# Are Shocks to the Terms of Trade Shocks to Productivity?

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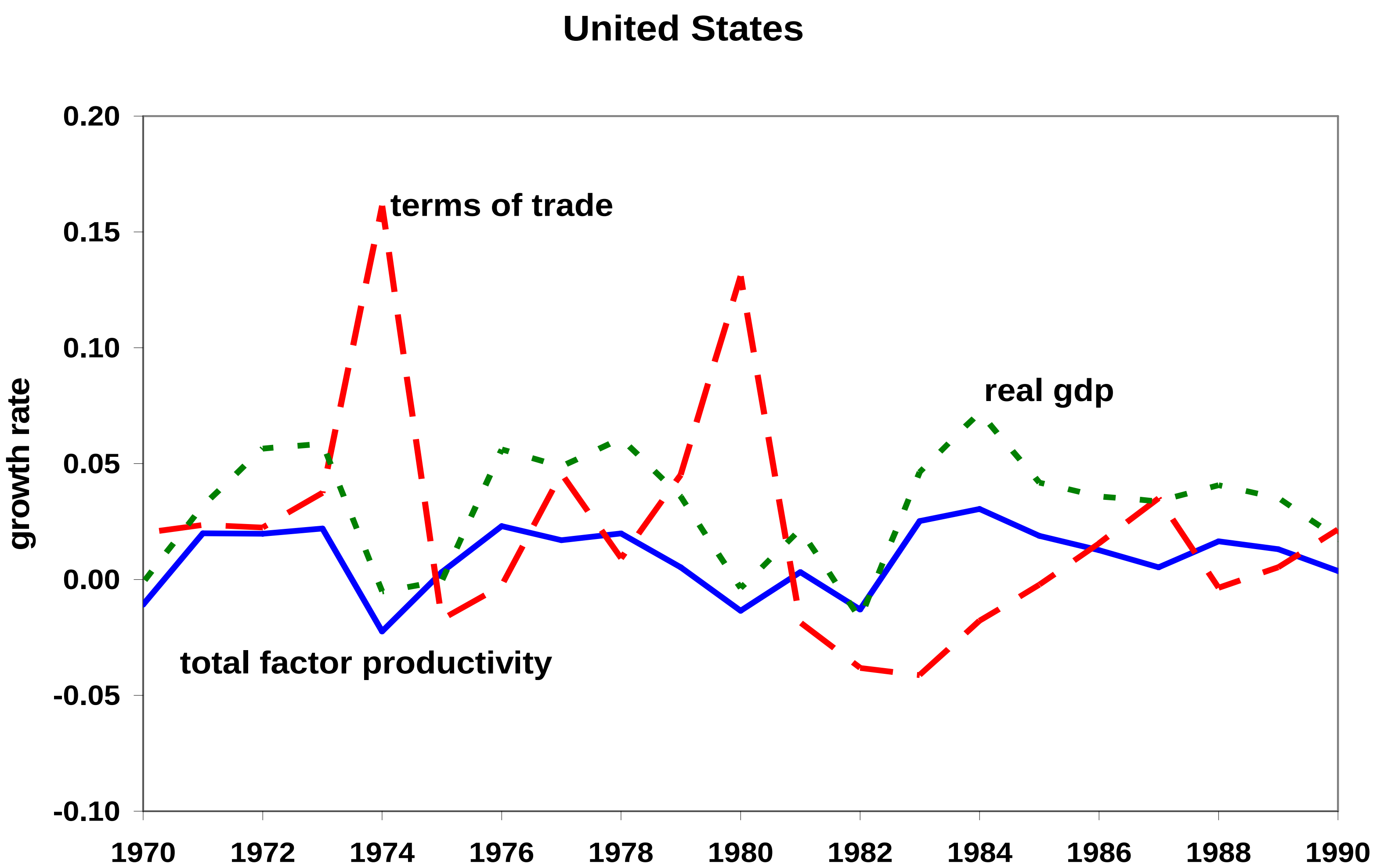
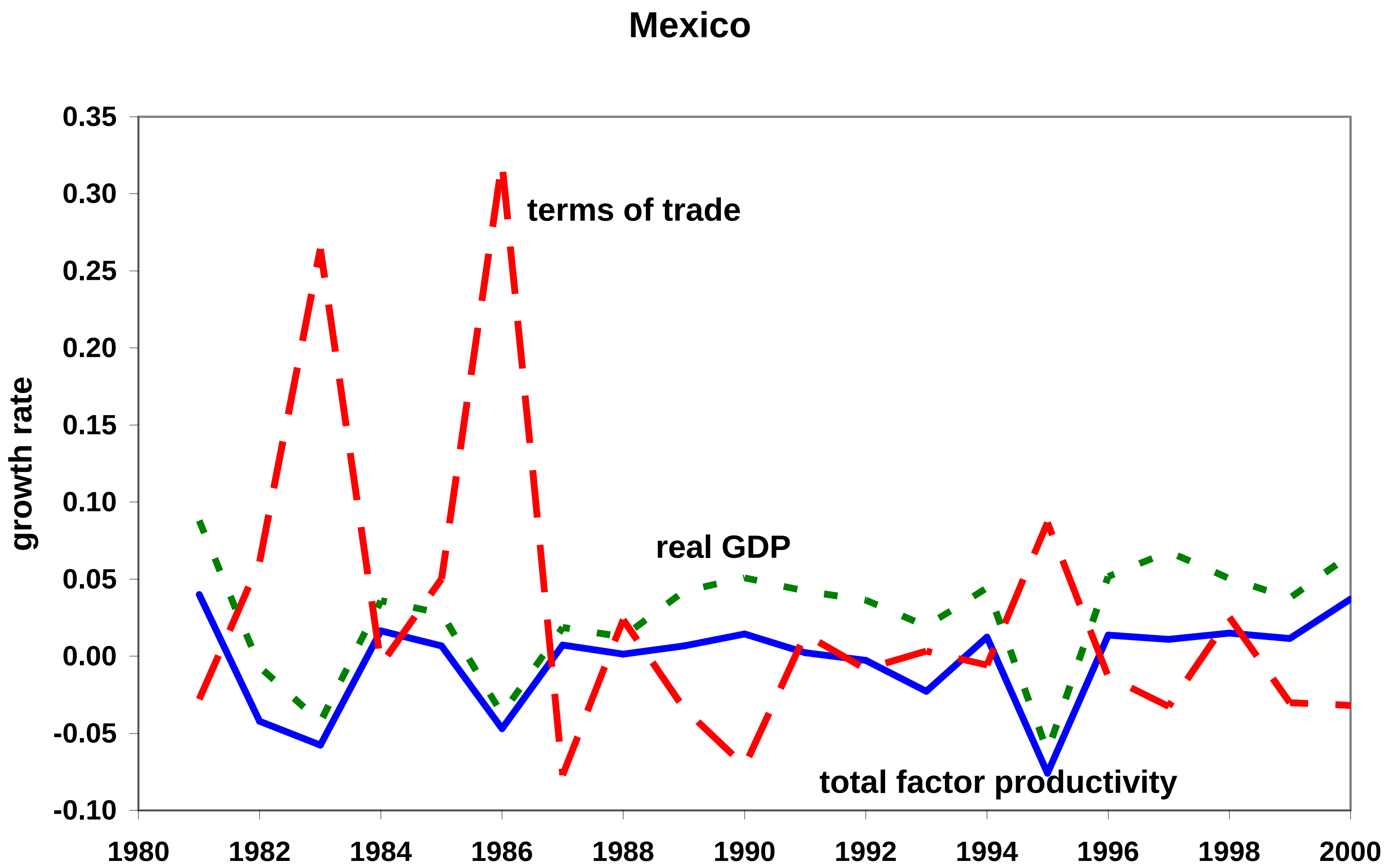
# Evidence on terms of trade, GDP, and TFP



