

Notes on Substitutability and Tariffs in Ricardian Models

Timothy J. Kehoe
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Environment:

Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$\begin{aligned} a_1(z) &= e^{\alpha z} \\ a_2(z) &= e^{\alpha(1-z)}. \end{aligned}$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor.

The utility function is

$$\left(\int_0^1 c_j(z)^\rho dz - 1 \right) / \rho = \int_0^1 \log c_j(z) dz \quad \text{if } \rho = 0.$$

The budget constraint is

$$\int_0^1 p_j(z) c_j(z) dz = w_j \bar{\ell}_j + T_j,$$

where T_j is the lump-sum rebate of tariff revenue.

Demand:

$$\begin{aligned} & \max \left(\int_0^1 c_j(z)^\rho dz - 1 \right) / \rho \\ & \text{s. t. } \int_0^1 p_j(z) c_j(z) dz = w_j \bar{\ell}_j + T_j \\ & c_j(z) = \frac{w_j \bar{\ell}_j + T_j}{p_j(z)^{\frac{1}{1-\rho}} \int_0^1 p_j(\zeta)^{-\frac{\rho}{1-\rho}} d\zeta} = \frac{w_j \bar{\ell}_j + T_j}{p_j(z)} \quad \text{if } \rho = 0 \end{aligned}$$

Suppose that labor endowments and tariffs are equal across countries. Then

$$p_1(z) = \min[a_1(z)w_1, (1 + \tau)a_2(z)w_2]$$

$$p_2(z) = \min[(1 + \tau)a_1(z)w_1, a_2(z)w_2].$$

Here $p_j(z)$ is the price paid for good z in country j .

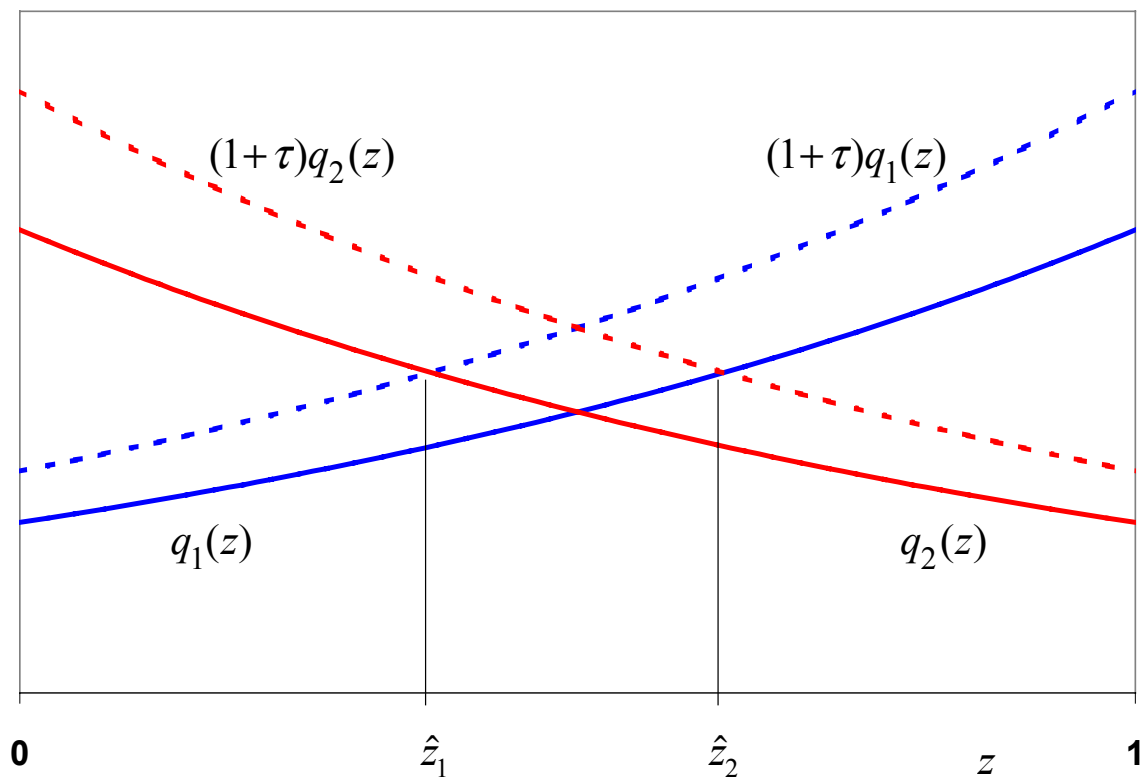
Symmetry implies that

$$w_1 = w_2 = 1$$

$$q_1 = q_2 = q$$

$$\begin{aligned} \hat{z}_1 &= 1 - \hat{z}_2 \\ (1 + \tau)e^{\alpha\hat{z}_1} &= e^{\alpha(1-\hat{z}_1)} \\ \hat{z}_1 &= \frac{\alpha - \log(1 + \tau)}{2\alpha}, \quad \hat{z}_2 = \frac{\alpha + \log(1 + \tau)}{2\alpha}. \end{aligned}$$

