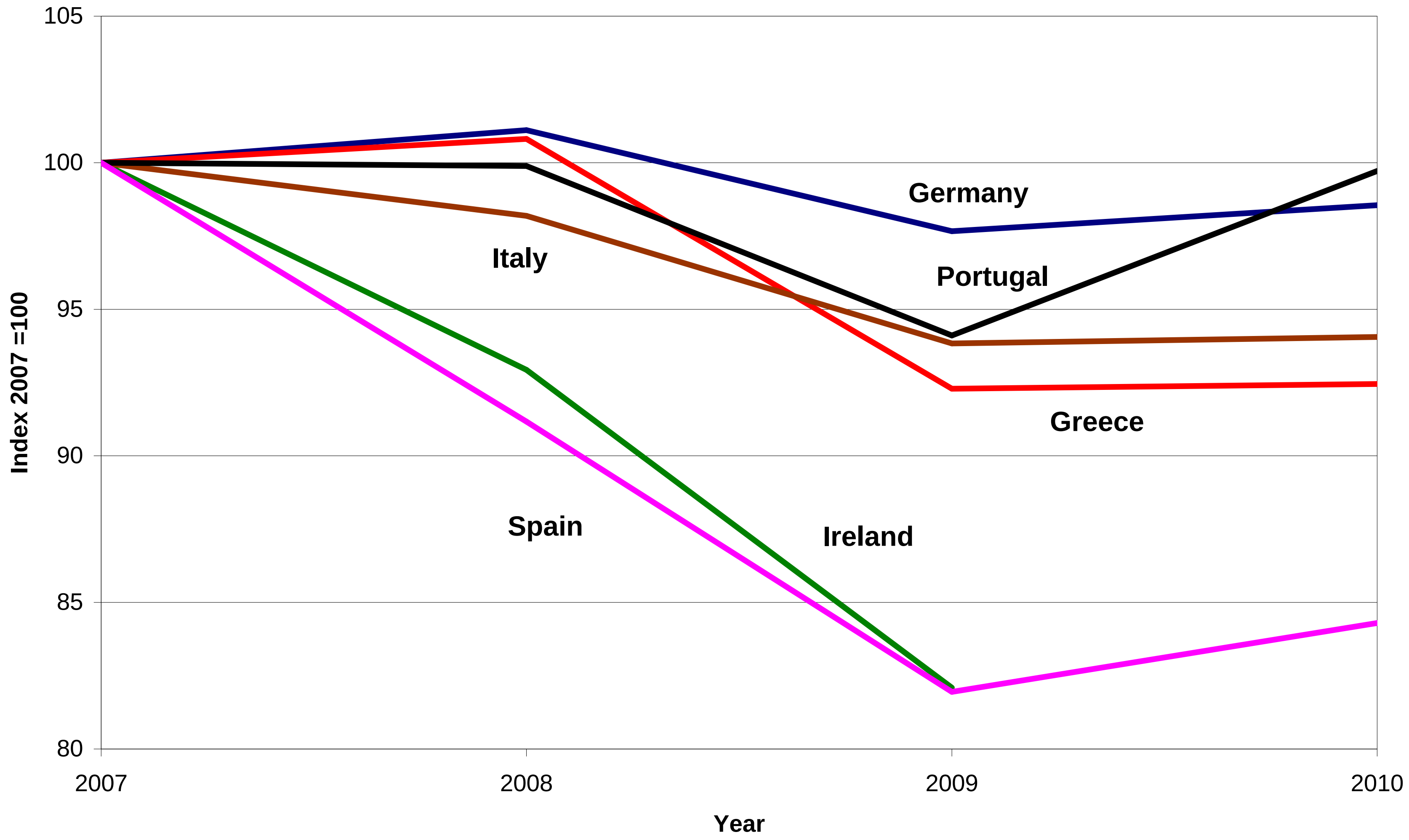
Recent recession (2007–09 in United States but ongoing in some European countries) has resulted in large drops in fiscal revenues.

Faced with low fiscal revenues and a large debt, the government has two conflicting incentives optimal debt policy:

Cut government spending rapidly and run down debt to avoid a financial crisis.

Cut government spending slowly and run up debt, gambling for a recovery of fiscal revenues.

Real tax revenues in select European countries



General model

State of the economy in every period $s = (B, a, z_{-1}, \zeta)$

B: government debt

a: condition of private sector, $a = 1$ normal, $a = 0$ recession

z_{-1}: whether or not default has occurred: $z_{-1} = 1$ no, $z_{-1} = 0$ yes

ζ: value of the sunspot variable

GDP

$$y(a, z) = A^{1-a} Z^{1-z} \bar{y}$$

where $1 > A, Z > 0$.

Before period 0, $a = 1$, $z = 1$.

Period 0, a unexpectedly becomes $a_0 = 0$

GDP drops from $y = \overline{y}$ to $y = A\overline{y} < \overline{y}$.

In every period t, $t = 1, 2, ...,$ a_t becomes 1 with probability p, $1 > p > 0$.

Once $a_t = 1$, it stays equal to 1 forever.

Drop in GDP by factor Z is default penalty. Once $z_t = 0$, it stays equal to 0 forever.

Government tax revenue is $\theta y(a,z)$.

To keep things simple, assume that the tax rate θ is fixed.

Government's problem is to choose c, g, B', z to solve

$$V(s;p,\pi) = \max u(c,g) + \beta EV(s';p,\pi)$$

$$\text{s.t. } c = (1-\theta)y(a,z)$$

$$g + zB = \theta y(a,z) + q(B',s;p,\pi)B'$$

$$z = 0 \text{ if } z_{-1} = 0.$$

Here $z = 1$ is the decision not to default, and $z = 0$ is the decision to default.

Some possibilities for $u(c, g)$ are

$$u(c,g) = (1-\gamma)\log c + \gamma \log g$$

$$u(c,g) = (1-\gamma)\log c + \gamma \log(g - \overline{g})$$

$$u(c,g) = \log(c + g - \overline{c} - \overline{g}),$$

or even more curvature than that of natural logarithm.

Sunspot

$\zeta_t \sim U[0,1]$

If $\zeta_t > 1-\pi$, bankers expect there to be a crisis and do not lend to the government if such a crisis would be self-fulfilling.

Probability of a self-fulfilling crisis π is arbitrary, $1 \geq \pi \geq 0$, if the level of debt is high enough for such a crisis to be possible.