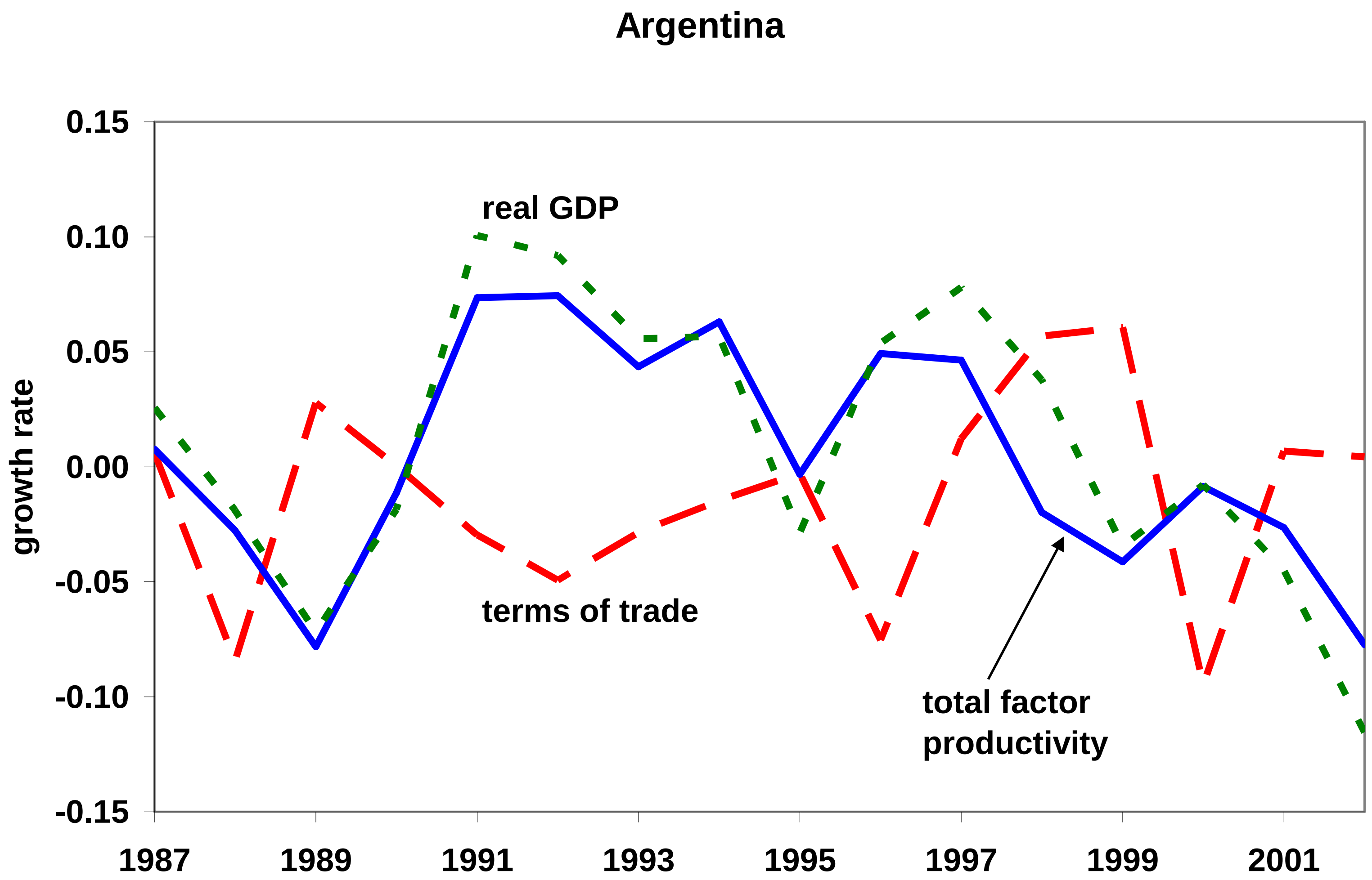

$$\rho\left(tot, tfp\right) = -0.46 \qquad \rho\left(tot, gdp\right) = -0.30$$

# Evidence on terms of trade, GDP, and TFP



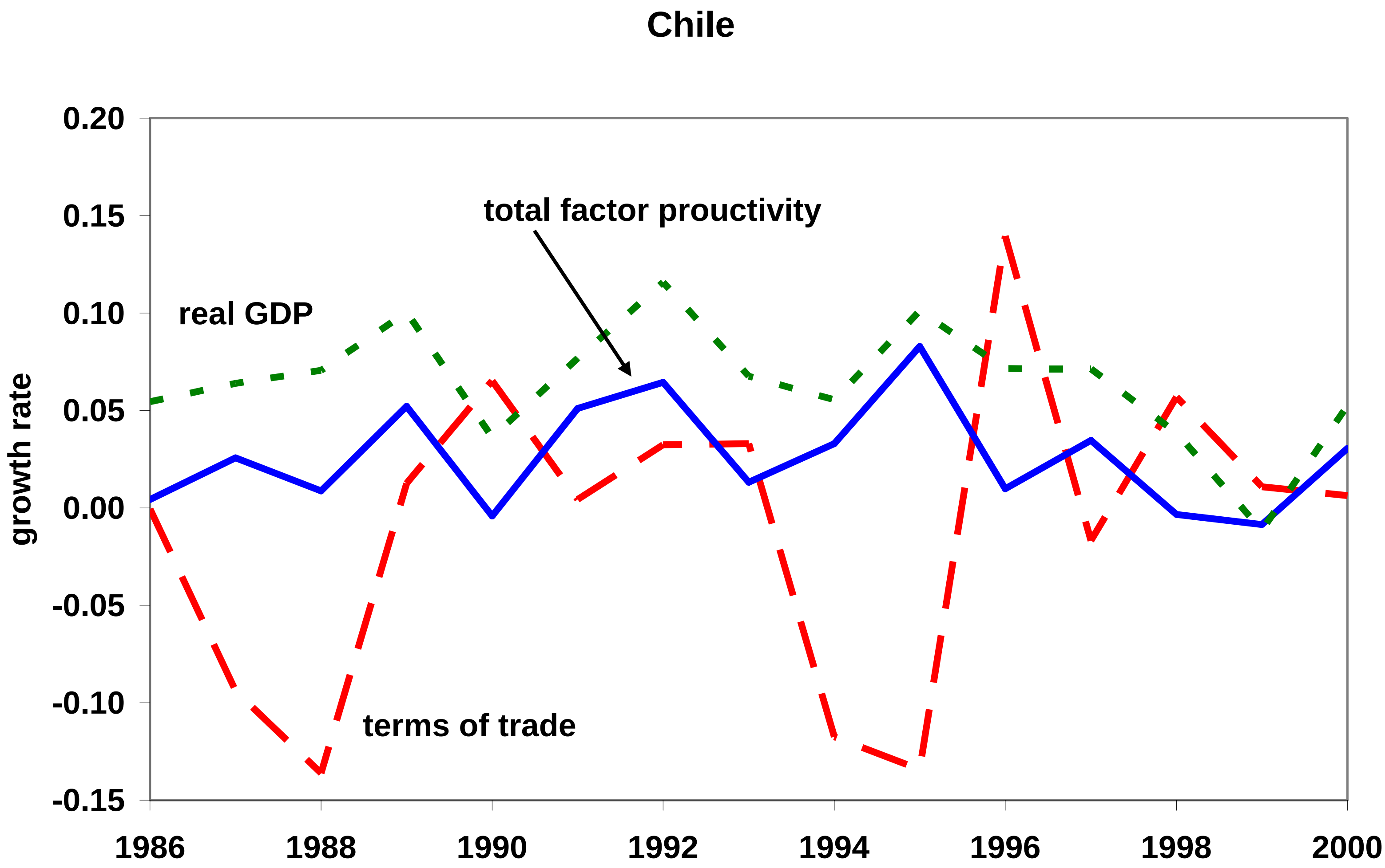

$$\rho\left(tot,tfp\right)=-0.73 \qquad \rho\left(tot,gdp\right)=-0.75$$

# Evidence on terms of trade, GDP, and TFP




$$\rho\left(tot, tfp\right) = -0.39 \qquad \rho\left(tot, gdp\right) = -0.26$$

# Evidence on terms of trade, GDP, and TFP




$$\rho(tot, tfp) = -0.37 \qquad \rho(tot, gdp) = -0.17$$

# Terms of trade volatility in the world

| | std(terms of trade) | std(TFP) |
|---|---|---|
| **Developing countries** | 0.132 | 0.026 |
| **Developed countries** | 0.053 | 0.017 |
| **Ratio** | 2.49 | 1.53 |

Hodrick-Prscott filtered annual data. source: Sengul (2006)

## International trade as a production technology

“For small *open* economies, adverse terms of trade shocks can have much the same effect as negative technology shocks, and this is one of the important differences between macroeconomics in these economies and that which underlies some of the traditional closed economy models.”