Patterns of production and trade



Cobb-Douglas case:

Let us do case where $\rho = 0$ first. Tariff revenues are

$$\begin{aligned}
 T &= \int_{\hat{z}_2}^1 \tau a_2(z) \frac{\bar{\ell} + T}{(1 + \tau)a_2(z)} dz = \frac{\tau(\bar{\ell} + T)}{(1 + \tau)} (1 - \hat{z}_2) \\
 T \left[1 - \frac{\tau(1 - \hat{z}_2)}{(1 + \tau)} \right] &= \frac{\tau \bar{\ell}}{(1 + \tau)} (1 - \hat{z}_2) \\
 T &= \frac{\tau \bar{\ell} [\alpha - \log(1 + \tau)]}{[2\alpha + \alpha\tau + \tau \log(1 + \tau)]}.
 \end{aligned}$$

GDP is

$$\begin{aligned}
 Y = \bar{\ell} + T &= \frac{\bar{\ell} [2\alpha + \alpha\tau + \tau \log(1 + \tau)] + \tau \bar{\ell} [\alpha - \log(1 + \tau)]}{[2\alpha + \alpha\tau + \tau \log(1 + \tau)]} \\
 Y &= \frac{2\alpha \bar{\ell} (1 + \tau)}{[2\alpha + \alpha\tau + \tau \log(1 + \tau)]}.
 \end{aligned}$$

Exports are

$$\begin{aligned}
 X &= \int_{\hat{z}_2}^1 a_2(z) \frac{\bar{\ell} + T}{(1 + \tau)a_2(z)} dz \\
 &= \frac{\bar{\ell} [\alpha - \log(1 + \tau)]}{[2\alpha + \alpha\tau + \tau \log(1 + \tau)]}.
 \end{aligned}$$

The ratio exports/GDP is

$$\frac{X}{Y} = \frac{\bar{\ell} [\alpha - \log(1 + \tau)]}{2\alpha \bar{\ell} (1 + \tau)} = \frac{[\alpha - \log(1 + \tau)]}{2\alpha (1 + \tau)}.$$

General case:

Now let us do case where $\rho \neq 0$:

$$\begin{aligned}
 \int_0^1 p_j(z) \frac{-\rho}{1-\rho} dz &= \int_0^{\hat{z}_2} e^{\frac{-\alpha \rho z}{1-\rho}} dz + \int_{\hat{z}_2}^1 (1 + \tau) \frac{-\rho}{1-\rho} e^{\frac{-\alpha \rho (1-z)}{1-\rho}} dz \\
 &= -\frac{1-\rho}{\alpha \rho} e^{\frac{-\alpha \rho z}{1-\rho}} \Big|_0^{\hat{z}_2} + \frac{1-\rho}{\alpha \rho} (1 + \tau) \frac{-\rho}{1-\rho} e^{\frac{-\alpha \rho (1-z)}{1-\rho}} \Big|_{\hat{z}_2}^1.
 \end{aligned}$$

$$\begin{aligned}
\int_0^1 p_j(z)^{\frac{\rho}{1-\rho}} dz &= -\frac{1-\rho}{\alpha\rho} e^{\frac{\alpha\rho z}{1-\rho}} \Big|_0^{\hat{z}_2} + \frac{1-\rho}{\alpha\rho} (1+\tau)^{\frac{\rho}{1-\rho}} e^{\frac{\alpha\rho(1-z)}{1-\rho}} \Big|_{\hat{z}_2}^1 \\
&= \frac{1-\rho}{\alpha\rho} \left[1 - e^{\frac{\alpha\rho\hat{z}_2}{1-\rho}} + (1+\tau)^{\frac{\rho}{1-\rho}} - (1+\tau)^{\frac{\rho}{1-\rho}} e^{\frac{\alpha\rho(1-\hat{z}_2)}{1-\rho}} \right] \\
&= \frac{1-\rho}{\alpha\rho} \left[1 + (1+\tau)^{\frac{\rho}{1-\rho}} - 2e^{\frac{\alpha\rho\hat{z}_2}{1-\rho}} \right] \\
&= \frac{1-\rho}{\alpha\rho} \left[1 + (1+\tau)^{\frac{\rho}{1-\rho}} - 2e^{\frac{\alpha\rho}{1-\rho} \left(\frac{1}{2} + \frac{\log(1+\tau)}{2\alpha} \right)} \right] \\
&= \frac{1-\rho}{\alpha\rho} \left[1 + (1+\tau)^{\frac{\rho}{1-\rho}} - 2 \left[(1+\tau)e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right].
\end{aligned}$$

