Self-Fulfilling Crises with Default and Devaluation[†]

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Abstract

We characterize optimal debt policy in a dynamic stochastic general equilibrium model of defaults and devaluations in which self-fulfilling crises can arise. When the government cannot commit to repay its debt and cannot commit to maintain the exchange rate, consumers' expectations of devaluation make the safe level of government debt very low. We show that, when the debt is in the crisis zone –where self-fulfilling crisis can occur– the government finds it optimal to reduce the debt in order to exit the zone. The lower the probability that consumers assign to devaluation, however, the greater is the number of periods that the government will choose to take to exit the crisis zone. We argue that this was the case in Argentina in 2001-2002.

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1 Introduction

Self-fulfilling crisis models of debt default study how much debt can be sustained when the government cannot commit to repay this debt. Cole and Kehoe (2000) show that, if the fundamentals of the economy fall within a particular range, which they call the crisis zone, then the probability of default is determined by the beliefs of international investors: if these investors expect a default, the government cannot rollover the debt, which induces a crisis. Cole and Kehoe characterize the fundamentals that determine the crisis zone in terms of levels of the government's debt and of the private capital stock. To remove the conditions that make crises possible, the government can reduce its debt and exit the crisis zone. The message of the Cole-Kehoe analysis is that the inability of the government to commit makes safe levels of debt lower.

Obstfeld (1996) studies self-fulfilling models of devaluations. He shows that, when consumers expect a devaluation, it can be optimal for the government to devalue. As a result, even pegged exchange rates that could be sustained can succumb to speculative attacks if the government cannot commit to defend the currency peg.

This paper constructs a dynamic stochastic general equilibrium model that combines the crucial mechanism in the Cole-Kehoe model with that in the Obstfeld model. We use this model to characterize optimal debt policy in an environment in which both defaults and devaluations are possible. The key feature of our model is that consumers' expectations of devaluations change the government's incentives to default. As in Obstfeld, expectations of a devaluation create economic conditions that make it more attractive for the government to devalue. In particular, we assume that expectations of devaluation lead consumers to reduce holdings of domestic capital and increase holdings of foreign securities. The government can eliminate these perverse effects of expectations of devaluation by devaluing. If government debt is denominated in the foreign currency, which we call US dollars, however, a devaluation can push the debt into the crisis zone, where with some probability the government can find optimal to default. This happens because we assume that a nominal devaluation results in a real devaluation, which makes the debt level larger as a fraction of GDP.

What do we learn from our model? Suppose that the government maintains a fixed exchange rate to lower inflation, and suppose that it has debt denominated in US dollars. The government faces two related commitment problems: It cannot commit not to devalue, and it cannot commit not to default. In this environment, consumers' expectations of devaluation make the safe level of government debt very low.

If the level of debt is sufficiently low, expectations of devaluation make it optimal for the government to devalue, but not to default. In this case, the benefits of eliminating the expectations of devaluation are higher than the future costs of repaying the increased real debt. On the other extreme, if the level of debt is sufficiently high, the government finds it optimal to default even if the private agents are not expecting a devaluation. In between these two extreme levels of debt, the government defaults only if the private agents expect a devaluation. That is, the expectations of devaluation enlarge the crisis zone.

Why do expectations of devaluation make default more likely? The benefits of a devaluation are independent of the level of debt, but the costs are increasing. Suppose first that the government can commit not to devalue. Then the government defaults only if the economy is in the Cole-Kehoe crisis zone. Suppose now that consumers believe that there is a small probability that the government will devalue. If the level of debt is sufficiently low, the government would find it optimal to devalue, but not to default. If the debt is near, but not within, the Cole-Kehoe crisis zone, however, the benefits of a defaults are high and less than the future costs. Consequently, the government now finds it optimal to default and to devalue.

Of course, when the government is inside this new crisis zone, it is optimal for the government to reduce the debt in order to exit this zone. The lower the probability that consumers assign to devaluation, however, the greater is the number of periods that the government will choose to take to exit the crisis zone.

We argue that our model can help us understand the situation in Argentina in 2001-2002. We calibrate the model to match the decrease in investment of domestic capital, the reduction in production, the increase in trade balance surplus, and the increase in debt levels observed throughout 2001 in Argentina. We demonstrate that, for a probability of devaluation consistent with the risk premium on the Argentine government bonds nominated in dollars issued in April 2001, the external debt of Argentina was in the crisis zone where the government found it optimal to default and to devalue.

Our paper is also related to other papers on self-fulfilling debt crisies (see, for example, Sachs, Tornell, and Velasco (1996)), the literature of self-fulfilling currency crises (see, for example Krugman (1979, 1996) and Obstfeld (1986)) and papers on the crisis in Argentina in 2001-2002 (see, for example, Calvo, Izquierdo, and Talvi (2003), Perry and Serven (2003) and De la Torre, Levi-Yeyati and Schmukler (2003)).

2 The economic environment

There are three agents in the economy – domestic consumers, international bankers, and government – and three assets – domestic capital, K, a foreign security, A, and public debt, B. Both A and B are denominated in US dollars and can be exchanged for domestic goods at the real exchange rate e.

The international bankers buy the government debt and sell the foreign security. The government, which is benevolent, provides a public good, raises taxes, and issues public debt. Finally, domestic consumers are the owners of the capital and foreign security holdings. They inelastically supply one unit of labor and derive utility from private consumption and the government good.

2.1 The consumers

There is a continuum with measure one of identical, infinitely lived consumers who consume, invest, and pay taxes. The representative consumer's utility function is

$$E \sum_{t=0}^{\infty} \beta^t \left(c_t + v(g_t) \right),$$

where c_t is private consumption and g_t is government consumption. The assumption of risk neutrality of consumers greatly simplifies the modeling of consumer behavior, as in Cole and Kehoe (1996). We assume that $0 < \beta < 1$ and that v is continuously differentiable, strictly concave, and monotonically increasing. We also assume that $v(0) = -\infty$. The households' income is devoted to paying taxes, to consuming domestically produced goods c, and to investing in domestic capital k' and in foreign securities a'. The consumer's budget constraint is

$$c_t + k_{t+1} + e_t[a_{t+1} + \Phi(a_{t+1})] = (1-\tau)\alpha(z_t)\theta(e_t, e_{t-1}) f(k_t) + e_t r^* a_t,$$

where z_t is an indicator taking the value 1 if the government repays its debt and the value 0 if it defaults and r^* is the international interest rate in US dollars. We assume that the real exchange rate e_t takes on one of two values, $\underline{e}$ and $\overline{e}$, where $\underline{e} < \overline{e}$. We normalize $\underline{e} = 1$. Here k_t is the consumer's individual capital stock; α is a multiplicative productivity factor that depends on whether or not the government has ever defaulted and $0 < \theta < 1$ is another productivity factor that depends on whether the government devalues or not; τ, $0 < \tau < 1$, is the constant proportional tax on domestic income; and f is a continuously differentiable, concave, and monotonically increasing production function that satisfies $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. Implicitly, of course, we are setting $f(k) = F(k, 1)$, where F is a constant returns production function in capital and the fixed supply of labor. The consumer is endowed with k_0 units of capital and a_0 units of foreign security at period 0. There is also an investment cost on international securities, represented by an increasing, convex function $\Phi(a_{t+1})$, with $\Phi'(0) = 0$. The existence of this function allows us to find the optimal allocations of the foreign security. [1]

[1] This trick is a common one in the small open economy literature. Otherwise, under arbitrage, it turns out to be difficult to compute the amount of resources devoted to the foreign assets. See, for example, Schmitt-Grohe and Uribe (2003).

There are three important assumptions. First, we are assuming that there is a technology that transforms domestic goods into foreign goods. The rate of transformation is the real exchange rate, e_t, and, in order to simplify the model, we assume that no changes in nominal prices of domestic goods are expected or reported, so that a nominal devaluation is also a real devaluation: the government, in choosing e_t also changes the real terms of trade between the domestic good and the foreign good. Later, we provide evidence to justify this assumption in the case of Argentina in 2001-2002.[2] We assume that the government can devalue the real exchange rate, but not revalue it. Second, we assume that, if the government decides to default, there is a permanent negative productivity shock, as in Cole and Kehoe (2000). Third, in order to determine the optimal level of a devaluation we assume that, in the period in which the devaluation happens, the economy is affected by a transitory negative shock in productivity

$$\theta(e_t, e_{t-1}) = \begin{cases} \theta < 1 & \text{if government devalues} \quad e_t > e_{t-1} \\ 1 & \text{if government not devalues} \quad e_t = e_{t-1}. \end{cases}$$

There are two models that would rationalize our assumption that productivity falls after the government devalues. In one model, firms must renegotiate contracts and, in the short term, firms cannot substitute foreign inputs. We could assume, for example, that there is a foreign produced intermediate good, which cannot be substituted, whose price increases after a devaluation.[3] In another model that would rationalize our assumption, after a devaluation the government increases trade taxes, sets different exchange rates for exports and imports, or establishes quotas on trade. In summary, government increases distortions in the economy and reduces output.

2.2 The international bankers

There is a continuum with measure one of identical, infinitely lived international bankers. The individual banker is risk neutral and has the utility function

$$E \sum_{t=0}^{\infty} \beta^t x_t,$$

where x_t is the banker's private consumption. As in Cole and Kehoe (1996), the assumption of risk neutrality of bankers captures the idea that the domestic economy is small compared to world financial markets. Each banker is endowed with $\bar{x}$ units of the consumption good in each period and faces the budget constraint

$$x_t + q_t B_{t+1} + r^* a_t \leq \bar{x} + z_t B_t + a_{t+1},$$

[2] This is a reduced form of a model where consumption and investment are composed of tradable and non-tradable goods.

[3] To see how we can incorporate this story into our model, denote by $g(n, k)$ the production function for output as a function the price of the intermediate good n and capital k. Abusing notation slightly, we set $\theta(e_t, e_{t-1})f(k) = g(n^*, k) - (e_t/e_{t-1})n^*$, where $n^* = \arg\max g(n, k) - n$.

where q_t is the price of one-period government bonds that pay B_{t+1} in period $t+1$ if $z_{t+1} = 1$, that is, the government decides to repay its debts, and 0 if $z_{t+1} = 0$, that is, if the government decides not to repay.

2.3 The government

There is a single government, which is benevolent in the sense that its objective is to maximize the welfare of the consumers. In every period, the government makes three decisions: (i) it chooses the level of government consumption, g_t, financed with household income taxes and with some of the dollars obtained from the US dollar denominated bonds issued new borrowing level B_{t+1}; (ii) it decides whether or not to default on its old debt, $z_t \in \{0, 1\}$; (iii) it chooses the real exchange rate, e_t.

The government budget constraint is

$$g_t = \tau\alpha(z_t)\theta(e_t, e_{t-1})f(k_t) + e_t[q_t B_{t+1} - z_t B_t].$$

The government decides to pay, $z_t = 1$, or to default public debt, $z_t = 0$, and whether devalue $e_t > e_{t-1}$ or not $e_t = e_{t-1}$. As in Cole and Kehoe (1996, 2000), productivity is affected by a default (that is, $\alpha(0) < \alpha(1)$) and the government losses access to international borrowing and lending after default. Finally, the market clearing condition for the government debt is $b_{t+1} = B_{t+1}$, and we also assume that $k_0 = K_0$ and $b_0 = B_0$, where capital letters denote aggregates and small letters denote individual decisions.

In each period, the value of an exogenous random variable ξ_t is realized. We assume that ζ is uniformly distributed on the unit interval. This variable is interpreted as a sunspot. We show that we can construct equilibria where, if the level of government debt B_t is above some crucial level and ζ_t is above another crucial level, then consumers anticipate a devaluation and reduce domestic investment. This creates a self-fulfilling debt crisis in the sense that, since the reduction in domestic investment changes the government incentive to honor its debt. The government chooses to default and then to devalue.