Timing within each period:

1. ζ_t is realized, $s_t = (B_t, a_t, z_{t-1}, \zeta_t)$, and government chooses B_{t+1}.
2. Each bankers chooses b_{t+1}. (In equilibrium, $b_{t+1} = B_{t+1}$.)
3. Government chooses default decision z_t, which determines y_t, c_t, and g_t.

Notes:

The equilibrium is perfect foresight —bankers do not lend if they know the government will default.

Bond price depends on B_{t+1}, crisis depends on B_t and ζ_t.

International bankers

$$W(b,B',s;p,\pi)=\max x+\beta EW((b',B'',s';p,\pi)$$

$$x+q(B',s;p,\pi)b'=w+z(B',s,q;p,\pi)b$$

$$x\geq 0,\ b\leq A.$$

$b\leq A$ eliminates Ponzi scheme's but A is large enough to not otherwise bind.

Endowment of consumption good w is large enough to rule out corner solutions in equilibrium.

First order condition and perfect foresight condition:

$$q(B',s;p,\pi)=\beta Ez(B',s,q(B',s;p,\pi);p,\pi).$$

Recursive equilibrium

Value function for government $V(s)$ and policy functions $B'(s)$ and $z(B',s,q;p,\pi)$ and $g(B',s,q;p,\pi)$,

value function for bankers $W(b,B',s;p,\pi)$

bond price function $q(B',s;p,\pi)$.

Bankers' problem:

$$q(B',s;p,\pi)=\beta Ez(B',s,q(B',s;p,\pi);p,\pi).$$

Government's problem at the beginning of the period: Choose B' to solve

$$V(B,a,z_{-1},\zeta;p,\pi)=\max u(c,g)+\beta EV(B',a',z,\zeta';p,\pi)$$

$$\text{s.t. } c=(1-\theta)y(a,z(B',s,q(B',s;p,\pi);p,\pi))$$

$$g(B',s,q(B',s;p,\pi);p,\pi)+z(B',s,q(B',s;p,\pi);p,\pi)B$$
$$=\theta y(a,z)+q(B',s;p,\pi)B'$$

Government's problem at the end of the period: Choose z and g to solve

$$\max u(c,g)+\beta EV(B',a',z,\zeta';p,\pi)$$

$$\text{s.t. } c=(1-\theta)y(a,z)$$

$$g+zB=\theta y(a,z)+qB'$$

$$z=0 \text{ or } z=1$$

$$z=0 \text{ if } z_{-1}=0.$$

Four cutoff levels of debt: $\bar{b}(a; p, \pi)$, $\bar{B}(a; p, \pi)$, $a = 0,1$:

If $B \leq \bar{b}(a; p, \pi)$, government repays even if bankers do not lend, it defaults if $B > \bar{b}(a; p, \pi)$.

If $B \leq \bar{B}(a; p, \pi)$, government repays if bankers lend, it defaults if $B > \bar{B}(a; p, \pi)$.

Assumption that a government is permanently excluded from borrowing after default.

Once default has occurred, bankers do not lend:

$$q(B',(B,a,0,\zeta);p,\pi)=0.$$

During a crisis, bankers do not lend: If $B>\bar{b}(a;p,\pi)$ and $\zeta>1-\pi$,

$$q(B',(B,a,1,\zeta);p,\pi)=0$$

This is how q depends on (B,a,z_{-1},ζ) and the (perfect foresight) expectations of z.

Otherwise, q depends on B' and the expectations of a' and z'.

$\bar{b}(0;p,\pi)<\bar{b}(1;p,\pi)$, $\bar{b}(0;p,\pi)<\bar{B}(0;p,\pi)$, $\bar{b}(1;p,\pi)<\bar{B}(1;p,\pi)$, and $\bar{B}(0;p,\pi)<\bar{B}(1;p,\pi)$.

More interesting case:

$$\bar{b}(0;p,\pi)<\bar{b}(1;p,\pi)<\bar{B}(0;p,\pi)<\bar{B}(1;p,\pi).$$

Other cases:

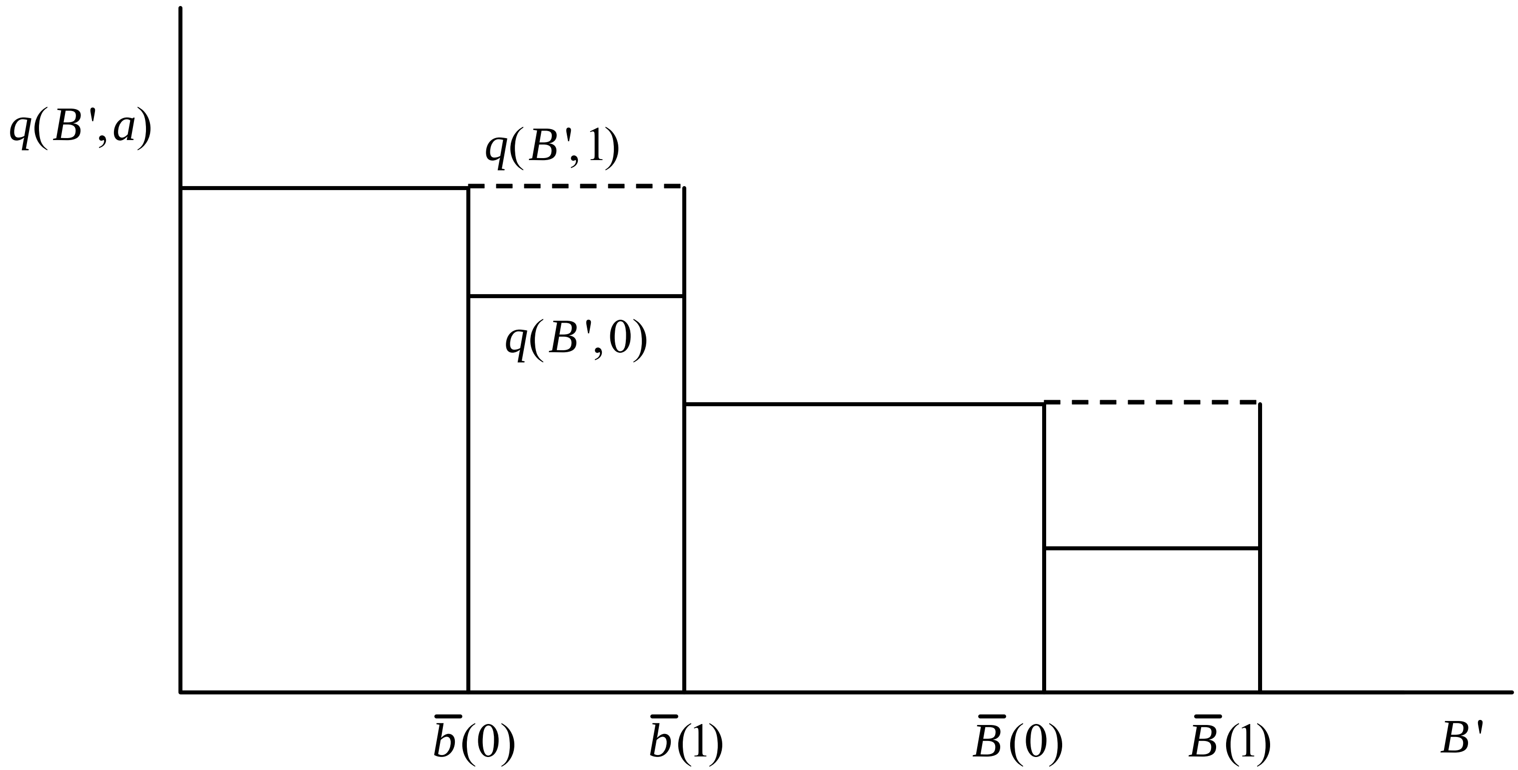
$$\bar{b}(0;p,\pi)<\bar{B}(0;p,\pi)<\bar{b}(1;p,\pi)<\bar{B}(1;p,\pi)$$