We define

$$P(\tau) = \left[\int_0^1 p_j(z)^{\frac{\rho}{1-\rho}} dz \right]^{\frac{1-\rho}{\rho}} = \left(\frac{1-\rho}{\alpha\rho} \left[1 + (1+\tau)^{\frac{\rho}{1-\rho}} - 2 \left[(1+\tau)e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right] \right)^{\frac{1-\rho}{\rho}}.$$

Tariff revenues are

$$\begin{aligned}
T &= \int_{\hat{z}_2}^1 \tau a_2(z) \frac{\bar{\ell} + T}{(1+\tau)^{\frac{1}{1-\rho}} a_2(z)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}} dz \\
&= \frac{\tau(\bar{\ell} + T)}{(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}} \int_{\hat{z}_2}^1 e^{\frac{\alpha\rho(1-z)}{1-\rho}} dz \\
&= \frac{\tau(\bar{\ell} + T)}{(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}} \left(\frac{1-\rho}{\alpha\rho} - \frac{1-\rho}{\alpha\rho} \left[(1+\tau)^{-1} e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right) \\
&= \frac{(1-\rho) \left(1 - \left[(1+\tau)^{-1} e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right) \tau}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}} (\bar{\ell} + T).
\end{aligned}$$

$$T \left[1 - \frac{(1-\rho) \left(1 - \left[(1+\tau)^{-1} e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right) \tau}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}} \right] = \frac{(1-\rho) \left(1 - \left[(1+\tau)^{-1} e^\alpha \right]^{\frac{\rho}{2(1-\rho)}} \right) \bar{\ell}}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{\frac{\rho}{1-\rho}}}.$$

$$\begin{aligned}
T &= \frac{(1-\rho) \left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \bar{\tau} \bar{\ell}}{\left[\alpha \rho (1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}} - (1-\rho) \left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \tau \right]} \\
&= \frac{\left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \bar{\tau} \bar{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}} \left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2[(1+\tau) e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right] - \left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \tau \right]}
\end{aligned}$$

GDP is

$$\begin{aligned}
Y = \bar{\ell} + T &= \frac{(1+\tau)^{\frac{1}{1-\rho}} \left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2[(1+\tau) e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right] \bar{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}} \left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2[(1+\tau) e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right] - \left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \tau \right]}
\end{aligned}$$