2.4 The timing

We assume that the following timing of actions within each period:

1. The government sells debt.

2. The international bankers, taking the price of debt as given, choose to buy or not to buy the debt.

3. The government decides to default or not, and chooses the exchange rate and government consumption.

4. The exogenous random variable, ζ, is realized.

5. Consumers choose consumption and investment on the domestic and the foreign securities.

3 Equilibrium

As in Cole and Kehoe (2000), the government cannot commit itself either to honoring its debt obligations or to following a fixed borrowing and spending path. It also cannot commit to modify or not the real exchange rate, e. We follow closely Cole and Kehoe's recursive equilibrium definition in which there is no commitment and the agents choose their actions sequentially.

We describe the decision making by the agents within a period starting with the consumers and working our way forward, following the timing presented above. Agents who act earlier in the timing can solve the problems of agents who act later to predict what they will do.

When an individual consumer acts, he knows the following: his individual capital k and foreign securities a, the aggregate state $s = (B, K, A, \alpha_1, e_{-1})$; the government's supply of new debt B'; the price that bankers are willing to pay for this debt q; the government's spending, g, and default and devaluation decisions, z and e, respectively; and the sunspot ζ. We define the state of the individual consumer as $(k, a, s, B', g, z, e, \zeta)$. We denote the government's policy functions by $B'(s)$, $g(s, B', q)$, $z(s, B', q)$ and $e(s, B', q)$; the price function by $q(s, B')$; and by $K'(s, B', g, z, e, \zeta)$, $A'(s, B', g, z, e, \zeta)$ the functions that describe the evolution of the aggregate capital and foreign securities stocks, all yet to be defined. The representative consumer's value function is defined by the functional equation

$$
\begin{aligned}
V_c(k, a, s, B', g, z, e, \zeta) \quad &= \quad \max_{c, k', a'} c + v(g) + \beta E\left[V_c(k', a', s', B'(s'), g', z', e', \zeta')\right] \\
s.t \quad & c + k' + e[a' + \Phi(a')] \leq (1 - \tau)\alpha(s, z)\theta(s, e) f(k) + e r^* a \\
& c, k' > 0 \quad a' \geq 0 \\
& s' = (B', K'(s, B', g, z, e, \zeta), A'(s, B', g, z, e, \zeta), \alpha(s, z), e) \\
& g' = g(s', B'(s'), q(s', B'(s'))) \\
& z' = z(s', B'(s'), q(s', B'(s'))) \\
& e' = e(s', B'(s'), q(s', B'(s')))
\end{aligned}
$$

The three policy functions of the consumers are $c(k, a, s, B', g, z, e, \zeta)$, $k'(k, a, s, B', g, z, e, \zeta)$ and $a'(k, a, s, B', g, z, e, \zeta)$. Because consumers are competitive, we need to distinguish between the individual decisions, k_{t+1} and a_{t+1}, and the aggregate values, K_{t+1} and A_{t+1}. In equilibrium, given that all consumers are identical, $k_{t+1} = K_{t+1}$ and $a_{t+1} = A_{t+1}$.

As explained, the production parameters satisfy $\alpha(s, z) = 1$ if the government has not defaulted in the past and has not defaulted this period and $\alpha(s, z) = \alpha < 1$ if it has defaulted in the past. Similarly, $\theta(s, e) = 1$ if the government has not devalued in this period, and $\theta(s, e) < 1$ if the government has devalued in this period.[4]

When an individual banker chooses his new debt level, he knows his individual holdings of government debt b, the aggregate state s, and the government's offering of new debt B'. The state of an individual banker is defined as (b, s, B'). The representative banker's value function is defined by the functional equation

$$\begin{aligned} V_b(b, s, A, B') \quad &= \quad \max_{b'} x + \beta E\left[V_b(b', s', A', B'(s'))\right] \\ s.t \quad & \quad x + q(s, B')b' + r^* A \leq \bar{x} + z(s, B', q(s, B'))b + A' \\ & \quad x > 0, \;\; q(s, B')b' \leq \bar{x} \\ & \quad s' = (B', K'(s, B', g, z, e, \zeta), A'(s, B', g, z, e, \zeta), \alpha(s, z), e) \end{aligned}$$

Bankers are relatively passive: if $\bar{x}$ is sufficiently large, they purchase the amount of bonds offered by the government as long as the price of these bonds satisfies

$$q(s, B') = \beta E \; z(s', B'(s'), q(s', B'(s'))),$$

and the assumption that they behave competitively guarantees that they sell the amount of foreign assets demanded by consumers if $r^* = 1/\beta$.

The only strategic agent in the model is the government. Its makes decisions at two points in time. At the beginning of the period, when the government chooses B', the government's state is simply the initial state s. Later, after it has observed the actions of the bankers, which are summarized in the price q, the government chooses whether or not to devalue, e, and whether or not to default, z. There decisions in turn determine the level of government spending, g, and the levels of productivity, α and θ. This choice is given by the policy functions $g(s, B', q)$, $e(s, B', q)$, and $z(s, B', q)$. In consequence, at the beginning of the period, the government knows how the price that its debt will bring, $q(s, B')$, depends on this state and on the level of new borrowing. The government also knows what its own optimizing choices $g(s, B', q(s, B'))$, $e(s, B', q(s, B'))$, and $z(s, B', q(s, B'))$ will be later. The government also realizes that it can affect consumption, c, domestic investment K', foreign securities, A', and the production parameters, α and θ, through its choices. The government's value function is defined by the functional equation

[4] Note that to define the equilibrium we write α and θ as functions of the state. In the next section, we return to our original notation.

$$
\begin{aligned}
V_g(s) \quad &= \quad \max_{B'} E\left\{c(K, A, s, B', g, z, e, \zeta) + v(g) + \beta V_g(s')\right\} \\
s.t \quad & g = g(s, B', q(s, B')) \\
& z = z(s, B', q(s, B')) \\
& e = e(s, B', q(s, B')) \\
& s = (B', K'(s, B', g, z, e, \zeta), A'(s, B', g, z, e, \zeta), \alpha(s, z), e).
\end{aligned}
$$

We denote by $B'(s)$ the government's debt policy. After the international bankers decide to buy or not the debt, the government makes its decisions on default, z, and devaluation, e, which determines the level of α and θ and the level of government spending, g. Given $V_g(s)$, we define the policy functions $g(s, B', q)$, $e(s, B', q)$ and $z(s, B', q)$ as the solutions to the following problem:

$$
\begin{aligned}
& \max_{g,z,e} E\left\{c(K, A, s, B', g, z, e, \zeta) + v(g) + \beta V_g(s')\right\} \\
s.t \quad & g - \tau\alpha(s, z)\theta(s, e)f(K) \leq e[qB' - zB] \\
& z = 0 \text{ or } z = 1 \\
& g \geq 0 \\
& \theta(s, e) = \begin{cases} \theta(\underline{e}, \overline{e}) & \text{if the government devalues} \\ 1 & \text{if government does not devalue} \end{cases} \\
& s' = (B', K'(s, B', g, z, e, \zeta), A'(s, B', g, z, e, \zeta), \alpha(s, z), e).
\end{aligned}
$$

3.1 Definition of an equilibrium

An equilibrium is a list of value functions V_c, for the representative consumer, V_b for the representative banker, and V_g for the government; policy functions c, k' and a' for the consumer, b' and a' for the banker, and B', g, z and e for the government; a price function q and an interest rate r^*; and equations of motion for the aggregate capital stock K' and foreign asset stock A' such that the following conditions hold:

1. Given B', g, z, e and ζ, V_c is the value function for the solution to the representative consumer's problem, and c, k' and a' are the maximizing choices.

2. Given B', A', q, and z, V_b is the value function for the solution to the representative banker's problem, and the value of B' chosen by the government solves the problem when $b = B$.

3. Given q, c, K', A', g, z and e, V_g is the value function for the solution to the government's problem first problem, and B' is the maximizing choice. Furthermore, given c, K', A', V_g, and B', then g, z and e maximize the consumer welfare subject to the government's budget constraint.

4. $q(s, B') = \beta E\ z(s', B'(s'), q(s', B'(s')))$, and $r^* = 1/\beta$.

5. $K'(s, B', g, z, e, \zeta) = k'(K, A, s, B', g, z, e, \zeta)$,
   $A'(s, B', g, z, e, \zeta) = a'(K, A, s, B', g, z, e, \zeta)$ and $B'(s) = b'(B, s, B')$.

In this definition of equilibrium, we are assuming that consumers and bankers know that the government solves its problem each period, and therefore understand that, under some circumstances, the government will choose to default and/or to devalue.

4 The optimal behavior of private agents

The model that we have presented has many equilibria. In this section, we present a method for constructing equilibria in which the probability of devaluation and the probability of default are constants. In these equilibria, the realizations of devaluation and default depend on the realizations of the sunspot variable ζ_t. Depending on the level of government debt and the level of the capital and foreign securities stocks, an unfavorable realization of the sunspot variable can lead to devaluation with or without default.

The bankers' optimal behavior depends upon the expectations that they have about the government's future repayment decision z'. If bankers expect that $z' = 0$, then they are not willing to buy any debt unless the price is 0. If bankers expect that $z' = 1$, then they are willing to buy any amount of the government debt up to $\overline{x}$ at price β. If bankers expect default to occur with probability π, they are willing to purchase whatever amount of bonds the government offers up to $\overline{x}$ at price $q = \beta(1 - \pi)$.

The consumers' optimal policy depends solely on what they expect the values of the productivity parameters α and θ will be next period. There are several cases.

4.1 No expectations of devaluation

We start first with the cases where consumers have no expectations of devaluation. Consumers believe that the government will not devalue in the next period ($\pi = 0$). Then the first-order conditions are

$$\begin{aligned}
&\beta(1 - \tau)\alpha(z')f'(k') = 1, \\
&\Phi(a') = 0, \\
&c + k' = (1 - \tau)\alpha(z')\theta(\underline{e}, \overline{e})f(k) + er^*a.
\end{aligned}$$

If devaluation has occurred in period t and the government has already defaulted, it is optimal for consumers to set the capital stock for next period to a level k^d that satisfies

$$\beta(1-\tau)\alpha(0)f'(k^d) = 1,$$

to set the level of foreign holdings $a' = 0$, and to eat whatever output is left over,

$$c^{dd}(K,a) = (1-\tau)\alpha(0)\theta(\underline{e},\overline{e})f(K) + \overline{e}r^*a - k^d.$$

Their consumption after devaluation and default occur is

$$c^{nd}(k^d,0) = (1-\tau)\alpha(0)f(k^d) - k^d.$$

If devaluation has occurred in period t and if the government has not defaulted, it is optimal for consumers to set the capital stock for next period to a level k^n that satisfies

$$\beta(1-\tau)\alpha(1)f'(k^n) = 1,$$

to set the level of foreign holdings $a' = 0$, and to eat whatever output is left over,

$$c^{dn}(K,a) = (1-\tau)\alpha(1)\theta(\underline{e},\overline{e})f(K) + \overline{e}r^*a - k^n.$$

Their consumption after devaluation and not default occur is

$$c_0^{nn}(k^n,0) = (1-\tau)\alpha(1)f(k^n) - k^n.$$

If the government does not devalue and has not defaulted, it is optimal for consumers to set the capital stock for the next period to the level k^n, to set the level of foreign holdings $a' = 0$ and eat whatever is left over,

$$c_0^{nn}(K,a) = (1-\tau)\alpha(1)f(K) + r^*a\underline{e} - k^n,$$

and their consumption thereafter $c_0^{nn}(k^n,0)$.