$$\bar{b}(0;p,\pi)<\bar{B}(0;p,\pi)=\bar{b}(1;p,\pi)<\bar{B}(1;p,\pi).$$

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B'\le \bar{b}(0;p,\pi)\\ \beta\left(p+(1-p)(1-\pi)\right) & \text{if } \bar{b}(0;p,\pi)<B'\le \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi)<B'\le \bar{B}(0;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{B}(0;p,\pi)<B'\le \bar{B}(1;p,\pi)\\ 0 & \text{if } \bar{B}(1;p,\pi)<B'\end{cases}$$

$$q(B',(B,1,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B'\le \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi)<B'\le \bar{B}(1;p,\pi)\\ 0 & \text{if } \bar{B}(1;p,\pi)<B'\end{cases}$$

Bond prices as a function of debt and conditions in the private sector



In case where

$$\overline{b}(0;p,\pi) < \overline{B}(0;p,\pi) < \overline{b}(1;p,\pi) < \overline{B}(1;p,\pi),$$

$$q(B',(B,0,1,\zeta);p,\pi) = \begin{cases} \beta & \text{if } B' \leq \overline{b}(0;p,\pi) \\ \beta\left(p+(1-p)(1-\pi)\right) & \text{if } \overline{b}(0;p,\pi) < B' \leq \overline{B}(0;p,\pi) \\ \beta p & \text{if } \overline{B}(0;p,\pi) < B' \leq \overline{b}(1;p,\pi) \\ \beta p(1-\pi) & \text{if } \overline{b}(1;p,\pi) < B' \leq \overline{B}(1;p,\pi) \\ 0 & \text{if } \overline{B}(1;p,\pi) < B' \end{cases}$$

Self-fulfilling liquidity crises

Cole-Kehoe (1996, 2000) without private sector capital.

Also limiting case where $a = 0$ and $p = 0$: Replace $\bar{y}$ with $A\bar{y}$.

Self-fulfilling crises are possible, but no incentive for gambling for redemption

Start by assuming that $\pi = 0$. When $s = (B, a, z_{-1}, \zeta) = (B,1,1,\zeta)$,

$$V(B,1,1,\zeta; p,0) = \frac{u((1-\theta)\overline{y}, \theta\overline{y} - (1-\beta)B)}{1-\beta}.$$

When default has occurred, $s = (B, a, z_{-1}, \zeta) = (B,1,0,\zeta)$,

$$V(B,1,0,\zeta; p,0) = \frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{1-\beta},$$

$\overline{b}(1;p,0)$:

The utility of repaying even if bankers do not lend is

$$V_n(B,1,0;p,0) = u((1-\theta)\overline{y},\theta\overline{y}-B) + \frac{\beta u((1-\theta)\overline{y},\theta\overline{y})}{1-\beta}.$$

The utility of defaulting if bankers do not lend is

$$V_d(B,1,0;p,0) = \frac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}.$$

$\overline{b}(1;p,0)$ is determined by

$$V_n(\overline{b}(1;p,0),1,0;p,0) = V_d(\overline{b}(1;p,0),1,0;p,0)$$

$$u((1-\theta)\overline{y},\theta\overline{y}-\overline{b}(1;p,0)) + \frac{\beta u((1-\theta)\overline{y},\theta\overline{y})}{1-\beta} = \frac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}$$

We can similarly determine $\bar{B}(1; p, 0)$.

Now let $\pi > 0$. Observe that

$$\bar{b}(1; p, \pi) = \bar{b}(1; p, 0).$$

Suppose that $B_0 > \bar{b}(1; p, \pi)$ and the government decides to reduce B to $\bar{b}(1; p, \pi)$ in T periods, $T = 1, 2, ..., \infty$. First order conditions imply

$$g_t = g^T(B_0; \pi).$$

The government's budget constraints are

$$g^T(B_0;\pi) + B_0 = \theta\overline{y} + \beta(1-\pi)B_1$$

$$g^T(B_0;\pi) + B_1 = \theta\overline{y} + \beta(1-\pi)B_2$$

$$\vdots$$

$$g^T(B_0;\pi) + B_{T-2} = \theta\overline{y} + \beta(1-\pi)B_{T-1}$$

$$g^T(B_0;\pi) + B_{T-1} = \theta\overline{y} + \beta\overline{b}(1;p,\pi).$$

Multiply each equation by $(\beta(1-\pi))^t$ and adding, we obtain

$$\sum_{t=0}^{T-1}(\beta(1-\pi))^t g^T(B_0;\pi) + B_0 = \sum_{t=0}^{T-1}(\beta(1-\pi))^t\theta\overline{y} + (\beta(1-\pi))^{T-1}\beta\overline{b}(1;p,\pi)$$

$$g^{T}(B_0;\pi)=\theta\overline{y}-\frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^{T}}\Big(B_0-(\beta(1-\pi))^{T-1}\beta\overline{b}(1;p,\pi)\Big).$$