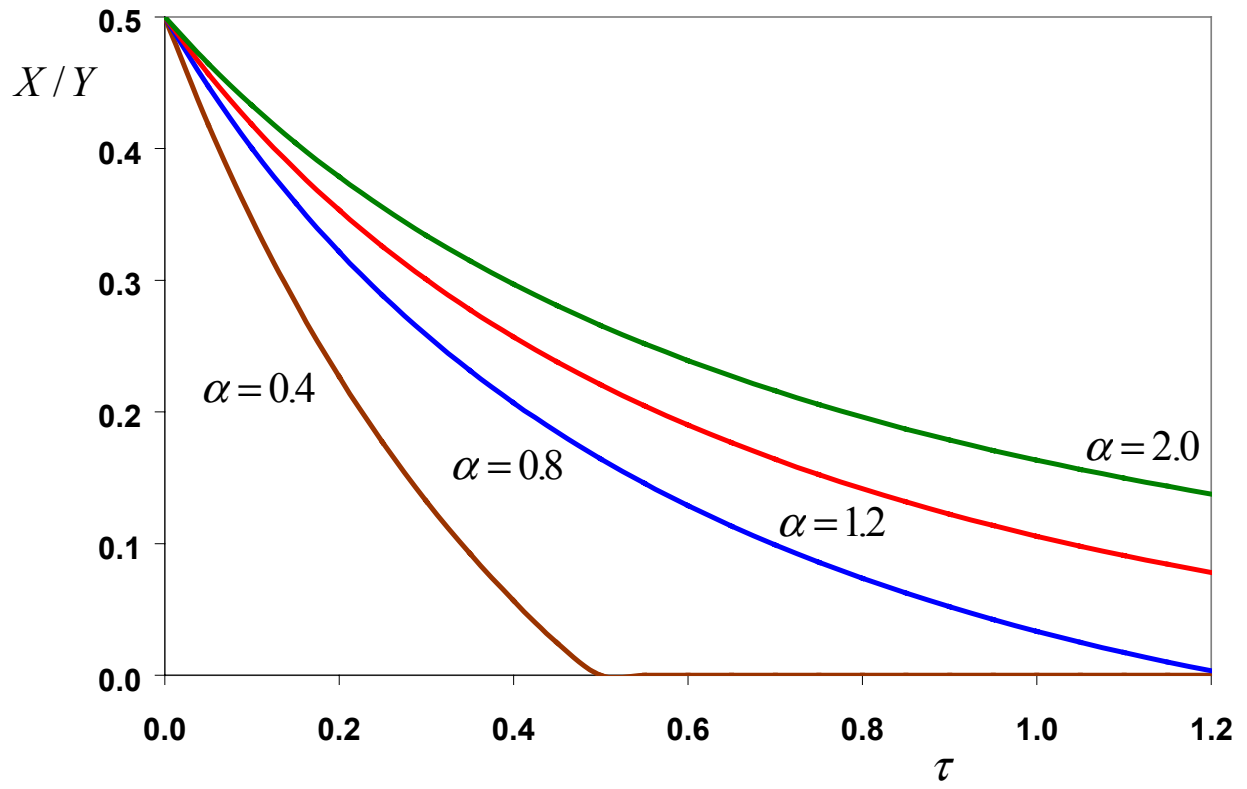
Exports are

$$\begin{aligned}
X &= \int_{z_2}^1 a_2(z) \frac{\bar{\ell} + T}{(1+\tau)^{\frac{1}{1-\rho}} a_2(z)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} dz \\
&= \frac{\left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \bar{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}} \left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2[(1+\tau) e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right] - \left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right) \tau \right]}
\end{aligned}$$