If the government does not devalue but has defaulted, it is optimal for consumers to set the capital stock for the next period to the level k^d, to set the level of foreign holdings $a' = 0$ and eat whatever is left over,

$$c^{nd}(K,a) = (1-\tau)\alpha(0)f(K) + r^*a\underline{e} - k^d.$$

Consumption thereafter is $c^{nd}(k^d,0)$.

4.2 Expectations of devaluation

Now, we are interested in studying the cases in which consumers believe that the productivity parameter θ will be equal to $\theta(\underline{e}, \overline{e})$ for the next period because the government has not previously devaluated, but consumers believe that the government will devalue during the next period ($\pi = 1$). Then the first-order conditions are

$$\begin{aligned}
&\beta(1-\tau)\alpha(z')\theta(\underline{e}, \overline{e})f'(k') = 1, \\
&1 + \Phi(a') = \frac{1}{\theta(\underline{e}, \overline{e})}, \\
&c + k' = (1-\tau)\alpha(z')f(k) + \left[r^* a - a' - \Phi(a')\right]\underline{e}.
\end{aligned}$$

If the government does not devalue and has not defaulted, it is optimal for consumers to set the capital stock for the next period to a level k^{dn} that satisfies

$$\beta(1-\tau)\alpha(1)\theta(\underline{e}, \overline{e})f'(k^{dn}) = 1,$$

to set the level of foreign holdings a^{dn} that satisfies

$$1 + \Phi'(a^{dn}) = \frac{\overline{e}}{\underline{e}},$$

and eat whatever is left over,

$$c_1^{nn}(K, a) = (1-\tau)\alpha(1)f(K) + \left[r^* a - a^{dn} - \Phi(a^{dn})\right]\underline{e} - k^{dn}.$$

If consumers believe that the government will devalue the next period ($\pi = 1$) and the government does not devalue and has defaulted, it is optimal for them to set the capital stock for the next period to a level k^{nd} that satisfies

$$\beta(1-\tau)\alpha(0)\theta(\underline{e}, \overline{e})f'(k^{nd}) = 1,$$

to set the level of foreign holdings a^{dn}, and to eat whatever is left over,

$$c_1^{nd}(K, a) = (1-\tau)\alpha(0)f(K) + \left[r^* a - a^{dn} - \Phi(a^{dn})\right]\underline{e} - k^{nd}.$$

5 Crisis zones

In this section we will show that, if the level of debt is sufficiently low, expectations of devaluation make it optimal for the government to devalue, but not to default. In this case, the benefits of eliminating the expectations of devaluation are higher than the future costs of repaying the increased real debt. On the other extreme, if the level of debt is sufficiently high, the government finds it optimal to default even if the private agents are not expecting a devaluation. In between these two extreme levels of debt, the government defaults only if the private agents are expecting a devaluation. That is, the expectations of devaluation enlarge the crisis zone.

5.1 Devaluation without default

For low levels of debt an equilibrium will exist where, if consumers expect a devaluation, the government prefers to devalue and to repay its debt. After the devaluation, the productivity of the economy increases and the government distributes the present cost of the devaluation across future periods. The maximum level of debt for which devalue and repay it is optimal is denoted by $\underline{b}$.

We suppose initially that the government always pays its debt and that the consumers believe that the government will devalue in the following period, i.e. $\pi = 1$. The bankers always buy all the debt issued, up to the level $\overline{x}$ (which we assume never binds) at the price $q = \beta$. We will compare the payoffs that the government obtains by devaluing and not devaluing to find the level of debt $\underline{b}$. The payment the government obtains after devaluing and not defaulting is

$$\begin{aligned} V^{dn}(s, B_0, B_1) &= c^{dn}(K_0, a_0) + v\Big(\tau\alpha(1)\theta(\underline{e}, \overline{e})f(K_0) + \overline{e}\,(\beta B_1 - B_0)\Big) + \\ & \frac{\beta}{1-\beta}\Big\{c_0^{nn}(k^n, 0) + v\Big(\tau\alpha(1)f(k^n) - \overline{e}(1-\beta)B_1\Big)\Big\}, \end{aligned} \tag{1}$$

while if not devaluing and not defaulting

$$\begin{aligned} V_1^{nn}(s, B_0, B_1) &= c_1^{nn}(K_0, a_0) + v\Big(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\Big) + \\ & \frac{\beta}{1-\beta}\Big\{c_1^{nn}(k^{dn}, a^{dn}) + v\Big(\tau\alpha(1)f(k^{dn}) - (1-\beta)B_1\Big)\Big\}. \end{aligned} \tag{2}$$

The threshold $\underline{b}$ will be the higher level of debt B_0 that verifies

$$V^{dn}(s, B_0, B_1) \geq V_1^{nn}(s, B_0, B_1). \tag{3}$$

That is to say, for greater levels of debt, despite consumer expectations of devaluation, the government does not devalue, and it repays its debt.

To determine the level of debt $\underline{b}$, however, it is necessary to characterize the behavior of the government relating to the new debt. It is optimal for the government to maintain a constant level of spending, $g_{t+1} = g_t$, and, hence, of its debt. Both depend on initial conditions (K_0, B_0).

If the government has chosen to devalue, government consumption, given that it is constant, is given by

$$g^d(B_0, K_0) = \tau\alpha(1)\Big[\beta f(k^n) + \theta(\underline{e}, \overline{e})(1-\beta)f(K_0)\Big] - \overline{e}(1-\beta)B_0,$$

while government debt stays constant at

$$B^d(B_0, K_0) = B_0 + \frac{\tau\alpha(1)}{\overline{e}}\Big[f(k^n) - \theta(\underline{e}, \overline{e})f(K_0)\Big].$$

In the case that the government does not devalue, the constant government consumption will be given by

$$g^n(B_0, K_0) = \tau\alpha(1)\Big[\beta f(k^{dn}) + (1-\beta)f(K_0)\Big] - (1-\beta)B_0,$$

while government debt stays constant at

$$B^n(B_0, K_0) = B_0 + \tau\alpha(1)\Big[f(k^{dn}) - f(K_0)\Big].$$

Given initial conditions (K_0, B_0, A_0), when government consumption is constant, the government's payoff from devaluing and not devaluing (1) and (2) is given, respectively, by

$$V^{dn}\left(s, B_0, B^d(B_0, K_0)\right),$$

and

$$V_1^{nn}\left(s, B_0, B^n(B_0, K_0)\right).$$

We now argue that, when government expenditure is constant and β is sufficiently high, there is a unique $b^* > 0$ such that

$$V^{dn}\left(s, b^*, B^d(b^*, K_0)\right) = V_1^{nn}\Big(s, b^*, B^n(b^*, K_0)\Big).$$

When the constraint $V^{dn} \geq V_1^{nn}$ is violated, i.e. $B_0 > b^*$, in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or it may choose to devalue with a non-stationary expenditure by issuing a new debt level B_1, to be different from $B^d(B_0, K_0)$, and then maintain this level thereafter. Let $B_1(B_0, K_0, A_0)$ be the value of B_1 that satisfies $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$, if such value exists. If no such B_1 exists, then it is optimal for the government not to devalue. We now present a characterization of the equilibrium.

Proposition 1. *For $\beta < 1$ sufficiently close to 1 and $\overline{e}$ sufficiently high, there exists a continuous function $\underline{b}(K, A)$, increasing in both arguments, and a positive debt level b^*, such that the following outcomes occur.*

(i) If $0 \leq B_0 \leq b^*$, then the economy has a stationary equilibrium with no default, devaluation, constant government expenditure

$$g_1 = g_2 = g^d = \tau\alpha(1)\Big[\beta f(k^n) + \theta(1-\beta)f(K_0)\Big] - \bar{e}(1-\beta)B_0,$$

and constant government bonds $B_1 = B_2 = B^d = B_0 + \tau\alpha(1)\Big[f(k^n) - \theta f(K_0)/\overline{e}\Big]$.

(ii) If $b^* \leq B_0 \leq \underline{b}(K_0, A_0)$, then the economy has a stationary equilibrium with no default and no devaluation. The dynamics for the government expenditure are $g_1^d < g^d < g_2^d$ and constant at this level thereafter, and for the government bonds $B_0 < B_1$ and constant at this level thereafter.

Proof. See the appendix.

[Figure 1 about here.]

The most interesting case is $K_0 \leq k^n$, where the government issues new debt before devaluing (Figure 1). The reason is that it tries to distribute the cost of the devaluation across periods and to smooth its expenditure. After the recovery, it can face the future higher payment of the debt with higher tax revenue. The highest level of debt for which it is possible to transfer the cost of the devaluation to the new debt and completely smooth the public expenditure is b^*.

If $B_0 > b^*$, it is not possible to distribute all the cost of the devaluation over time by issuing new debt, and therefore the government must transfer part of the cost to a reduction of the public expenditure in the first period. If it tried to maintain the public expenditure constant, the cost of repaying the debt in the future would be so high that the government would prefer not to devalue.

[Figure 2 about here.]

Proposition 1 establishes that there exists a level of debt that equalize the benefits of the devaluation with the cost that this devaluation causes.[5] That is to say, on one hand, a devaluation increases the future cost of repaying the debt and also increases the future levels of debt because the government issues more debt today to smooth the public expenditures (these two effects are collected into the term $\overline{e}(1-\beta)B_1$ of (1)). On the other hand, the devaluation eliminates the expectations of consumers and increases investment from k^{dn} to k^n, with the consequent increase in consumption and income. Note that the benefits are independent of the level of debt, while the costs are increasing in it. The figure 2 shows this intuition.

5.2 Default with devaluation

When the level of debt is very high, the government has no incentive to repay it. Thus, if consumers expect a devaluation, it also devalues, since the future costs of repaying the debt do not exist.

For lower levels of debt, we characterize two critical levels of debt. The first one, denoted by $\overline{b}$, determines a zone where the government always repays and therefore never devalues. The second, denoted by $\overline{B}$ determines another zone where the government never repays and therefore always devalues.

[5] Note that if $\beta \to 1$, the benefits and costs relevant in this case are the ones that are produced in the future.

In order to show that a crisis zone exists, we show that: first, if the level of debt is lower than the critical level $\overline{b}$, even if consumers expect a devaluation and bankers decide not to buy the new debt (observing the lower level of domestic investment), the government does not devalue and it repays its debt; second, if the level of debt is higher than the other critical level $\overline{B}$, even if consumers do not expect a devaluation and bankers buy all the debt issues, the government defaults and devalues. These two levels of debt determine three zones: (1) the zone where the government does not devalue and it repays its debt; (2) the crisis zone without expectations of devaluation, where the government always defaults and devalues; (3) the self fulfilling crisis zone, where, if consumers expect a devaluation, the government defaults and then devalues.

5.2.1 No default when consumers have expectations of devaluation

To see how the government does not devalue and repays its debt, we study the case where the government repays even if bankers do not buy government bonds and consumers expect a devaluation.

To show that this equilibrium exists, we must show that the level of debt satisfies two conditions. First, the government prefers not to default and not to devalue rather than to default and to devalue. Second, the government prefers not to default and not to devalue rather than to devalue and to repay. Finally, we must show that the upper bound found in proposition 1 is lower than the threshold found in this case.

The payoff to the government if it devalues and defaults is given by

$$\begin{aligned} V^{dd}(s, B_0, B_1) \quad &= \quad c^{dd}(K_0, A_0) + v\Big(\tau\alpha(0)\theta(\underline{e}, \overline{e})f(K_0) + \overline{e}\beta B_1\Big) + \\ &\quad \frac{\beta}{1-\beta}\left[c^{nd}(k^d, 0) + v\Big(\tau\alpha(0)f(k^d)\Big)\right], \end{aligned} \tag{4}$$

with $B_1 = 0$ because bankers do not buy any government bonds, i.e., $V^{dd}(s, B_0, 0)$.