Easterly, Islam, and Stiglitz (2001)

**International trade as a production technology**

Inputs are exports and outputs are imports.

$$p_t M_t = X_t \;\Rightarrow\; M_t = \frac{1}{p_t} X_t$$

A deterioration in the terms of trade (an increase in $p_t$) acts as a productivity shock.

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**Or does it?**

# Overview of results

- Changes in $p_t$ have no first order effect on chain weighted GDP or measured productivity.

- With fixed proportions production, result is exact even for large shocks. (Forget about calculus!)

- Without chain weighting, effect involves $p_t - p_0$. (Effect goes either way!)

- With elastically supplied factors of production, effect goes either way.

- Results generalize to changes in tariffs and other trade barriers.

What drives the correlation between $p_t$ and real GDP and TFP?

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Not the mechanism we have discussed!

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These ideas are well understood by economists interested in index numbers and national income accounting.

Diewert and Morrison (1986)

Kohli (1983, 2004)

**Roadmap**

1. Simple closed economy
2. Model reinterpreted as an open economy
3. Chain weighting
4. Elasticity of substitution
5. Extension: endogenous labor choice
6. Extension: taxes and tariffs
7. Quantitative effects in Mexico

**Simple model: Closed economy**

$$\ell_t = \overline{\ell}$$

$$y_t = f(\ell_t, m_t)$$

$$m_t = \frac{x_t}{a_t}$$

$$c_t + x_t = y_t$$

Normalize the price of the $y$ good to be 1.

$$p_t = a_t$$

Real GDP:

expenditure side

$$Y_t = c_t = y_t - x_t$$

output side

$$Y_t = \left( y_t + p_0 m_t \right) - \left( p_0 m_t + x_t \right) = y_t - x_t$$

where $p_0 = a_0$

Firms solve

$$\max\ f(\overline{\ell}, m_t) - a_t m_t$$

$$f_m(\overline{\ell}, m_t) = a_t$$

$$m'(a_t) = \frac{1}{f_{mm}(\overline{\ell}, m(a_t))} < 0$$

With fixed proportions, $y_t = \min\left[\ell_t, m_t / b\right]$,

$$m'(a_t) = 0$$

How does real GDP change?

$$Y(a_{t+1}) - Y(a_t) \approx Y'(a_t)(a_{t+1} - a_t)$$

where

$$Y(a_t) = f(\bar{\ell}, m(a_t)) - a_t m(a_t)$$

$$Y'(a_t) = f_m(\bar{\ell}, m(a_t)) m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0.$$

With fixed proportions, $y_t = \min\left[\ell_t, m_t / b\right]$,

$$Y(a_t) = \bar{\ell} - a_t b \bar{\ell}$$

$$Y'(a_t) = -b\bar{\ell} = -m_t.$$

Real GDP and productivity decline.

## Simple model: Open economy

$m_t$ is an imported intermediate input

$x_t$ are exports of the $y$ good

$p_t$ is the terms of trade

we assume balanced trade,

$$p_t m_t = x_t$$

Real GDP

$$Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\overline{\ell}, m_t) - p_0 m_t$$

An increase in $p_t$ has the identical impact on consumption and welfare as the decline in productivity in the closed economy.

But what happens to real GDP and productivity?

$$Y(p_t) = f(\overline{\ell}, m(p_t)) - p_0 m(p_t)$$

$$Y'(p_t) = f_m(\overline{\ell}, m(p_t)) m'(p_t) - p_0 m'(p_t) = (p_t - p_0) m'(p_t)$$

With fixed proportions,

$$Y(p_t) = \overline{\ell} - p_0 b \overline{\ell}$$

$$Y'(p_t) = 0,$$

but

$$c(p_t) = (1 - p_t b)\overline{\ell}.$$

This is the case where consumption, and therefore welfare, falls the most in response to a deterioration in the terms of trade.