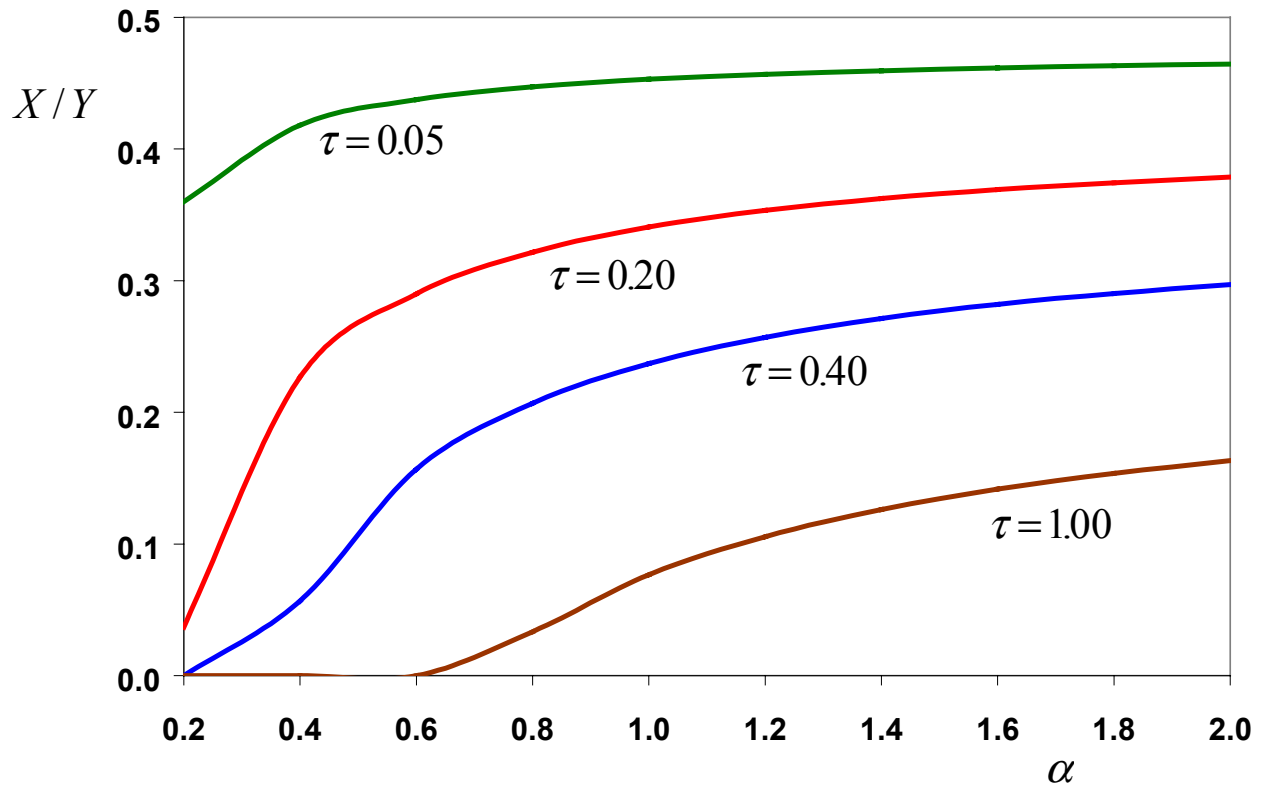
The ratio exports/GDP is

$$\frac{X}{Y} = \frac{\left(1 - [(1+\tau)^{-1} e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right)}{(1+\tau)^{\frac{1}{1-\rho}} \left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2[(1+\tau) e^\alpha]^{-\frac{\rho}{2(1-\rho)}} \right]}$$

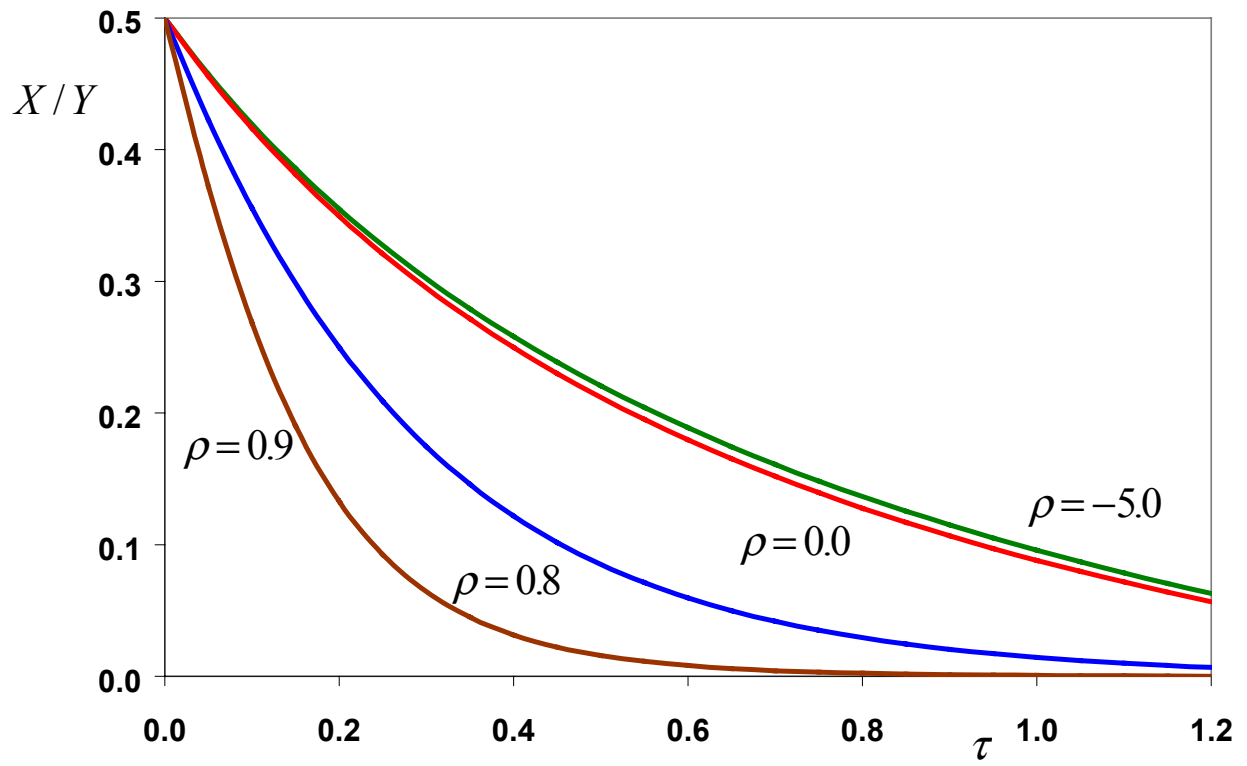
Fraction of world trade ($\rho=0$)



Fraction of world trade ($\rho=0$)



Fraction of world trade ($\alpha=1$)



Fraction of world trade ($\alpha=1$)