The two constraints on government debt must be satisfied simultaneously in any equilibrium with no default and no devaluation are

$$\begin{aligned} V^{dn}(s, B_0, 0) \quad &\leq \quad V_1^{nn}(s, B_0, 0), \\ V^{dd}(s, B_0, 0) \quad &\leq \quad V_1^{nn}(s, B_0, 0). \end{aligned}$$

Let us define

$$(V^{dd} - V_1^{nn})(B_0, B_1) = H_1^{dd-nn} + v\Big(\tau\alpha(0)\theta(\underline{e}, \overline{e})f(K_0) + \overline{e}\beta B_1)\Big) -$$
$$v\Big(\tau\alpha(1)f(K_0) + \beta B_1 - B_0\Big) + \frac{\beta}{1-\beta}\left\{v\Big(\tau\alpha(0)f(k^d)\Big) - v\Big(\tau\alpha(1)f(k^{dn}) - (1-\beta)B_1\Big)\right\},$$

where $H_1^{dd-nn} \equiv c^{dd}(K_0, A_0) - c_1^{nn}(K_0, A_0) + \beta/(1-\beta)\left[c^{nd}(k^d, 0) - c_1^{nn}(k^{dn}, a^{dn})\right]$. We let $\overline{b}(K, A)$ be the largest value of B_0 for which the government weakly prefers to repay its debt, even if it cannot sell new bonds at a positive price, i.e.,

$$(V^{dd} - V_1^{nn})(\bar{b}(K_0, A_0), 0) = 0.$$

We refer to the range of debt value for which both constraints are satisfied as the no crisis zone, $B \in (\underline{b}(K), \overline{b}(K)]$. The following proposition establishes when we can have a non-empty zone and shows that, in the equilibrium characterized in proposition 1, the government always repays its debt.

Proposition 2. *For $\beta < 1$ sufficiently close to 1 and $\bar{e}$ sufficiently high, there exists a continuous function $\overline{b}(K, A) > \underline{b}(K, A)$, increasing in both arguments, such that there exists a non-empty interval of levels of government debt B, $\underline{b}(K_0, A_0) < B < \bar{b}(K_0, A_0)$ where the government does not devalue and repays its debt.*

Proof. See the appendix.

5.2.2 Default when consumers have no expectations of devaluation

In order to determine the zone were self-fulfilling crisis with default can be possible, we show that there exists a level of debt for which, even in the case that consumers have no expectations of devaluation ($\pi = 0$), the government default.

When consumers believes that the government will not devalue in the next period, the government's payoff of not devaluing and not defaulting is

$$\begin{aligned} V_0^{nn}(s, B_0, B_1) &= c_0^{nn}(K_0, A_0) + v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) + \\ &\quad \frac{\beta}{1-\beta}\left\{c_0^{n}(k^n, 0) + v\left(\tau\alpha(1)f(k^n) - (1-\beta)B_1\right)\right\}. \end{aligned} \tag{5}$$

and the government's payoff of not devaluing and defaulting is

$$\begin{aligned} V_0^{nd}(s, B_0, B_1) &= c_0^{nd}(K_0, A_0) + v\left(\tau\alpha(0)f(K_0) + (\beta B_1)\right) + \\ &\quad \frac{\beta}{1-\beta}\left\{c_0^{nd}(k^n, 0) + v\left(\tau\alpha(0)f(k^n) - (1-\beta)B_1\right)\right\}. \end{aligned} \tag{6}$$

Let be $\bar{B}(K_0, A_0)$ the lower level of debt for which the payoff to the government if it default, given by (6), is greater than the payoff given by (5). Formally

$$V^{nd}(s, B_0, B_1) \geq V_0^{nn}(s, B_0, B_1).$$

Proposition 3. *For $\beta < 1$ sufficiently close to 1, there exists a continuous and increasing function $\bar{B}(K_0, A_0) > \bar{b}(K_0, A_0)$ for all K_0, A_0, such that if $B_0 > \bar{B}(K_0, A_0)$, then the economy has an equilibrium with default and devaluation that is stationary.*

Proof. See the appendix.

5.2.3 The self-fulfilling crisis zone

As a consequence of proposition 2 and 3, for an intermediate level of government bonds, the government will default and devalue if consumers expect a devaluation for sure ($\pi = 1$), and repay and does not devalue if consumers believe that no devaluation will occur ($\pi = 0$).

Proposition 4. *For $\beta < 1$ sufficiently close to 1, there is a non-empty interval of levels of government debt B, $\bar{b}(K_0, A_0) < B \leq \bar{B}(K_0, A_0)$, where the government defaults and devalues if consumers expect a devaluation for sure ($\pi = 1$).*

Proof. See the appendix.

Propositions 1, 2, 3 and 4 establish that there exist four zones: (1) If $B \leq \underline{b}$ and if consumers expect a devaluation, the government devalues and repays its debt; (2) If $\underline{b} < B \leq \bar{b}$, the government does not devalue and repays its debt; (3) If $\bar{b} < B \leq \bar{B}$and if consumers expect a devaluation, the government devalues and does not repay its debt; (4) If $\bar{B} < B$, the government always defaults and, if consumers expect a devaluation, the government always devalues.

Figure 3 shows us the conditions under which the self-fulfilling crisis exists. The cost of the default is a fraction $(\alpha(1) - \alpha(0))$ of the gross national product and therefore independent of the level of debt, while the benefits of the default are increasing in the level of debt issued today. Figure 3 represents this intuition. The level of debt $\bar{B}$ is the level of debt such that benefits and costs of a default are equals. It is important to remark that, for levels of debt lower than $\bar{B}$, the government always repays its debt if the consumers does not expect a devaluation ($\pi = 0$), since, in contrast to Cole and Kehoe (2000), the panic does not start with the international bankers.

[Figure 3 about here.]

When the government can commit not to devalue, then, for levels of debt lower than $\bar{B}$, it finds optimal to repay its debt. If the government can commit not to default, for levels of debt higher than $\underline{b}$, it never devalues. But when the government cannot commit not to default, and cannot commit not to devalue, a default eliminates the future cost of repaying the debt and increases the benefits of a devaluation. For levels of debt close to $\bar{B}$, the net benefits of a default with devaluation are higher than the net cost of the default without devaluation. Moreover, this is true for levels of debt higher than $\bar{b}$ and lower than $\bar{B}$.

Finally, note that the benefits of the devaluation depends on consumers' expectations. If the consumers have not expectations of devaluation ($\pi = 0$), the benefits of the devaluation are zero and the government never defaults for levels of debt lower than $\bar{B}$. In summary, for levels of debt between $\bar{b}$ and $\bar{B}$, consumers' expectations of devaluation change the government incentives to default and enlarge the crisis zone.

6 Self-fulfilling crises

We can now characterize the optimal government behaviour in an equilibrium in which devaluation can occur with a positive probability $0 < \pi < 1$ depending on realizations of the sunspot variable.

A self-fulfilling devaluation crisis arises when there are two possible equilibrium outcomes, one in which the government does not devalue and chooses to repay its debt, and another in which the government defaults and devalue. Self-fulfilling crises are possible in these equilibria for certain values of the fundamentals (K, B, A); the realization of the sunspot variable determines which of these two outcomes ensues.

Because ζ is uniformly distributed on the unit interval, π is both the crucial value of ζ, and the probability that $\zeta \leq \pi$. In equilibrium, if $\zeta \leq \pi$ and B is greater than the crucial level $\overline{b}(K, A)$, then consumers predict that the government will devalue. Consumers reduce their investment in domestic capital and increase their foreign securities holdings. This reduces output and tax revenues in the next period and bankers are therefore not willing to pay a positive price for the new debt offered and thus provoke a default. If, however, $\zeta > \pi$, then consumers predict that the government will not devalue. If B is less than or equal to the crucial level $\overline{b}(K, A)$, however, then no crisis can occur, no matter what the realization of ζ. Because ζ is uniformly distributed on the unit interval, π is both the crucial value of ζ, and the probability that $\zeta \leq \pi$. If $\zeta \leq \pi$, a crisis takes place if the debt is above $\overline{b}(K, A)$ and below the upper bound, which we now denote $\overline{B}(K, A, \pi)$ since this bound also will vary with π. In the previous sections we have analyzed the limiting cases where $\pi = 0$ and $\pi = 1$.

Before characterizing the government's behavior in this equilibrium, we need to know for what regions of (B, K, A) values a self-fulfilling devaluation crisis is possible and for what regions devaluation and default are the only outcome.

The lower bound $\overline{b}(K, A)$ does not change. No crisis equilibrium is possible if the government weakly prefers to repay its debt, even if it cannot sell new bonds and consumers predict a devaluation. Explicitly characterizing the upper bound on debt $\overline{B}(K, A, \pi)$ is more difficult here because, as we shall see, optimal government policy will not, in general, be stationary in the crisis zone. We can explicitly characterize the upper bound on debt under a stationary debt policy where the capital stock is equal to k^n. Let $B^s(\pi)$ be the largest value of B for which

$$
\begin{aligned}
& c_0^{nn}(k^n, a) + v\left[\tau\alpha(1)f(k^n) - (1-\hat{\beta})B\right] + \\
& \frac{\hat{\beta}}{1-\hat{\beta}}\left\{c_0^{nn}(k^n, 0) + v\left[\tau\alpha(1)f(k^n) - (1-\hat{\beta})B\right]\right\} + \\
& \frac{\beta\pi\left(c_1^{nn}(k^n, 0) + v\left[\tau\alpha(1)f(k^n) - (1-\hat{\beta})B\right]\right)}{1-\hat{\beta}} + \\
& \frac{\beta^2\pi\left(c^{dd}(k^{dn}, a^{dn}) + v\left[\tau\alpha(0)f(k^{dn})\right]\right)}{(1-\hat{\beta})} + \\
& \frac{\beta^3\pi\left(c_0^{nd}(k^d, 0) + v\left[\tau\alpha(0)f(k^d)\right]\right)}{(1-\hat{\beta})(1-\beta)} \geq \\
& c^{dd}(k^n, a) + v\left[\tau\alpha(0)\theta(\underline{e}, \overline{e})f(k^n) + \hat{\beta}\overline{e}B\right] + \\
& \frac{\beta}{1-\beta}\left(c_0^{nd}(k^d, 0) + v\left[\tau\alpha(0)f(k^d)\right]\right),
\end{aligned}
$$

where we have denoted $\hat{\beta} = \beta(1-\pi)$. As π tends to 0 this constraint tends to $V^{dd} - V_0^{nn}(B, B) \leq 0$ in proposition 3; hence $B^s(0) = B^s$.

Proposition 5. *If $B^s(0) > \overline{b}(k^n)$ then for any probability π and for $K_0 = k^n$, there is a non-empty region of debt levels $\overline{b}(k^n, A) < B < \overline{B}(k^n, a, \pi)$ where self-fulfilling crises are possible.*

Proof. See the appendix.