## Chain weighted real GDP

NIPA: Fisher chain weights, (UN SNA: Laspeyres chain weights)

$$Y_t(p_t) = \frac{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}{P_t}$$

$$P_{t+1} = \left( \frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})} \right)^{\frac{1}{2}} \left( \frac{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} P_t$$

$$Y(p_{t+1}) = \left( \frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)} \right)^{\frac{1}{2}} \left( \frac{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} Y(p_t)$$

How does real GDP change with $p$?

$$Y(p_{t+1}) - Y(p_t) \approx Y'(p_t)(p_{t+1} - p_t)$$

$$\frac{d\log Y(p_{t+1})}{dp_{t+1}} = -\frac{m(p_{t+1})}{2\left(f(\overline{\ell}, m(p_t)) - p_t m(p_t)\right)} + \frac{m(p_t)}{2\left(f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)\right)} + \frac{(p_{t+1} - p_t) m'(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right)}$$

$$\frac{d\log Y\left(p_t\right)}{dp_{t+1}} = 0$$

With any method of chaining, effect involving $p_t - p_0$ disappears.

**Elasticity of substitution**

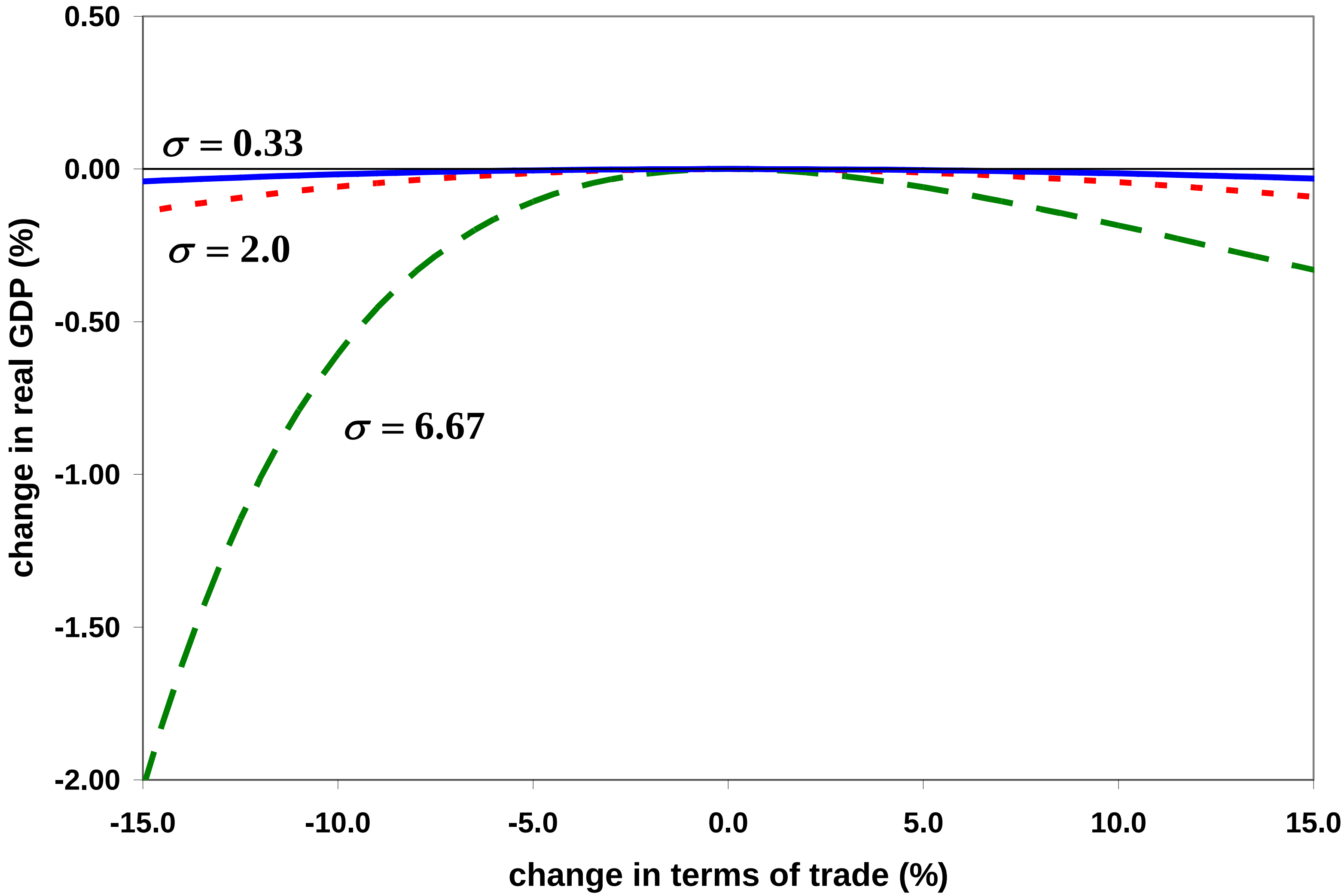
$$f\left(\ell_t, m_t\right) = \left(\left(1-\beta\right)\ell_t^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}$$