Notice that

$$g^{\infty}(B_0;\pi)=\lim_{T\to\infty} g^{T}(B_0;\pi)=\theta\overline{y}-(1-\beta(1-\pi))B_0.$$

Compute $V^{T}(B_0;\pi)$:

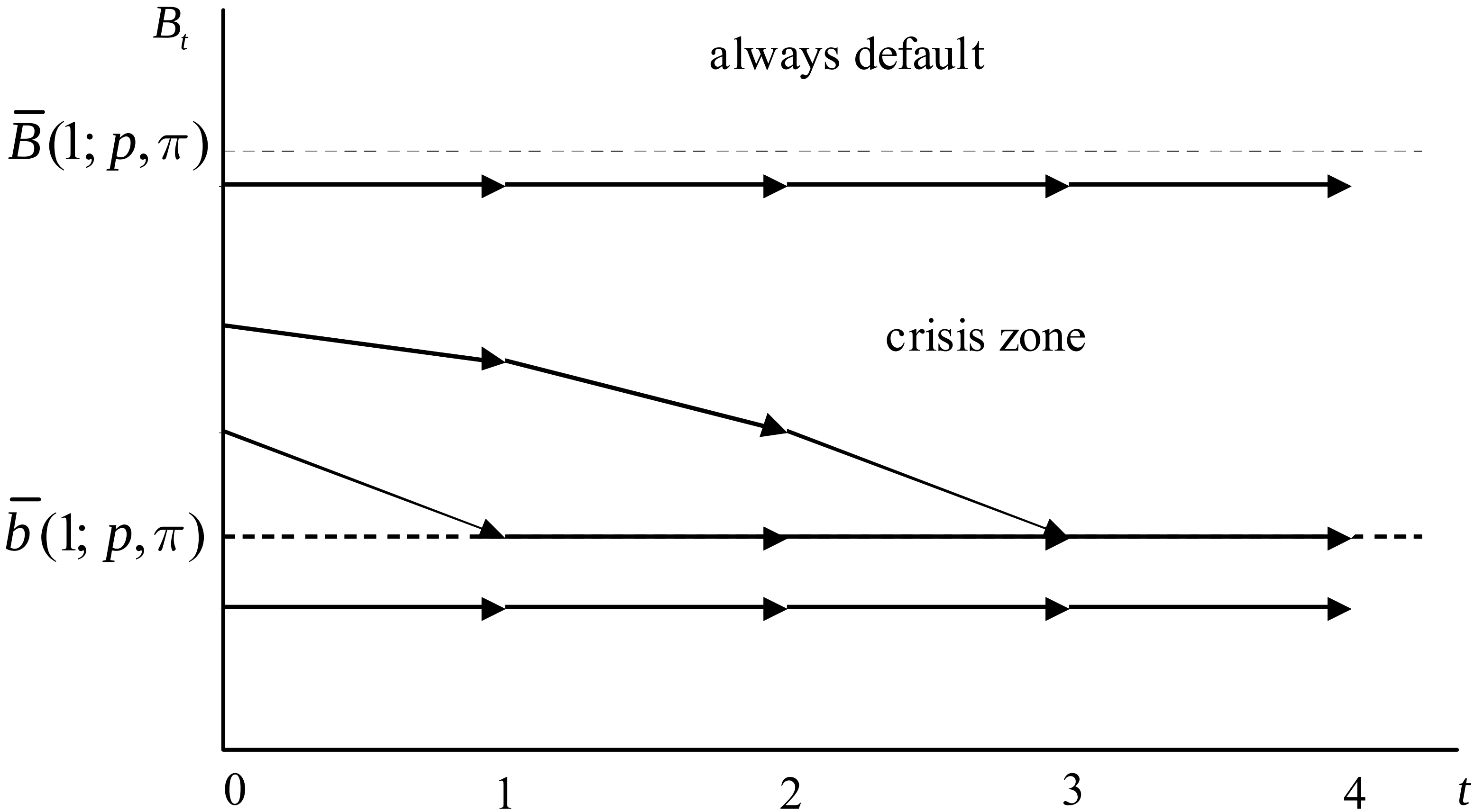
$$\begin{aligned}V^{T}(B_0;\pi)=&\frac{1-(\beta(1-\pi))^{T}}{1+\beta(1-\pi)}u((1-\theta)\overline{y},g^{T}(B_0;\pi))\\&+\frac{1-(\beta(1-\pi))^{T-1}}{1+\beta(1-\pi)}\frac{\beta\pi u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}\\&+(\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\overline{y},\theta\overline{y})}{1-\beta}\end{aligned}$$

To find $\overline{B}(1;p,\pi)$, we solve

$$\max\left[V^{1}(\overline{B}(1;p,\pi)),V^{2}(\overline{B}(1;p,\pi)),\ldots,V^{\infty}(\overline{B}(1;p,\pi);\pi)\right]$$
$$=u((1-\theta)Z\overline{y},\theta Z\overline{y}+\beta(1-\pi)\overline{B}(1;p,\pi)))+\frac{\beta u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}\cdot$$

$$V(B,1,1,\zeta;p,\pi)=\begin{cases}\dfrac{u((1-\theta)\overline{y},Z\overline{y})}{1-\beta} & \text{if } B\le\overline{b}(1;p,\pi)\\ \max\left[V^{1}(B;\pi),V^{2}(B;\pi),\ldots,V^{\infty}(B;\pi)\right] & \text{fi}\,\overline{b}(1;p,\pi)<B\le\overline{B}(1;p,\pi),\ \zeta\le 1-\pi\\ \dfrac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta} & \text{if } \overline{b}(1;p,\pi)<B\le\overline{B}(1;p,\pi),\ 1-\pi<\zeta\\ \dfrac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta} & \text{if } \overline{B}(1;p,\pi)<B\end{cases}$$