We now construct an equilibrium in which devaluation and default occur with positive probability. Suppose that $K_0 = k^n$ and $B_0 > \overline{b}(k^n, A)$, and the government is faced with the following choices in period 0: devaluate and default now; plan to run the debt down to $\overline{b}(k^n, A)$ or less in T periods if no devaluation occurs; or never run the debt down. For each of these choices, we can calculate the expected payoff. The equilibrium is determined by the choice that yields the maximum expected payoff. Assuming that $B_0 \leq B^s(\pi)$, the government maintains a constant level of government spending if a devaluation does not occur but is possible. If the government plans to run its debt down to $\overline{b}(k^n, A)$ in T periods, we can use the government's budget constraints to calculate that level of government spending:

$$
g^T(B_0) = \tau\alpha(1)f(k^n) + \frac{\hat{\beta}^{T-1}\beta(1-\hat{\beta})}{1-\hat{\beta}^T}\overline{b}(k^n, A) - \frac{1-\hat{\beta}}{1-\hat{\beta}^T}B_0.
$$

If the government chooses to never run its debt down to $\overline{b}(k^n, A)$, then government spending is

$$
g^\infty(B_0) = \tau\alpha(1)f(k^n) - (1-\hat{\beta})B_0.
$$

We can now calculate the expected payoff of running the debt down to $\overline{b}(k^n, A)$ in T periods

$$
\begin{aligned}
V^T(B_0) = \quad & c_0^{nn}(k^n, a) + v\left[g^T(B_0)\right] + \frac{\hat{\beta} - \hat{\beta}^T}{1-\hat{\beta}} \left\{(1-\pi)\left[c_0^{nn}(k^n, 0) + v\left(g^T(B_0)\right)\right] + \right. \\
& \left. \pi c_1^{nn}(k^n, 0) + \pi v\left[g^T(B_0)\right] + \pi V_\pi^{dd}\right\} + \\
& \frac{\hat{\beta}^{T-1}\beta}{1-\beta}\left[c_0^{nn}(k^n, 0) + v\left(\tau\alpha(1) f(k^n) - (1-\beta)\overline{b}(k^n, A_0)\right)\right],
\end{aligned}
$$

where $\hat{\beta} = \beta(1-\pi)$ and

$$V_\pi^{dd} = \beta\left(c^{dd}(k^{dn}, a^{dn}) + v\left[\tau\alpha(0) f(k^{dn})\right]\right) + \beta^2\left(c_0^{nd}(k^d, 0) + v\left[\tau\alpha(0) f(k^d)\right]\right)/(1-\beta).$$

To determine T, we merely choose the maximum of $V^1(B_0), V^2(B_0), \ldots V^\infty(B_0)$, where

$$
\begin{aligned}
& V^\infty(B_0) = c_0^{nn}(k^n, a) + v\left[g^T(B_0)\right] + \frac{\hat{\beta}}{1-\hat{\beta}}\left\{(1-\pi)\left[c_0^{nn}(k^n, 0) + v\left(g^\infty(B_0)\right)\right] + \right. \\
& \left. + \pi c_1^{nn}(k^n, 0) + \pi v\left[g^T(B_0)\right] + \pi V_\pi^{dd}\right\}.
\end{aligned}
$$

Applying the same variational argument using in Cole and Kehoe (2000, Proposition 6), we know that for any K_0 and $B_0 \leq B^s(\pi)) - \tau\alpha(1)(f(K_0) - f(k^n))$, if we denote by V^T the government's payoff when its policy is to lower its debt to $\overline{b}(k^n, A)$ in T periods while keeping g constant, then values of $T \in \{1, 2, \ldots, \infty\}$ that maximizes $\{V^1(B_0), V^2(B_0), \ldots, V^\infty(B_0)\}$ exists, and the following are true:

(i) If $K_0 \geq k^n$, as B_0 increases, $T(B_0)$ passes through critical points where it increases by one period. Furthermore, for π close enough to 0, there necessarily are regions of $B_0 \leq B^s(\pi)$ with the full range of possibilities $T(B_0) = 1, 2, \ldots, \infty$;

(ii) If $K_0 < k^n$, then the debt may increase in the first period, but afterwards follows the same characterization as in (i) since $K_1 = k^n$ and $B_1 \leq B^s(\pi)$.

Proposition 6. *For any $\pi > 0$ for which there exists a non-empty self-fulfilling crisis zone $\overline{b}(k^n, A) < B \leq \overline{B}(k^n, A, \pi)$, there exist a equilibrium in which the transition function for capital and the price function on government debt are given by*

$$
K(B') = \begin{cases} k^n & \text{if } B' \leq \overline{B}(k^n, A, \pi) \text{ and } \alpha = \alpha(1)\ \zeta > \pi \\ k^{dn} & \text{if } B' \leq \overline{B}(k^n, A, \pi) \text{ and } \alpha = \alpha(1)\ \zeta < \pi \\ k^d & \text{otherwise} \end{cases}
$$

$$
q(B') = \begin{cases} \beta & \text{if } B' \leq \overline{b}(k^n, A) \text{ and } z(s, B', \beta) = 1 \\ \hat{\beta} & \text{if } \overline{b}(k^n) < B' \leq \overline{B}(k^n, A, \pi) \text{ and } z(s, B', \hat{\beta}) = 1 \\ 0 & \text{otherwise} \end{cases}
$$

and, depending on B_0, the following outcomes occur:

(i) If $K_0 \geq k^n$ and $B_0 \leq \bar{b}(k^n, a)$ then $c_0 = c^{nn}(K_0, A_0)$ and all other equilibrium variables are stationary: $K = k^n$, $A = 0$, $c_t = c^{nn}(k^n, 0)$ for $t \geq 1$, $B = B_0 - \tau\alpha(1)\left(f(K_0) - f(k^n)\right)$, $g = \tau\alpha(1)f(k^n) - (1-\beta)B$ $q = \beta$ and $e = \underline{e}$. In this case no devaluation occurs;

(ii) If $\bar{b}(k^n, A) < B_0 \leq \overline{B}(k^n, A, \pi)$, then a default and devaluation occurs with probability π in the first period and every subsequent period in which $B > \bar{b}(k^n, A)$. If $B_0 \leq B^s(\pi) - \tau\alpha(1)\left(f(K_0) - f(k^n)\right)$, optimal government policy involves running down the debt to $\bar{b}(k^n, A)$ in $T(B_0)$ periods, while smoothing government expenditures as described in Proposition 6. If $T(B_0)$ is finite and a crisis does not occur, then following period $T(B_0)$, the equilibrium outcomes are those in (i). For $B_0 > B^s(\pi) - \tau\alpha(1)\left(f(K_0) - f(k^n)\right)$, the equilibrium reaches the outcome described in proposition 6 in at most two periods.

(iii) If $K_0 < k^n$ and $B_0 \leq \bar{b}(k^n, A)$, then there is no devaluation in period 0, and from period 1 onward, the outcomes correspond to those described in (i) if under the government's optimal policy, $B_1 \leq \bar{b}(k^n, A)$ or in (ii) if not.

(iv) If $B_0 > \overline{B}(K_0, a, \pi)$, then the government defaults and devalues. Then $c_0 = c^{dd}(K_0, A_0)$, $g_0 = \tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0)$, and all other equilibrium variables are stationary: $K = k^d$, $c = c^{dd}(k^d, 0)$, $B = 0$, $A = 0$, $g = \tau\alpha(0)f(k^d)$, $q = 0$, and $e = \bar{e}$.

Proof. See the appendix.

7 The Argentine crisis in 2001-2002

In this section, we use the model to help us to understand events in Argentina in 2001-2002. We show that, while Argentina's debt/GDP ratio was not high by international standards, it was in the crisis zone where, with some probability, the government find optimal to default and devalue.

Let us remember what happened in Argentina during 2001. During 2001 GDP fell by more than 20 percent and investment decreased by more than 5 percent of GDP. At the same time, the trade balance yielded a surplus in 2001, and foreign reserves fell dramatically. The ratio of external debt to the GDP increased so much that it forced the Argentine government to default in December 2001. Afterwards, in January 2002, the government devalued the peso by 40 percent. Figure 4 document these facts.

[Figure 4 about here.]

On March 16, President De La Rúa rejected the plan presented by Economics Minister López Murphy to reduce the fiscal deficit. The new minister, Domingo Cavallo presented a new economic plan, but on March 28, the congress refused to allow Cavallo to cut government salary and pension expenditure, and the government sold debt to cover the deficit. Between April and August, several announcements on changes in the exchange rate policy were made. First, on April 12, Cavallo announced that the peso would be pegged to the euro (and possibly to the yen). In May, the government announced economic plans that included currency changes, and, on June 18, the Argentine government announced a complex set of new economic policies, including the installation of multiple exchange rates to help the country's exporters. On October 30, the government could not sell new debt and started to restructure its debt, and finally on December 23, the government defaulted, and, on January 11, 2002, the government devalued the peso.

As we assume in the model, the nominal devaluation generated a real devaluation (see Figure 5). In March 2001, 92.3 percent of the nominal devaluation was a real devaluation. Even two years later, in March 2003, 71.7 percent of the nominal devaluation that had taken place since March 2001 was a real devaluation.

[Figure 5 about here.]

To show that the external debt of Argentina was in the crisis zone where the government found it optimal to default and to devalue, we calibrate the model to match the decrease in investment of domestic capital, the reduction in production, the increase in trade balance surplus, and the increase in debt levels observed throughout 2001 in Argentina.

We write the utility function for the consumers and the government as

$$E \sum_{t=0}^{\infty} \beta^t \left(c_t + \log(g_t) \right),$$

and the technology the adjustment cost function is given by

$$\begin{aligned} f(K) &= \gamma K^s, \\ \Phi(a) &= \phi_1 + \frac{\phi_2 a^2}{2}. \end{aligned}$$

The capital share in GDP was taken from Kydland and Zarazaga (2002), $s = 0.4$. The discount factor $\beta = 0.963$ corresponds to an international interest rate of 3.84% that was taken from the interest rate in 2001 of one year U.S. government treasury bills. The permanent drop of the productivity associated with a default is taken from Cole and Kehoe (2000) and implies a fall in productivity of 5%, $\alpha(0) = 0.95$. The temporary drop of the productivity related to a devaluation is set to reproduce the reduction in the investment rate observed between the year 2000 and 2001, $\theta(\underline{e}, \overline{e}) = 0.9892$ that represents a fall in productivity of 1.92%.

Setting the probability of devaluation $\pi = 0.0473$, we find that the yield of the Argentine government bonds nominated in dollars with a year of maturity is $0.09 = [\beta(1-\pi)]^{-1} - 1$ that corresponds with the government bonds issued with those characteristics on April 19, 2001. This means a risk premium of a 5.16% on the Argentine government bonds. The previous exchange rate to the crisis is fixed in $\underline{e} = 1$ and the exchange rate after the devaluation in $\overline{e} = 1.4$, which corresponds to the exchange rate set by the Argentine government on January 11, 2002. Table 1 shows the values of the parameters calibrated without solving the model.

The next five parameters ϕ_1, ϕ_2, τ, γ, δ and A_0 are calibrated solving the model. The adjustment cost parameters ϕ_1 and ϕ_2 are fixed to reproduce the the investment rate in the Argentine GDP 2000, $i/y = 0.18$, and the reduction in international reserves of the Central Bank that during the year 2001 reached, 9200 million of dollars 3.42% of the 2001 output. The tax rate and the TFP, γ, are calibrated from the steady state budget constraint of the government to reproduce the shares of government spending and public debt in the Argentine GDP 2000: , $g/y = 0.19$ and $B/y = 0.45$ respectively. We obtain a depreciation rate of $\delta = 0.0815$ for a capita-output ratio of $K/Y = 3$. Finally, the initial value of the foreign assets A_0 is chosen to reproduce the share in GDP of the trade balance in 2000, that is to say, a surplus of 0.41%. Table 2 shows the values of the parameters.

[Table 1 about here.]

[Table 2 about here.]

With these values of the parameters and with $K_0 = k^n$ the levels of debt that determine the different zones of the model are presented in Table 3. Consequently, the initial values of debt for which a self-fulfilling devaluation crisis can occur in the first period are between 19.59% and 236.78% of the output. The level of debt/GDP in 2000 reached 45% of the output, which means that it was in the crisis zone.

[Table 3 about here.]

8 Conclusions

In this paper, we present a model to characterize optimal debt policy in a environment in which defaults and devaluations are possible We show that, when the government cannot commit not to default, and it cannot commit not to devalue, consumer's expectations of devaluation make the safe level of government debt low.

Our model helps us to understand the crisis in Argentina in 2001-2002. While Argentina's debt/GDP ratio was not high by international standards, the announcements of devaluation increased consumers' expectations of devaluation, and reduced the safe level of government debt. When we calibrate the model to match the key features of the Argentina economy in 2001, we show that the debt/GDP ratio was consistent with Argentina being in the crisis zone where with some probability the government defaults and devalues, as it did.