For each value of $\rho$, choose $\beta$ so that

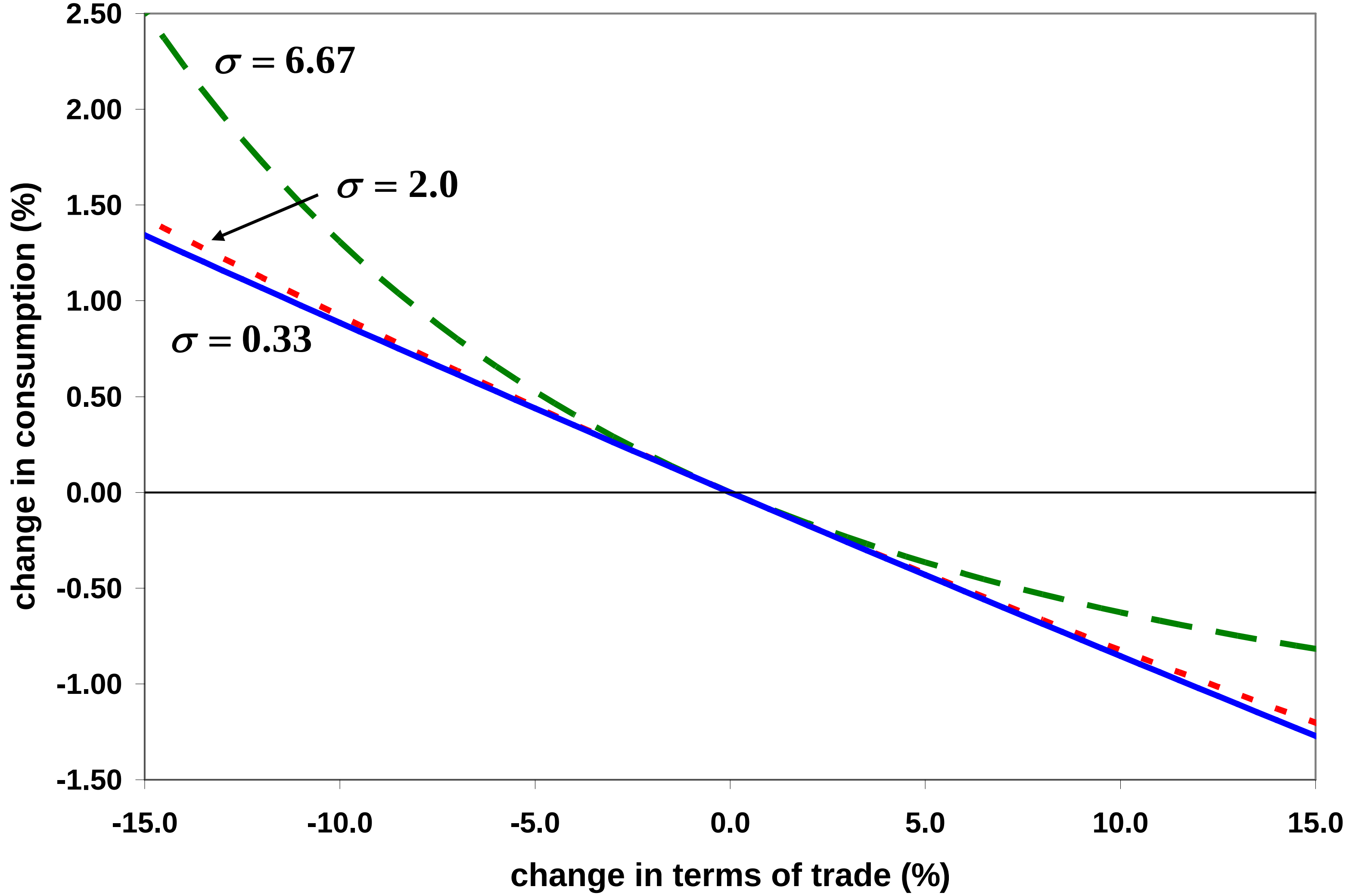
$$\frac{m_t}{\left(\left(1-\beta\right)\ell_t^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}} = 0.08$$

(U.S. data, 1998-2005).

