RICARDIAN MODEL

Utility of the representative consumer/worker:

$$u(c_1, c_2) = a_1 \log c_1 + a_2 \log c_2.$$

Endowment

$$\bar{\ell} \text{ units of labor.}$$

Production technology:

$$y_1 = \ell_1 / b_1$$
$$y_2 = \ell_2 / b_2.$$

An **autarky equilibrium** is a set of
prices $\hat{p}_1$, $\hat{p}_2$,
a wage $\hat{w}$,
a consumption plan $\hat{c}_1$, $\hat{c}_2$,
and production plans $\hat{y}_1$, $\hat{y}_2$, $\hat{\ell}_1$, $\hat{\ell}_2$
such that

- Given $\hat{p}_1$, $\hat{p}_2$, $\hat{w}$, the consumer chooses $\hat{c}_1$, $\hat{c}_2$ to solve

$$\max \ a_1 \log c_1 + a_2 \log c_2$$
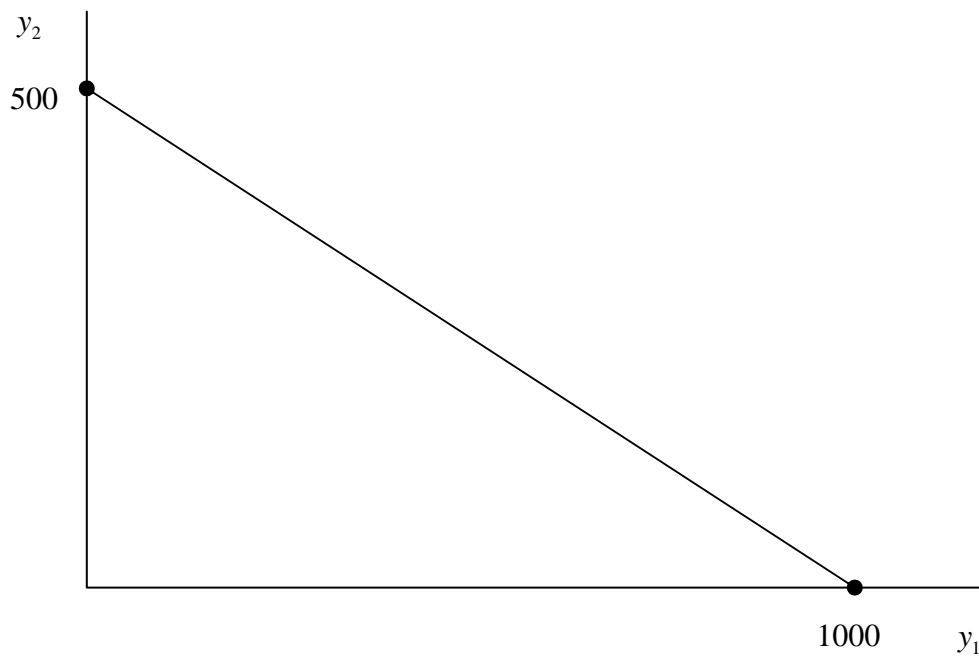$$\text{s. t. } \hat{p}_1 c_1 + \hat{p}_2 c_2 = \hat{w}\bar{\ell}.$$

- $\hat{p}_1 - b_1 \hat{w} \le 0$, $= 0$ if $\hat{y}_1 > 0$,
  $\hat{p}_2 - b_2 \hat{w} \le 0$, $= 0$ if $\hat{y}_2 > 0$.

- $\hat{c}_1 = \hat{y}_1$,
  $\hat{c}_2 = \hat{y}_2$.

- $\hat{y}_1 = \hat{\ell}_1 / b_1$,
  $\hat{y}_2 = \hat{\ell}_2 / b_2$.

- $\hat{\ell}_1 + \hat{\ell}_2 = \bar{\ell}$.

Example: $a_1 = 1$, $a_2 = 1$, $\bar{\ell} = 1000$, $b_1 = 1$, $b_2 = 2$.