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Appendix

Proof of Proposition 1

Proof. We first consider the case where the equilibrium is stationary after period 1. In this case, if the government devalues and repay its debt, government consumption is given by

$$g^d(B_0, K_0) = \tau\alpha(1)\Big[\beta f(k^n) + \theta(\underline{e}, \overline{e})(1-\beta) f(K_0)\Big] - \overline{e}(1-\beta)B_0,$$

and government debt becomes constant after one period at $B^d(B_0, K_0) = B_0 + \tau\alpha(1)\Big[f(k^n) - \theta(\underline{e}, \overline{e}) f(K_0)\Big]/\overline{e}$. If the government does not devalues and repay its debt, government consumption is given by

$$g^n(B_0, K_0) = \tau\alpha(1)\Big[\beta f(k^{dn}) + (1-\beta) f(K_0)\Big] - (1-\beta)B_0,$$

and government debt becomes constant after one period at $B^n(B_0, K_0) = B_0 + \tau\alpha(1)\Big[f(k^{dn}) - f(K_0)\Big]$. If these payoffs satisfy the constraint $V^{dn}\left(s, B_0, B^d(B_0, K_0)\right) \geq V_1^{nn}\left(B_0, B^n(B_0, K_0)\right)$, then this is an equilibrium outcome.

To show that there is a unique $b^* > 0$ such that

$$V^{dn}\left(s, b^*, B^d(b^*, K_0)\right) = V_1^{nn}\Big(s, b^*, B^n(b^*, K_0)\Big),$$

let us write this constraint as $(V^{dn} - V_1^{nn})(b^*) \geq 0$ where

$$\begin{aligned}(V^{dn} - V_1^{nn})(b^*) &= H_1^{dn-nn} + v\Big(\tau\alpha(1)\theta(\underline{e}, \overline{e}) f(K_0) + \overline{e}(\beta B^d(b^*, K_0) - b^*)\Big) - \\ &-v\Big(\tau\alpha(1) f(K_0) + (\beta B^n(b^*, K_0) - b^*)\Big) + \frac{\beta}{1-\beta}\Big\{v\left(\tau\alpha(1) f(k^n) - (1-\beta)\overline{e}B^d(b^*, K_0)\right) - \\ &-v\Big(\tau\alpha(1) f(k^{dn}) - (1-\beta)B^n(b^*, K_0)\Big)\Big\},\end{aligned}$$

and

$$H_1^{dn-nn} \equiv c^{dn}(K_0, A_0) - c_1^{nn}(K_0, A_0) + \beta\left[c_0^{nn}(k^n, 0) - c_1^{nn}(k^{dn}, a^{dn})\right]/(1-\beta).$$

Notice that $(V^{dn} - V_1^{nn})(0) > 0$ as $\beta \to 1$, and that $(V^{dn} - V_1^{nn})(b) \to -\infty$ as

$$b \to \tau\alpha(1)/\overline{e}\left[\theta(\underline{e}, \overline{e}) f(K_0) + \beta f(k^n)/(1-\beta)\right] \equiv B_0^{dn-nn}(B^d(b, K_0), \beta),$$

i.e., $g^d(b^*, K_0)$ goes to zero. Finally, differentiating $V^{dn} - V_1^{nn}$ yields

$$\frac{d\,(V^{dn} - V_1^{nn})(b)}{d\,b} < 0.$$

Consequently, since $(V^{dn} - V_1^{nn})$ is continuous in b, there is a unique b^* such that $0 < b < B_1^{dn-nn}(B^n(b, K_0), \beta)$. That is, $(V^{dn} - V_1^{nn})(b^*) = 0$ and $(V^{dn} - V_1^{nn})(b) > 0$ for all $B < b^*$, while $(V^{dn} - V_1^{nn})(b) < 0$ for all $B > b^*$.

Whenever the constraint $V^{dn} \geq V_1^{nn}$ is violated, i.e. $B_0 > b^*$, in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or it may choose to devalue with a non-stationary expenditure by issuing a new debt level B_1, to be different from $B^d(B_0, K_0)$, and then maintain this level thereafter. Let $B_1(B_0, K_0, A_0)$ be the value of B_1 that satisfies $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$, if such value exists. If no such B_1 exists, then it is optimal for the government not to devalue.

We now argue that there is a continuous increasing function $\underline{b}(K, A)$ such that for all $b^* \leq B_0 \leq \underline{b}(K_0, A_0)$ it is optimal for the government to devalue in period 0 and maintain a constant level of government expenditure different from period 1 on. In this case, the government maintains a level of debt that differs from B_0. For all $B_0 > \underline{b}(K_0, A_0)$, it is optimal for the government not devalue. We then let

$$\begin{aligned}\underline{b}(K_0, A_0) &= \max\ B_0(B_1, K_0, A_0)\\ &\quad \text{subject to}\\ &\quad 0 \leq B_1 \leq B_1(B_0, K_0, A_0),\end{aligned}$$

the constraint $B_1 \leq B_1(B_0, K_0, A_0)$ binds if and only if the constraint $V^{dn} \geq V_1^{nn}$ binds in period 0 when $B_0 = \underline{b}(K_0, A_0)$, i.e. $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$. Differentiating $\left(V^{dn} - V_1^{nn}\right)(B_0, B_1)$, we obtain

$$\frac{\partial\left(V^{dn} - V_1^{nn}\right)(B_0, B_1)}{\partial B_0} < 0.$$

Furthermore, since $(V^{dn} - V_1^{nn})(0, B_1) > 0$ as $\beta \to 1$ and $(V^{dn} - V_1^{nn})(B_0, B_1) \to -\infty$ as $B_0 \to B_0^{dn-nn}$, then there is a unique $B_0(B_1)$ for which the constraint holds with equality. Since $\partial(V^{dn} - V_1^{nn})(B_0, B_1)/\partial B_0 \neq 0$ the implicit function theorem implies that $B_0(B_1, K_0, A_0)$ is continuous. Since $B_0(B_1, K_0, A_0)$ is continuous in B_1, it achieves a maximum on the compact constraint set. Finally, the dynamics of the government expenditure and government bonds are the following:

$$\begin{aligned}g_1^d &= \tau\alpha(1)\theta f(K_0) + \bar{e}\beta B_1 - \bar{e}B_0,\\ g_2^d &= \tau\alpha(1)f(k^n) - \bar{e}(1-\beta)B_1.\end{aligned}$$

In order to prove part (ii), note that $\partial(V^{dn} - V^{nn})(B_0, B_1)/\partial B_0 < 0$, and that $\partial(V^{dn} - V^{nn})(B_0, B_1)/\partial B_1 > 0$ if $g_2^d - g_1^d > 0$, and $\partial(V^{dn} - V^{nn})(B_0, B_1)/\partial B_1 < 0$ if $g_2^d - g_1^d < 0$. Moreover, observe that as B_0 is greater than b^*, $(V^{dn} - V^{nn})(B_0, B_1(B_0))$ is positive, but as B_0 is lower than b^*, $(V^{dn} - V^{nn})(B_0, B_1(B_0))$ is negative. Having reached the threshold $B_0 = b^*$, with $B^d = B^{d*} = b^* + \tau\alpha(1)[f(k^n) - \theta f(K_0)]/\bar{e}$, the dynamics of the public expenditure is given by

$$g_2^d - g_1^d = \tau\alpha(1)\Big(f(k^n) - \theta f(K_0)\Big) + \bar{e}(B_0 - B_1) = \bar{e}\Big[(B_0 - b^*) - (B_1 - B^{d*})\Big]. \tag{7}$$

So the public expenditure can only be increased in this case by increasing government debt B_1, and lower the difference $B_0 - b^*$. Finally it is easy to show that $g_1^d < g^d < g_2^d$, since $g_1^d - g^d = -\bar{e}\beta\Big[(B_0 - b^*) - (B_1 - B^{d*})\Big]$ and $g_2^d - g^d = \bar{e}(1-\beta)\Big[(B_0 - b^*) - (B_1 - B^{d*})\Big]$. □

Proof of Proposition 2

Proof. To proof that the zone where the government find it optimal not to default and not to devalue exists, we must show that

$$\underline{b}(K_0, A_0) < \bar{b}(K_0, A_0, \beta). \tag{8}$$

From proposition 1, the government, after deciding not to default, will devalue if the initial government bond level B_0 is lower than $\underline{b}(K_0, A_0)$, beyond which there will be no devaluation. Next, the government will default and devalue if the initial government bond level B_0 is higher than $\bar{b}(K_0, A_0)$, below which there will be no devaluation and no default.

Now, let us define

$$\begin{aligned}(V^{dd} - V_1^{nn})(B_0, 0) = H_1^{dd-nn} &+ v\Big(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0)\Big) - v\Big(\tau\alpha(1)f(K_0) - B_0\Big)\\ &+ \frac{\beta}{1-\beta}\left\{v\Big(\tau\alpha(0)f(k^d)\Big) - v\Big(\tau\alpha(1)f(k^{nn})\Big)\right\},\end{aligned}$$

where $H_1^{dd-nn} \equiv c^{dd}(K_0, A_0) - c_1^{nn}(K_0, A_0) + \beta/(1-\beta)\left[c^{nd}(k^d, 0) - c_1^{nn}(k^{nn}, a^{nn})\right]$. Observe that $(V^{dd} - V_1^{nn})(0, 0) < 0$ as $\beta \to 1$, which implies that $c^{nd}(k^d, 0) - c_1^{nn}(k^{nn}, a^{nn}) + v\Big(\tau\alpha(0)f(k^d)\Big) - v\Big(\tau\alpha(1)f(k^{nn})\Big) < 0$; and that $(V^{dd} - V_1^{nn})(B_0, 0) \to +\infty$ as $B_0 \to \tau\alpha(1)f(K_0) \equiv B_0^{dd-nn}(0, \beta)$.

Here, it is easy to prove that $\partial(V^{dd} - V_1^{nn})(B_0, 0)/\partial B_0 = v'(\tau\alpha(1)f(K_0) - B_0) > 0$ so that there exists a threshold $(V^{dd} - V_1^{nn})(\bar{b}(K_0, A_0, \beta), 0) = 0$.

Then, condition (8) holds (see Figure 8) since in proposition 1 was proved that $(V^{dn} - V_1^{nn})(0) > 0$ as $\beta \to 1$, and that $(V^{dn} - V_1^{nn})(b) \to -\infty$ as

$$b \to \tau\alpha(1)/\bar{e}\left[\theta(\underline{e}, \bar{e})f(K_0) + \beta f(k^n)/(1-\beta)\right] \equiv B_0^{dn-nn}(B^d(b, K_0), \beta).$$

In addition, $\bar{b}(K_0, A_0, \beta)$ is strictly increasing in β and we can set $\bar{b}(K_0, A_0, \beta)$ as close as B_0^{dd-nn} as wished. This means that the zone where the government find it optimal not to default and not to devalue exists.

[Figure 6 about here.]