Optimal debt policy with self-fulfilling crises

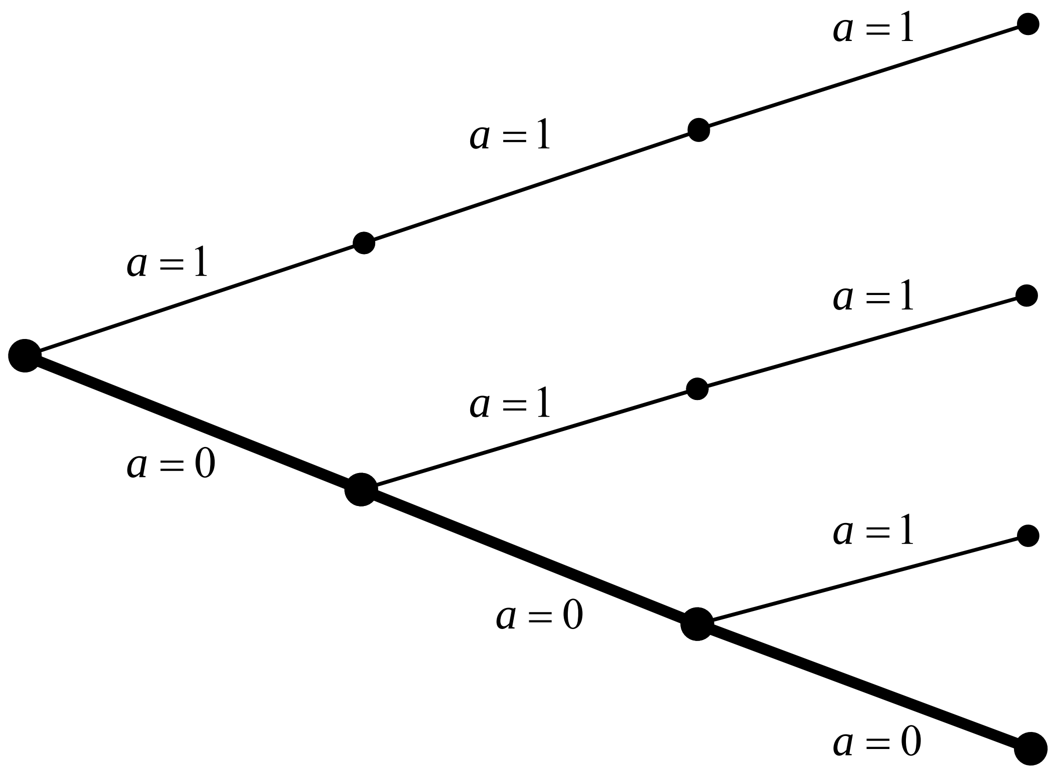


Gambling for redemption without self-fulfilling crises

$a = 0$ and $\pi = 0$.

no self-fulfilling crises are possible, but the private sector is in a recession and faces the possibility p, $1 > p > 0$, of recovering in every period.

Uncertainty tree with recession path highlighted

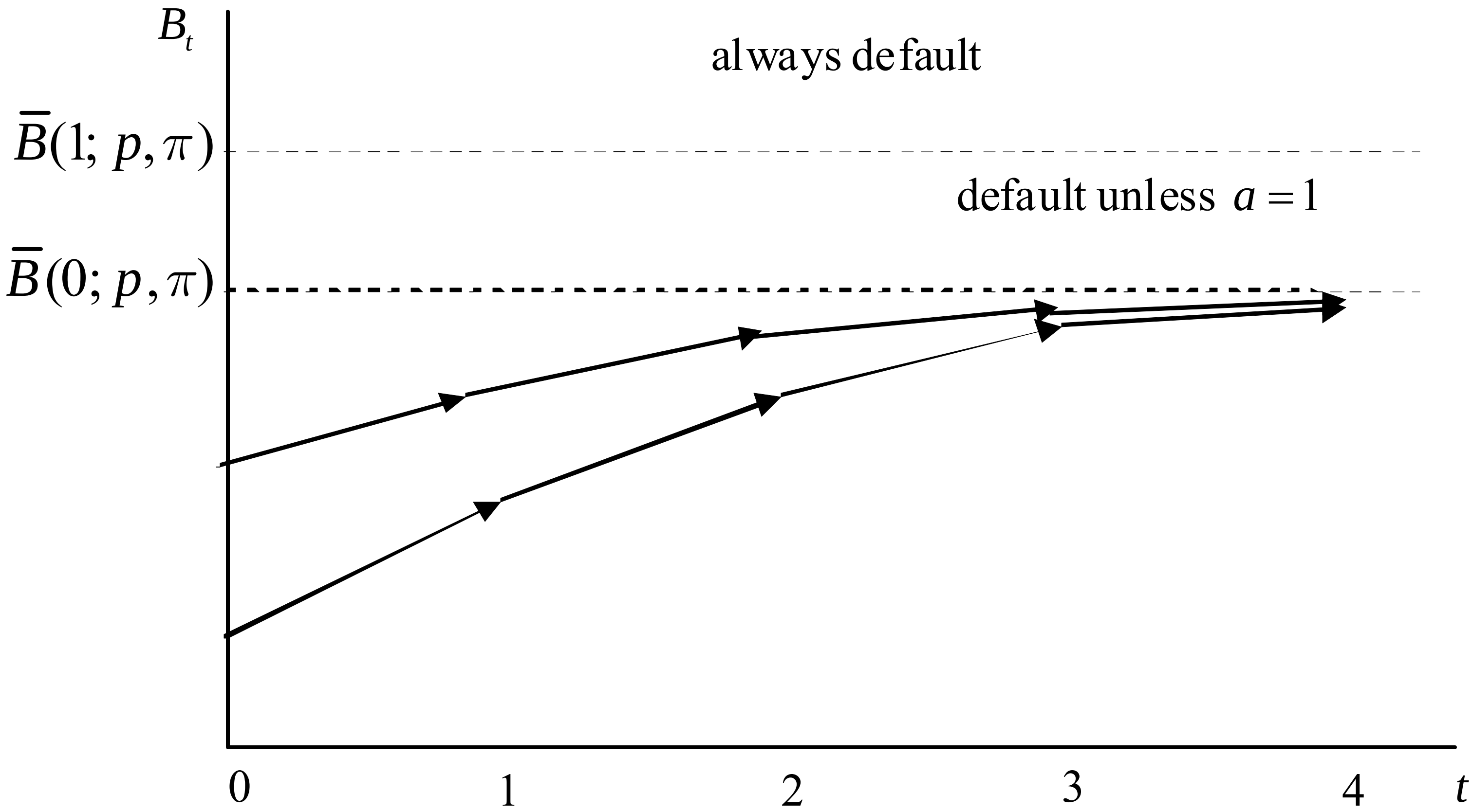


There are two cases:

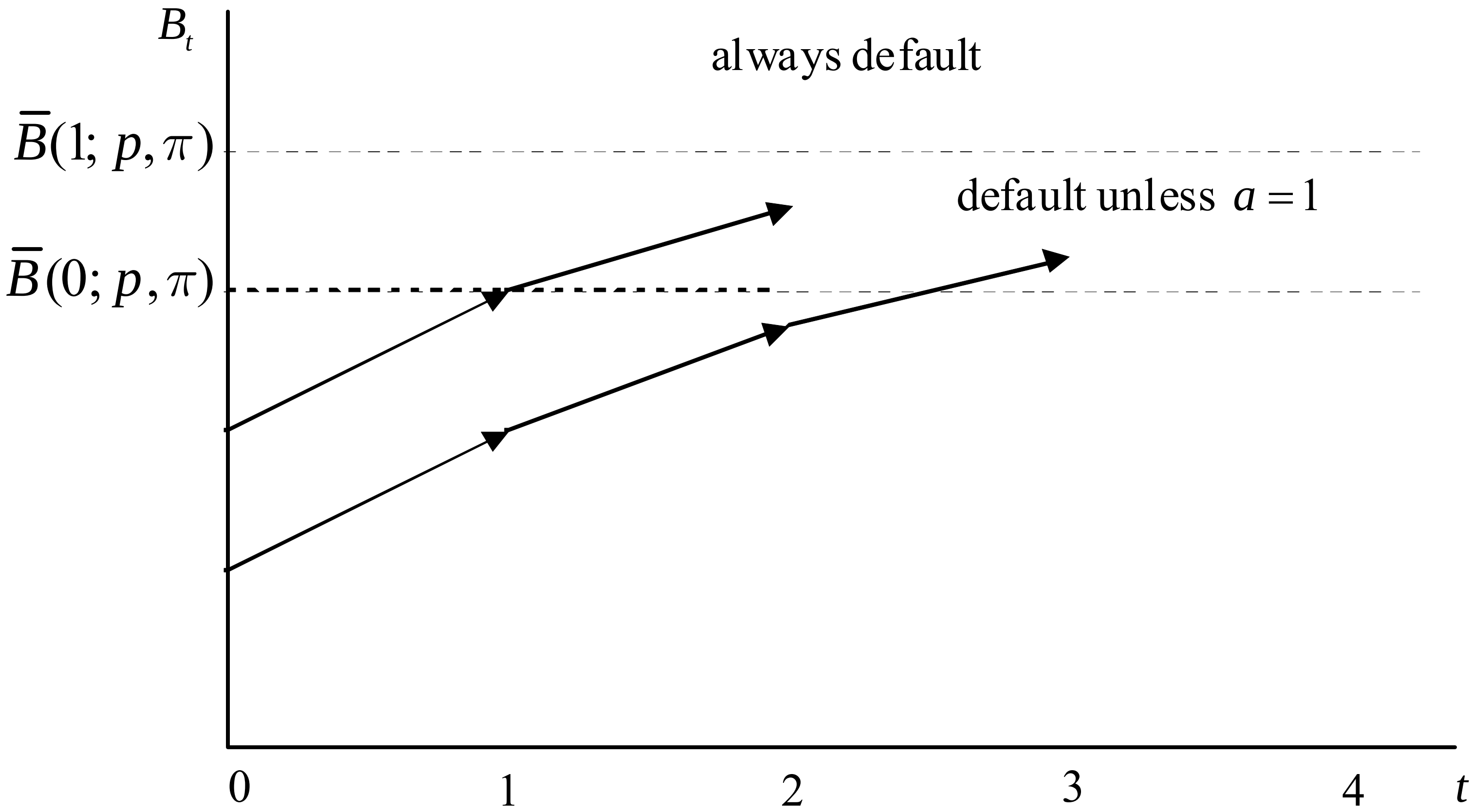
1. The government chooses to never violate the constraint $B \leq \overline{B}(0; p, 0)$, and the optimal debt converges to $\overline{B}(0; p, 0)$ if $a = 0$ sufficiently long.