Production possibility frontier:

$$\ell_1 + \ell_2 = \overline{\ell} = 1000$$
$$y_1 + 2y_2 = 1000.$$

From profit maximization, we know that, if $\hat{y}_1$, $\hat{y}_2 > 0$,

$$\hat{p}_1 = \hat{w}, \ \hat{p}_2 = 2\hat{w}.$$

Setting $\hat{w} = 1$ (numeraire), we obtain

$$\hat{p}_1 = 1, \ \hat{p}_2 = 2.$$

Solution to the consumer's problem:

$$\hat{c}_1 = \frac{\hat{w}\overline{\ell}}{2\hat{p}_1}, \ \hat{c}_2 = \frac{\hat{w}\overline{\ell}}{2\hat{p}_2}.$$

Plugging in prices, we obtain

$$\hat{y}_1 = \hat{c}_1 = \frac{1000}{2} = 500, \text{ which implies } \hat{\ell}_1 = 500,$$
$$\hat{y}_2 = \hat{c}_2 = \frac{1000}{2 \times 2} = 250, \text{ which implies } \hat{\ell}_2 = 500.$$

Two countries

Identical utilities:

$$u(c_1^i, c_2^i) = a_1 \log c_1^i + a_2 \log c_2^i, \ i = 1, \ 2.$$

Endowments:

$$\overline{\ell}^i, \ i = 1, \ 2.$$

Different production technologies:

$$y_1^i = \ell_1^i / b_1^i, \ i = 1, \ 2,$$
$$y_2^i = \ell_2^i / b_2^i, \ i = 1, \ 2.$$

A **free trade equilibrium** is a set of
prices $\hat{p}_1$, $\hat{p}_2$,
wages $\hat{w}^1$, $\hat{w}^2$,
consumption plans $\hat{c}_1^1$, $\hat{c}_2^1$, $\hat{c}_1^2$, $\hat{c}_2^2$,
and production plans $\hat{y}_1^1$, $\hat{y}_2^1$, $\hat{\ell}_1^1$, $\hat{\ell}_2^1$, $\hat{y}_1^2$, $\hat{y}_2^2$, $\hat{\ell}_1^2$, $\hat{\ell}_2^2$
such that

- Given $\hat{p}_1$, $\hat{p}_2$, $\hat{w}^i$, the consumer in country i, $i = 1, \ 2$, chooses $\hat{c}_1^i$, $\hat{c}_2^i$ to solve

$$\max \ a_1 \log c_1^i + a_2 \log c_2^i$$
$$\text{s. t. } \hat{p}_1 c_1^i + \hat{p}_2 c_2^i = \hat{w}^i \overline{\ell}^i.$$

- $\hat{p}_1 - b_1^i \hat{w}^i \leq 0$, $= 0$ if $\hat{y}_1^i > 0$, $i = 1, \ 2$,
  $\hat{p}_2 - b_2^i \hat{w}^i \leq 0$, $= 0$ if $\hat{y}_2^i > 0$, $i = 1, \ 2$.

- $\hat{c}_1^1 + \hat{c}_1^2 = \hat{y}_1^1 + \hat{y}_1^2$,
  $\hat{c}_2^1 + \hat{c}_2^2 = \hat{y}_2^1 + \hat{y}_2^2$.

- $\hat{y}_1^i = \hat{\ell}_1^i / b_1^i$, $i = 1, \ 2$,
  $\hat{y}_2^i = \hat{\ell}_2^i / b_2^i$, $i = 1, \ 2$.

- $\hat{\ell}_1^i + \hat{\ell}_2^i = \overline{\ell}^i$, $i = 1, \ 2$.

Example: $a_1 = 1$, $a_2 = 1$, $\overline{\ell}^1 = 1000$, $b_1^1 = 1$, $b_2^1 = 2$,
$\overline{\ell}^2 = 1800$, $b_1^2 = 2$, $b_2^2 = 3$.

Notice that

$$\frac{\hat{p}_1^2}{\hat{p}_2^2} = \frac{2}{3} \text{ in autarky in country 2}$$

compared to

$$\frac{\hat{p}_1^1}{\hat{p}_2^1} = \frac{1}{2} \text{ in autarky in country 1.}$$

Since good 1 is relatively less expensive in country 1, country 1 exports good 1 and country 2 exports good 2. There are three logical possibilities for patterns of production and specialization:

$$\hat{y}_1^1 > 0, \ \hat{y}_2^1 = 0 \text{ in country 1, } \hat{y}_1^2 = 0, \ \hat{y}_2^2 > 0 \text{ in country 2;}$$
$$\hat{y}_1^1 > 0, \ \hat{y}_2^1 > 0 \text{ in country 1, } \hat{y}_1^2 = 0, \ \hat{y}_2^2 > 0 \text{ in country 2;}$$
$$\hat{y}_1^1 > 0, \ \hat{y}_2^1 = 0 \text{ in country 1, } \hat{y}_1^2 > 0, \ \hat{y}_2^2 > 0 \text{ in country 2.}$$