□

Proof of Proposition 3

Proof. Compare the constraints on the government's debt that must be satisfied in any equilibrium with default and no devaluation, in the case that consumers have no expectations on devaluation, is so close to Cole and Kehoe (2000, Proposition 1) that can be followed in their proof. Finally, to prove that the crises zone exists, i.e., $\bar{B}(K_0, A_0) > \bar{b}(K_0, A_0)$, we define

$$(V^{dd} - V_0^{nn})(B_0, B_1) = H_0^{dd-nn} + v\Big(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \beta\bar{e}B_1\Big) - v\Big(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\Big)$$
$$+\frac{\beta}{1-\beta}\left\{v\Big(\tau\alpha(0)f(k^d)\Big) - v\Big(\tau\alpha(1)f(k^n) - (1-\beta)B_1\Big)\right\},$$

where $H_0^{dd-nn} \equiv c^{dd}(K_0, A_0) - c_0^{nn}(K_0, A_0) + \beta/(1-\beta)\left[c^d(k^d) - c^n(k^n)\right]$. Next, we will compare the constraints that set both levels $(V^{dd} - V_1^{nn})(B_0, 0)$ and $(V^{dd} - V_0^{nn})(B_0, B_1)$. We show that $(V^{dd} - V_1^{nn})(B_0, 0) > (V^{dd} - V_0^{nn})(B_0, B_1)$ for all B_1. Given any $B_1 > 0$, it is verified that $(V^{dd} - V_0^{nn})(B_0, B_1) \to +\infty$ as $B_0 = \tau\alpha(1)f(K_0) + \beta B_1$ is greater than B^{dd-nn} stated in proposition 2, and in addition it is easy to show that

$$(V^{dd} - V_1^{nn})(B_0, 0) > (V^{dd} - V_0^{nn})(B_0, 0),$$

given that subtracting both we find

$$(V^{dd} - V_0^{nn})(B_0, 0) - (V^{dd} - V_1^{nn})(B_0, 0) = c_1^{nn}(K_0, A_0) - c_0^{nn}(K_0, A_0) +$$
$$\beta/(1-\beta)\left[c_1^{nn}(k^{nn}, a^{nn}) - c_0^{nn}(k^n, 0) + v\Big(\tau\alpha(1)f(k^{nn})\Big) - v\Big(\tau\alpha(1)f(k^n)\Big)\right],$$

which is negative for $\beta < 1$ sufficiently close to one. Then, by continuity of $(V^{dd} - V_0^{nn})(B_0, B_1)$ this means that the zone were the government default and devalues if consumers expect a devaluation for sure exists. □

Proof of Proposition 4

Proof. The proof consists of showing that, when consumers expect a devaluation, the government prefers to default and devalue rather than to default and not to devalue for all B_1, i.e., $(V^{dd} - V_1^{nd})(B_0, B_1) > 0$, with

$$(V^{dd} - V_1^{nd})(B_0, B_1) = H_1^{dd-nd} + v\Big(\tau\alpha(0)f(K_0) + \beta B_1\Big) - v\Big(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \beta\bar{e}B_1\Big),$$

where $H_1^{dd-nd} \equiv c^{dd}(K_0, A_0) - c_1^{nd}(K_0, A_0) + \beta/(1-\beta)\left[c^{nd}(k^d, 0) - c_1^{nd}(k^{nd}, a^{nn})\right]$, $c_1^{nd}(K, A) = (1-\tau)\alpha(0)f(K_0) + [r^*a - a^{nn} - \phi(a^{nn})]\,\underline{e} - k^{nd}$ and k^{nd} satisfies $\beta(1-\tau)\alpha(0)\theta(\underline{e}, \bar{e})f'(k^{nd}) = 1$. □

Proof of Proposition 5

Proof. If the government prefers not to devalue and not to default rather than to devalue and to default, conditional on keeping a constant debt level, then it certainly does so under the optimal debt policy; hence, $B^s(\pi) \leq \overline{B}(k^n, A, \pi)$. As π increases, we can use the implicit function theorem to show that $B^s(\pi)$ decreases, making it more difficult for a nonempty interval $\bar{b}(k^n, A) < B \leq B^s(\pi)$ to exist. Notice that $B^s(0) > \bar{b}(k^n, A)$ implies that, if $K_0 = k^n$ and $B_0 = B_1 = \bar{b}(k^n, A)$, then the constraint $V^{dd} - V_0^{nn}(B, B) \leq 0$ with $q = \beta$ and $K_1 = k^n$ is strictly satisfied, and hence it is also satisfied by B_0 slightly larger than $\bar{b}(k^n, A)$. Since this holds for any π, $\overline{B}(k^n, A, \pi) > \bar{b}(k^n, A)$. $\square$

Proof of Proposition 6

Proof. The characterization of the crisis equilibrium works similarly to that of Proposition 4. In the no devaluation equilibrium, the stationary debt policy characterizes optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind. In the crisis equilibrium, $T(B_0)$ and $V_g^T(B_0)$ characterize optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind. When the participation constraint does bind, we can use the identical logic as that in the proof of proposition 3 to argue that, if $K = k^n$ then the equilibrium adjusts to that characterized by $T(B)$ and $V_g^T(B)$ in at most one period; in particular, if $B_1 > B^s(\pi)$, then $B_2 < B^s(\pi)$ and the government runs down its debt in $T(B_2)$ periods starting in the period after $K = k^n$. If $K_0 = k^n$, this is period 1, but if $K_0 \neq k^n$ and if the participation constraint binds in period 1, it is period 2. We need to also allow for the possibility that $K = k^n$ if the government needs to lower either B_1 or B_2 so much as to satisfy the participation constraints in period 0 or period 1 so that B_1 or B_2 is less than or equal to $\bar{b}(k^n, a)$. Otherwise, the proof follows the identical logic as that of proposition 3. The notation involved in writing out the expressions for $V_g^{dn} - V_g^{nn}$ analogous to those found in the proofs in proposition 3 and proposition 4 is straightforward, but tedious. We omit it here. $\square$

Table 1: **Parameters Calibrated without Solving the Model**

Parameter value	**Parameter meaning**	**Source**
$s = 0.400$	Capital Share	Kydland and Zarazaga (2000)
$\beta = 0.963$	Discount Factor	1 year Gov. Securities Treasury bills
$\alpha(0) = 0.950$	Permanent drop in productivity	Cole and Kehoe (2000)
$\theta(\underline{e}, \overline{e}) = 0.981$	Temporal drop in productivity	Investment rate reduction in 2001
$\pi = 0.047$	Devaluation Probability	Risk premium of Argentine Debt
$\underline{e} = 1.000$	Exchange rate pegged to US dollar	
$\overline{e} = 1.400$	Exchange rate set on January 11, 2002	

Table 2: **Parameters Calibrated by Solving the Model**

Parameters		**Calibration Targets**
$\phi_1 = 71.54$	Adjustment cost	Investment rate in 2000 ($i/y = 0.18$)
$\phi_2 = 4.3478 \times 10^{-5}$	Adjustment cost	Reduction in reserves over GDP (3.42%)
$\tau = 0.2593$	Tax Rate	Government spending over GDP in 2000 ($g/y = 0.19$)
$\gamma = 1206$	T.F.P.	External Debt over GDP in 2000 ($B/y = 0.45$)
$\delta = 0.0815$	Depreciation rate	Capital-Output ratio ($k/y = 3$)
$A_0 = 3436.9$	Initial foreign assets	Trade balance surplus (0.41%)

Table 3: **Crisis Zones**

	Debt/GDP (%)
$\underline{b}(k^n, A_0)$	8.66%
$\overline{b}(k^n, A_0)$	19.59%
$B^s(\pi)$	236.78%

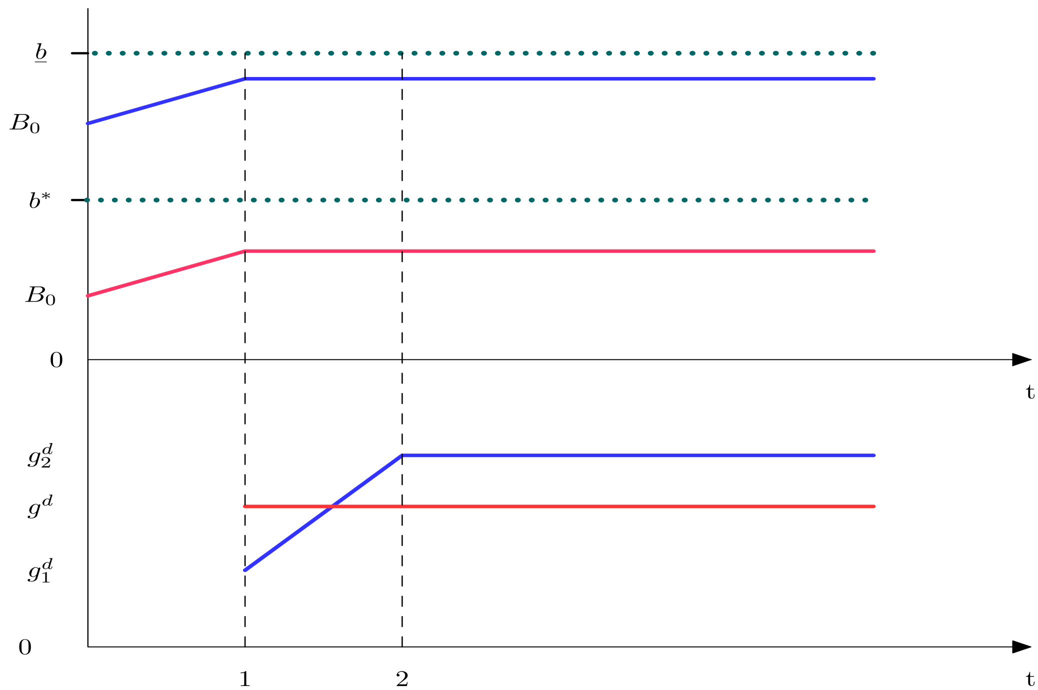


Figure 1: Bonds and government expenditure paths described IN proposition 1 when $K_0 \leq k^n$.

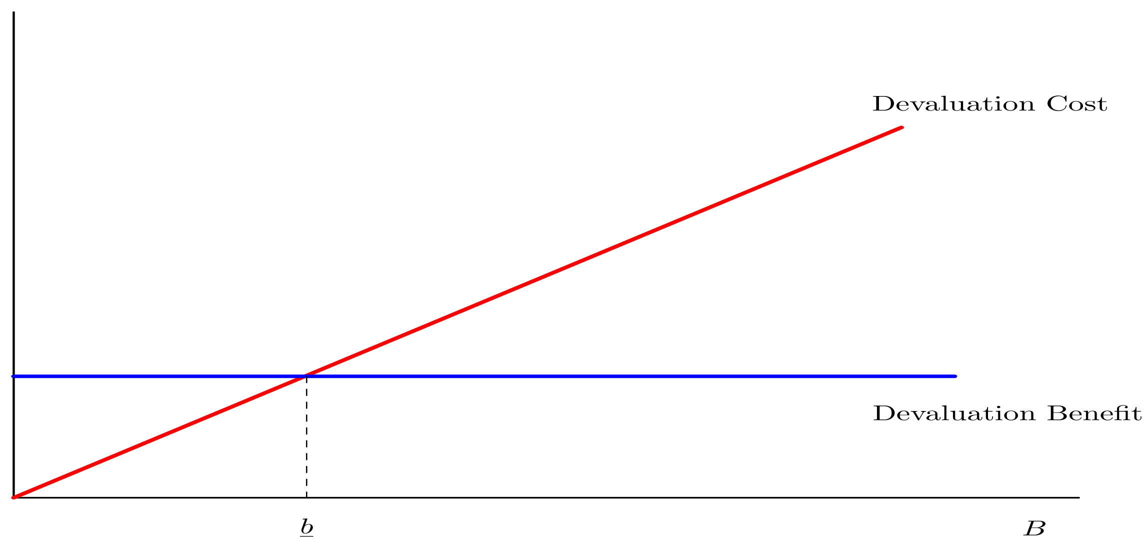


Figure 2: Devaluation without default.