2. The government chooses to default in T periods if $a = 0$ sufficiently long.

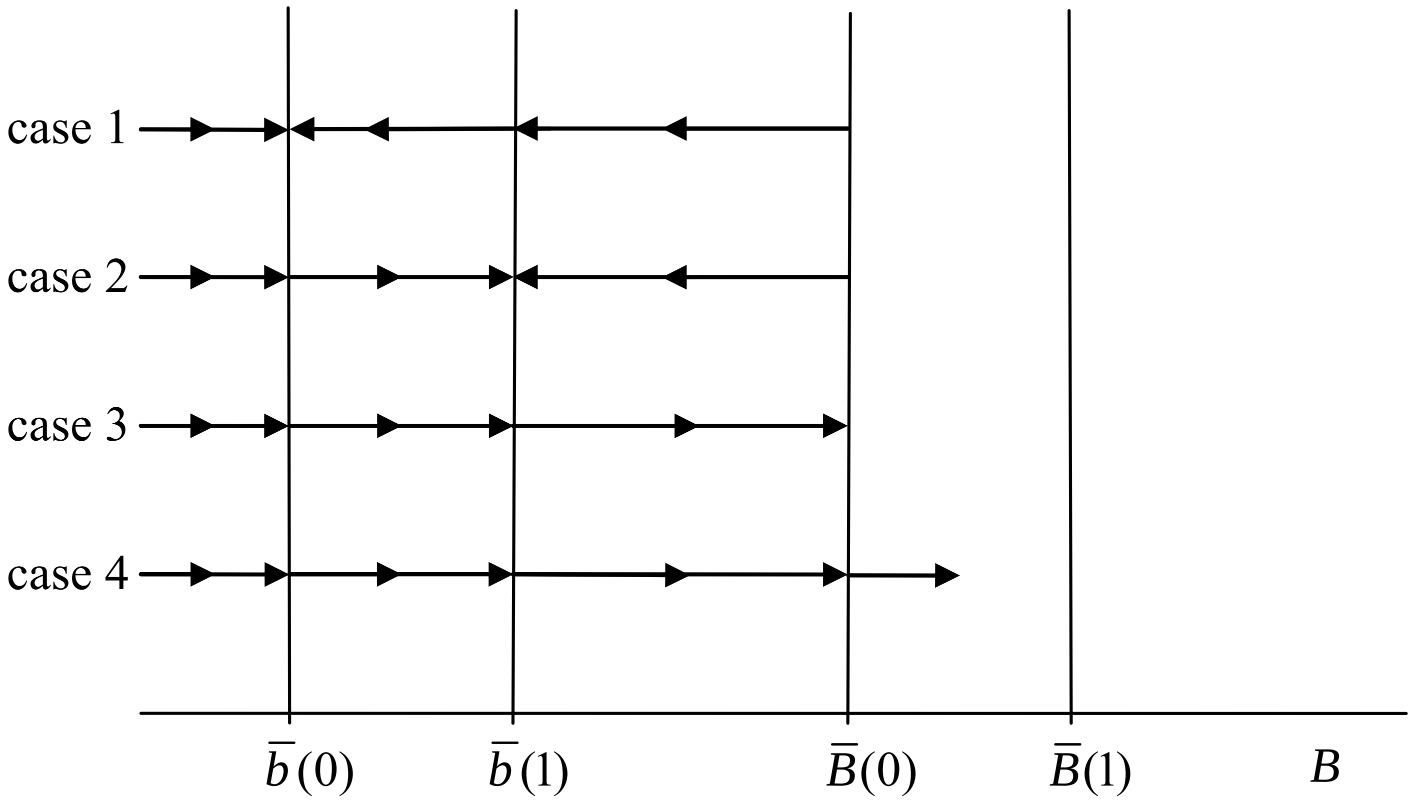
Equilibrium with no default



Equilibrium with eventual default



Some possible phase diagrams in general model



Suppose we are in case 1. Then, if realization of the sunspot variable signals that a crisis will take place that period, the provision of a loan from a third party an interest rate higher than

$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

can prevent a crisis but leave the government in case 1.

Suppose that we are in case 1 or case 2. Then, the provision of a loan from a third party an interest rate lower than

$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

can push the government into case 3 or 4.

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U.S. President Bill Clinton's 1995 loan package for Mexico

Suppose that we are in case 1 or case 2. Then, the provision of a loan from a third party at an interest rate lower than

$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

can push the government into case 3 or 4.

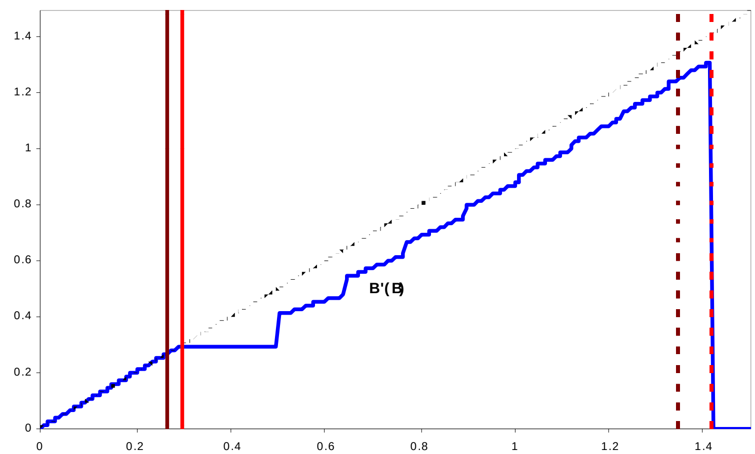
European Union's 2010 rescue package for Greece

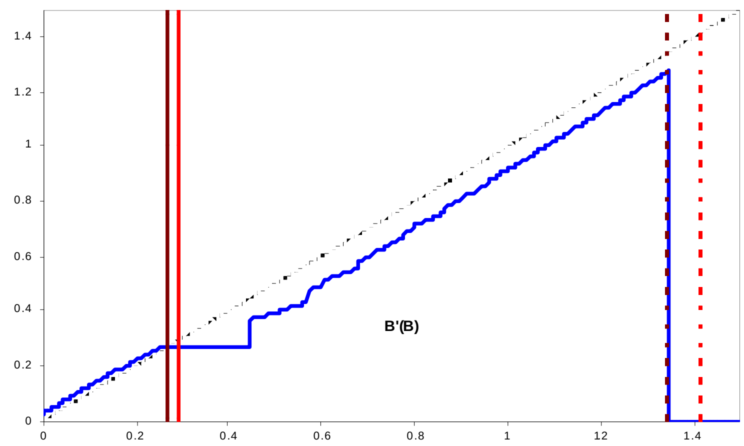
Calibrated model

$$u(c,g) = (1-\gamma)\log c + \gamma \log g$$

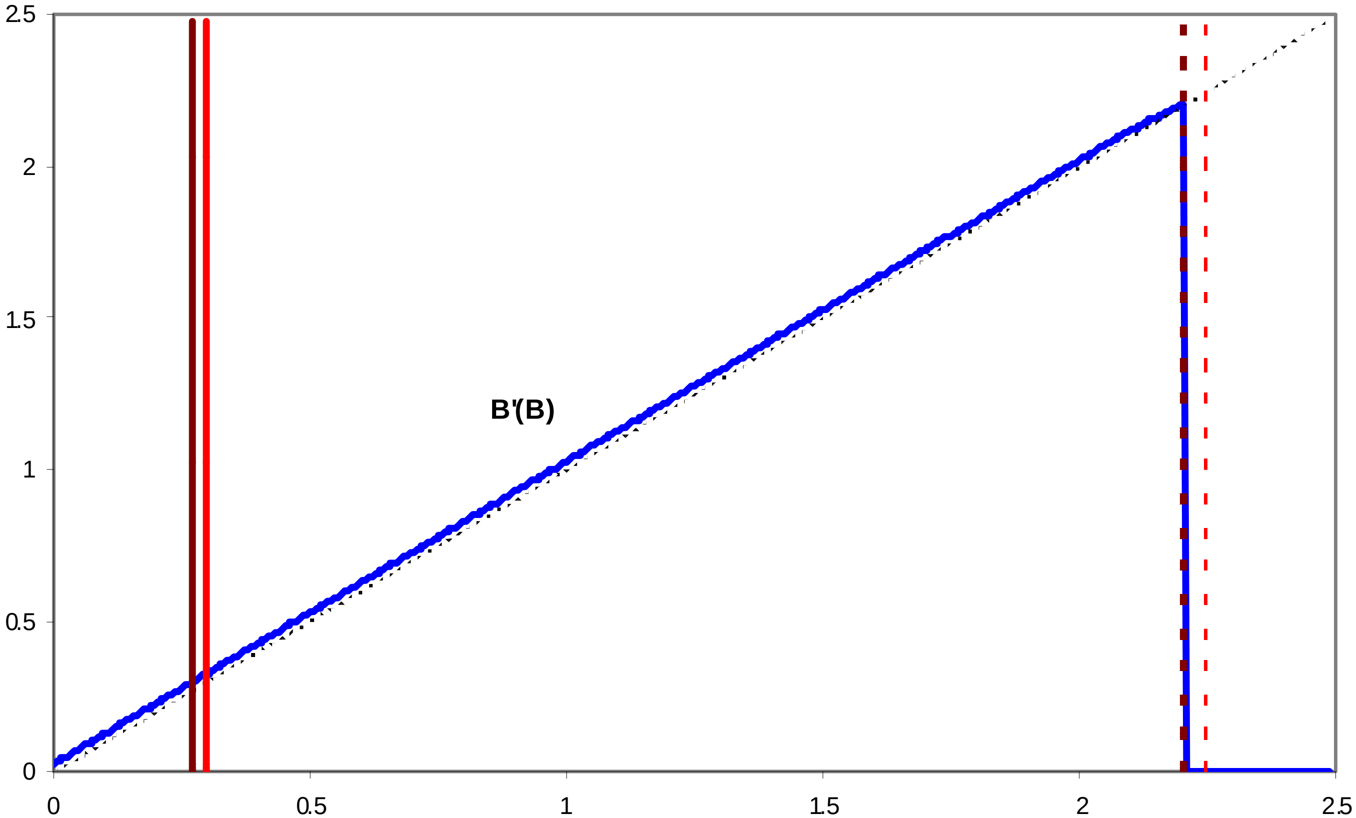
Parameter	Value	Target
A	0.90	Average government revenue loss
Z	0.95	Cole and Kehoe (1996)
p	0.20	Average recovery in 5 years
β	0.98	Real interest rate of safe bonds 2%
π	0.03	Real interest rate in crisis zone 5%
γ	0.25	Consumers value c 3 times more than g
θ	0.30	Government revenues as a share of output