Let us guess that $\hat{y}_1^1 > 0$, $\hat{y}_2^1 = 0$ in country 1, $\hat{y}_1^2 = 0$, $\hat{y}_2^2 > 0$ in country 2:

$$\hat{\ell}_1^1 = 1000, \ \hat{y}_1^1 = 1000,$$
$$\hat{\ell}_2^2 = 1800, \ \hat{y}_2^2 = 600.$$

We know that $\hat{p}_1 = \hat{w}^1$ and $\hat{p}_2 = 3\hat{w}^2$. We can normalize $\hat{w}^1 = 1$. We still need to calculate $\hat{w}^2$ and to check that

$$\hat{p}_2 - 2 \leq 0$$
$$\hat{p}_1 - 2\hat{w}^2 \leq 0.$$

(It is in checking these conditions that an inactive industry cannot earn profits by operating that we check if our guess about the pattern of production and specialization is, in fact, correct.) To find $\hat{w}^2$, let us use the condition that demand equal supply for good 1. (We get the same result if we use the condition that demand equal supply for good 2.

$$\hat{c}_1^1 + \hat{c}_1^2 = \hat{y}_1^1$$
$$\frac{1000\hat{w}^1}{2\hat{p}_1} + \frac{1800\hat{w}^2}{2\hat{p}_1} = 1000$$
$$500 + 900\hat{w}^2 = 1000$$
$$900\hat{w}^2 = 500$$
$$\hat{w}^2 = \frac{5}{9}, \text{ which implies } \hat{p}_2 = \frac{5}{3}.$$

Check:

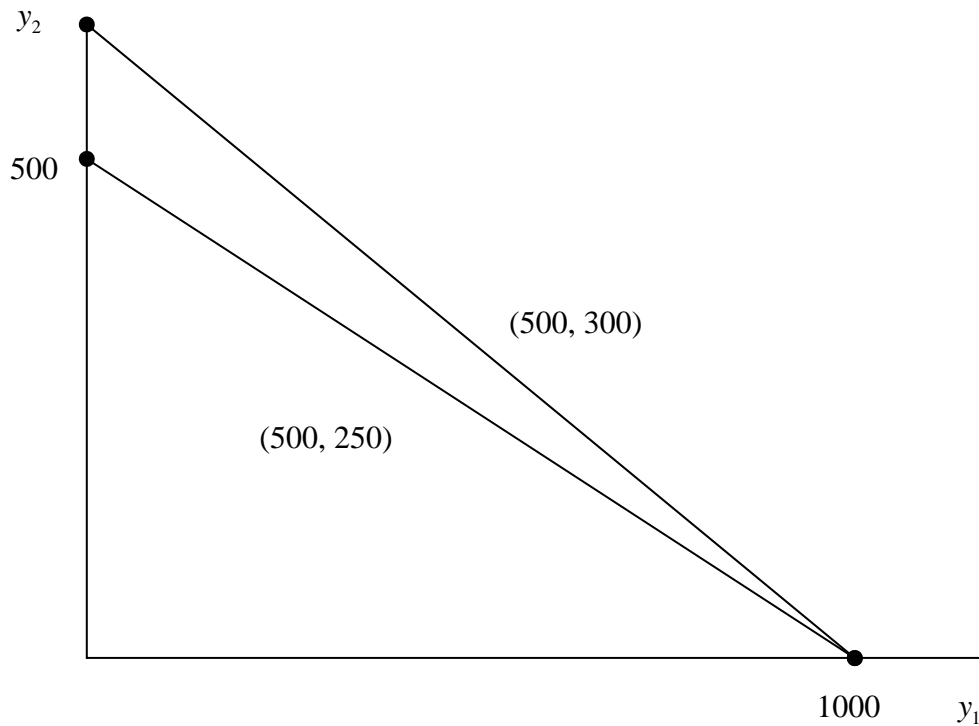$$\hat{p}_2 - 2 = \frac{5}{3} - 2 \leq 0$$

$$\hat{p}_1 - 2\hat{w}^2 = 1 - 2\frac{5}{9} \leq 0.$$

Calculate consumption:

$$\hat{c}_1^1 = 500, \ \hat{c}_2^1 = 300$$
$$\hat{c}_1^2 = 500, \ \hat{c}_2^2 = 300.$$

Gains from trade for country 1:

Exercise: Check the case where we set $\bar{\ell}^2 = 5400$, but keep all other parameter the same. You will find, in this case, that supposing that $\hat{y}_1^1 > 0$, $\hat{y}_2^1 = 0$ in country 1, $\hat{y}_1^2 = 0$, $\hat{y}_2^1 > 0$ in country 2 does not work.