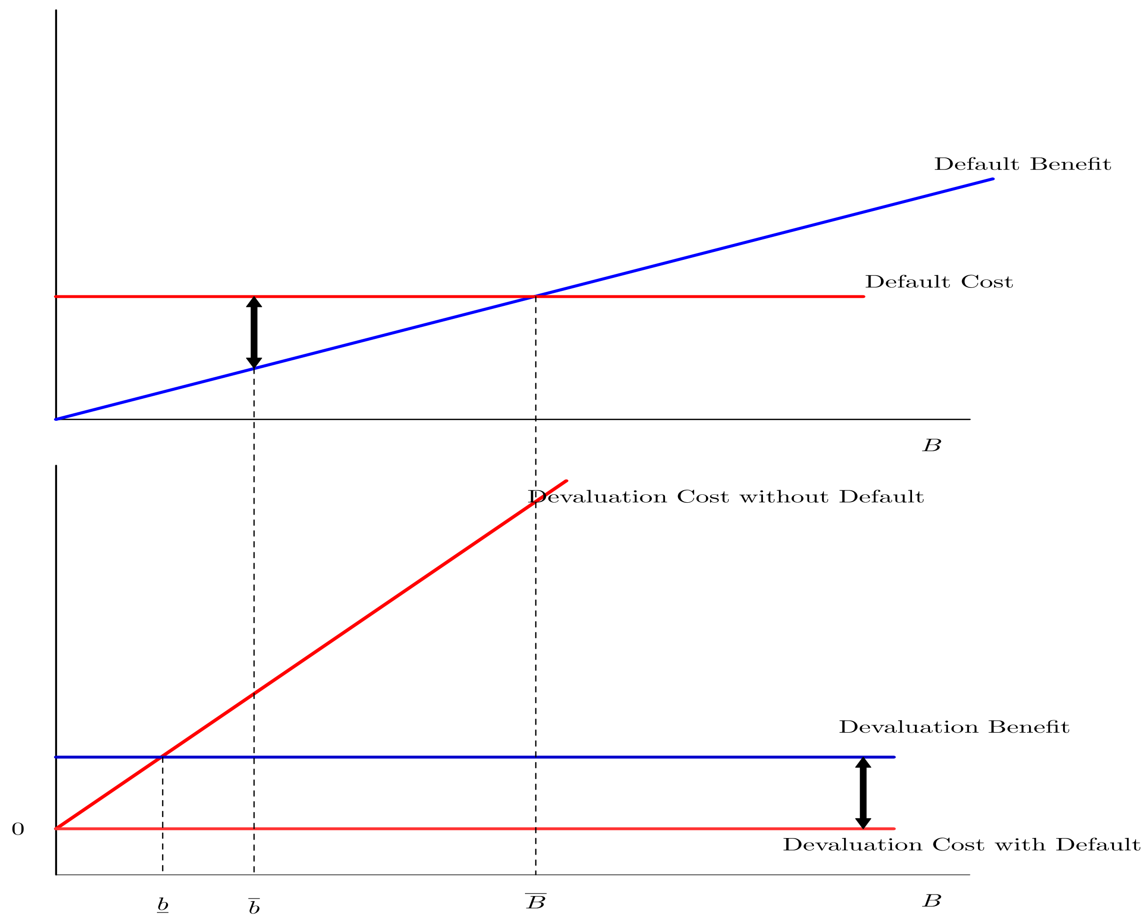
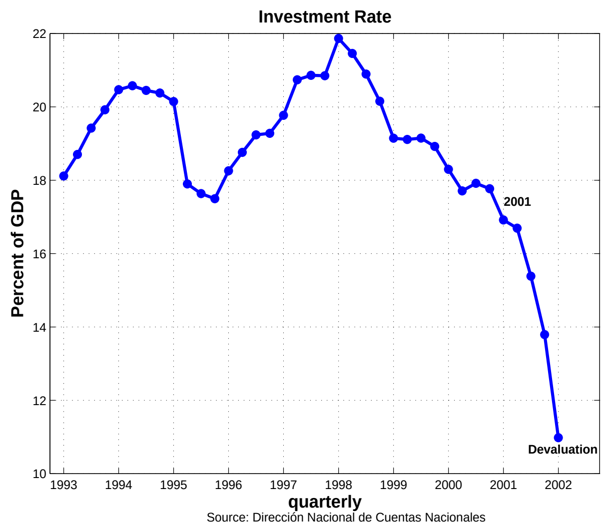
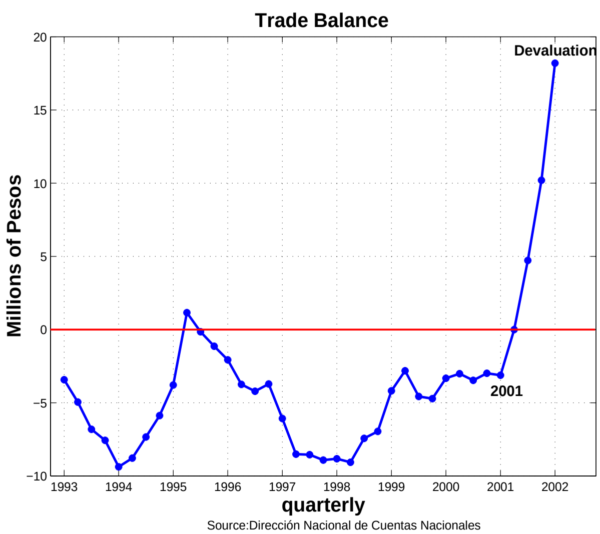


Figure 3: Existing zones

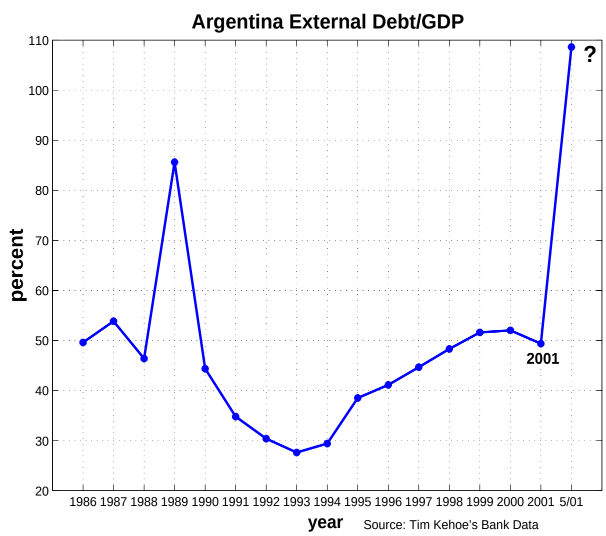
(a) Output.



(b) Investment Rate.



(c) Trade Balance over output.



(d) Debt over output.

Figure 4: Argentinian facts.

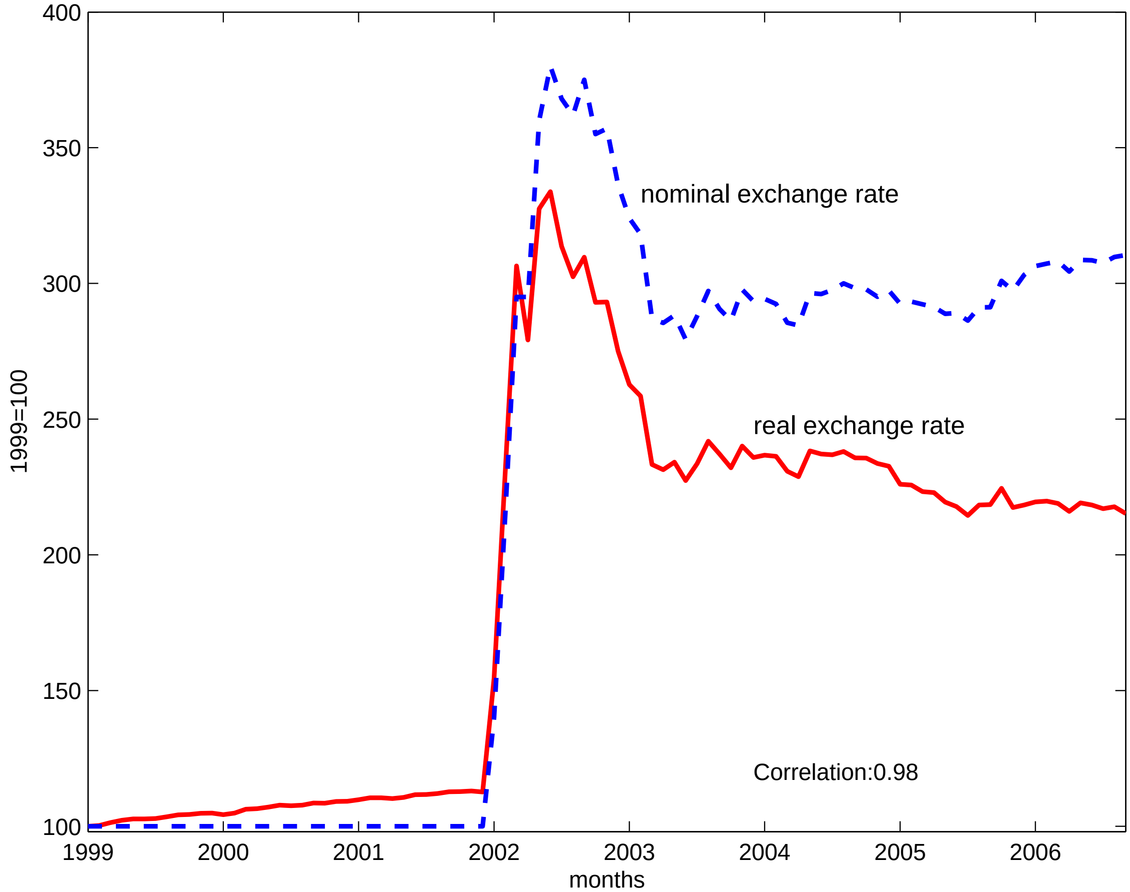


Figure 5: Exchange rates.

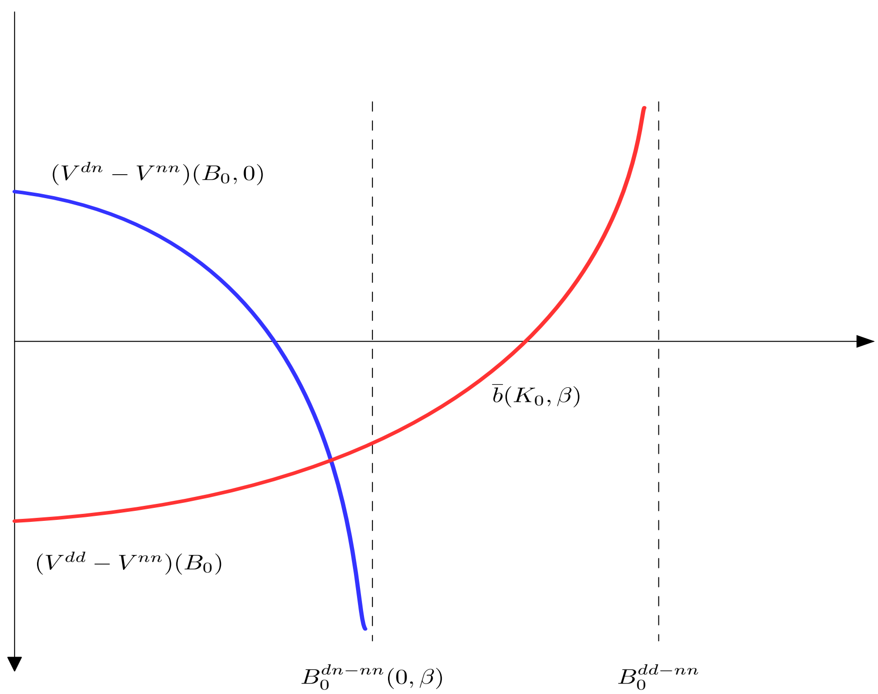


Figure 6: The no crises without devaluation zone upper bound $\bar{b}(K_0, \beta)$, and the $(V^{dn} - V_1^{nn})(B_0, B_1)$ function evaluated at $B_1 = 0$.