Policy functions in good (left panel) and bad (right) times





Policy function in bad times for small π



Maturity of debt

Maturity structure makes a difference, not just average maturity!

Suppose that every period, the government sells the 310 of bonds, divided between 300 1 year bonds and 10 30 year bonds. Then the government has total debt of

$$300 + (30)10 = 600$$

Notice that the average maturity is

$$\frac{300(30+29+...+1)/30+300}{600} = \frac{150(30)+300}{600} = \frac{15(3)+3}{6} = 8.$$

Every period the fraction of debt that becomes due is

$$\frac{310}{600} = 0.5167.$$

Suppose, in contrast, the government sells 40 15 year bonds every period. Then the government has debt of

$$(15)40 = 600,$$

and the average maturity is

$$\frac{15 + 14 + ... + 1}{2} = 8,$$

but every period the fraction of debt that becomes due is

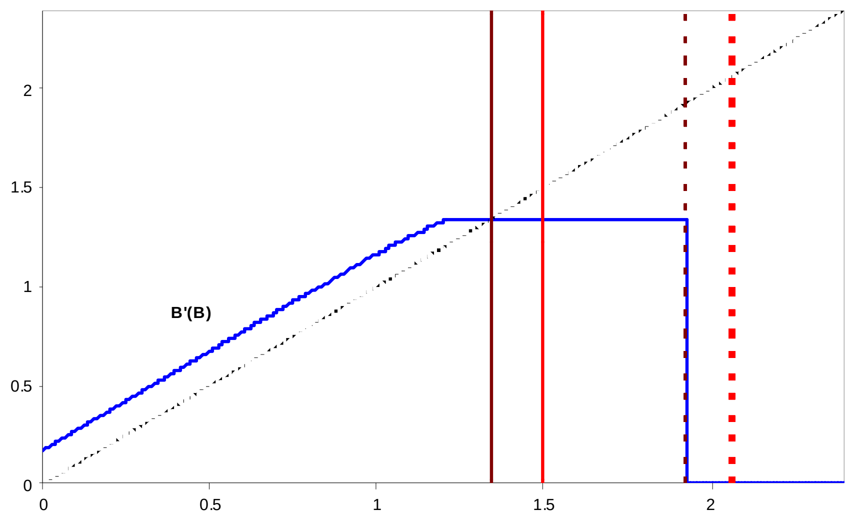
$$\frac{40}{600} = 0.0667.$$

Maturity of debt

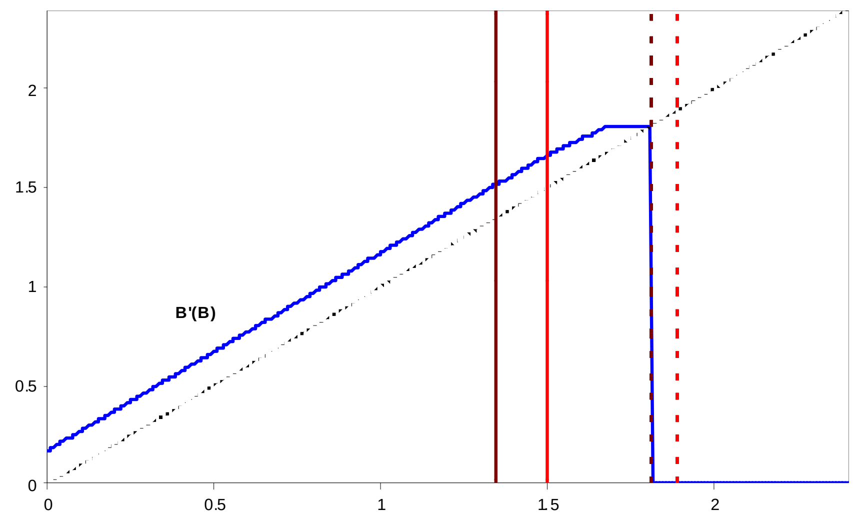
	Weighted average years until maturity	Percent debt with one year or less maturity at issuance
Germany	6.8	7.2
Greece	7.1	11.9
Ireland	6.4	0.0
Italy	7.1	19.2
Portugal	6.0	12.6
Spain	6.8	16.1

Policy functions in bad times with 6-year periods

Normal



Low π



Model with long maturity bonds

Assume that

- At $t = 0$, debt is equally divided among bonds of maturity $1, 2, ..., N$;
- New sales are similarly divided among bonds of these maturities;
- If one period bonds are sold at price q, then n period bonds are sold at price q^n.

Average maturity 6 years