# Real GDP and the elasticity of substitution



# Consumption and the elasticity of substitution



# Extensions to the simple model

Variable labor supply

$$\max u(c_t, \overline{\ell} - \ell_t)$$

$$\text{s.t. } c_t = w_t \overline{\ell}$$

where $w_t = f_\ell(\ell_t, m_t)$.

$$w_t u_c(c_t, \overline{\ell} - \ell_t) = u_z(c_t, \overline{\ell} - \ell_t)$$

which implicitly defines the function $\ell(w)$:

$$w_t u_c(w_t \ell(w_t), \overline{\ell} - \ell(w_t)) = u_z(w_t \ell(w_t), \overline{\ell} - \ell(w_t))$$

$$\ell'(w_t) = -\frac{u_c(c_t, \overline{\ell} - \ell_t) + u_{cc}(c_t, \overline{\ell} - \ell_t) w_t \ell_t - u_{cz}(c_t, \overline{\ell} - \ell_t)\ell_t}{u_{cc}(c_t, \overline{\ell} - \ell v) w_t^2 - 2u_{cz}(c_t, \overline{\ell} - \ell_t) w_t + u_{zz}(c_t, \overline{\ell} - \ell_t)}.$$

C. E. S. case

$$u(c,z)=\begin{cases}\left(c^{\rho}+\gamma z^{\rho}-1-\gamma\right)/\rho & \text{for } \rho\le 1,\ \rho\ne 0\\ \log c+\gamma\log z & \text{for } \rho=0\end{cases}$$

$$\ell'(w)=\frac{\rho c^{\rho-1}}{(1-\rho)\left(w^2c^{\rho-2}+\gamma(\overline{\ell}-\ell)^{\rho-2}\right)}$$

$\ell'(w)$ has same sign as $\rho$.

How do $w$ and $m$ vary with $p$?

$$f_\ell(\ell(w(p)), m(p)) = w(p)$$

$$f_m(\ell(w(p)), m(p))) = p$$

$$f_{\ell\ell}(\ell, m)\ell'(w)w'(p) + f_{\ell m}(\ell, m)m'(p) = w'(p)$$

$$f_{\ell m}(\ell, m)\ell'(w)w'(p) + f_{mm}(\ell, m)m'(p) = 1$$

$$w'(p) = \frac{f_{\ell m}(\ell, m)}{f_{mm}(\ell, m) - \left( f_{mm}(\ell, m) f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2 \right)\ell'(w)}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell, m)\ell'(w)}{f_{mm}(\ell, m) - \left( f_{mm}(\ell, m) f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2 \right)\ell'(w)}$$

Consumer welfare:

$$c(p) = f(\ell(w(p)), m) - pm(p)$$

$$\frac{d}{dp}u(c(p_t), \overline{\ell} - \ell(w(p_t))) = -u_c(c_t, \overline{\ell} - \ell_t)m_t < 0.$$

Real GDP:

$$Y(p_t) = f(\ell(w(p_t)), m(p_t)) - p_0 m(p_t)$$

$$Y'(p_t) = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t)$$

Real GDP can either rise or fall with $p_t$

If $\ell'(w_t) > 0$, which implies that $w'(p_t) < 0$, and if $(p_t - p_0)m'(p_t)$ is small, real GDP falls.

Productivity:

$$Y(p_t)/\ell(w(p_t))$$

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))} = \frac{\ell(w_t)Y'(p_t) - Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}$$

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))} = \frac{(p_t - p_0)\left(\ell_t m'(p_t) - m_{t-1}\ell'(w_t)w'(p_t)\right)}{\ell_t^2}.$$

with fixed proportions case,

$$Y(p_t) = (1 - p_0 b)\ell(w(p_t))$$

Tariffs

$$\max f(\overline{\ell}, m_t) - (1+\tau_t) p_t m_t$$

Real GDP:

$$Y'(p_t) = ((1+\tau) p_t - p_0) m'(p_t)$$

$$Y'(\tau_t) = ((1+\tau_t) p_t - p_0) m'(\tau_t)$$

$\approx 0$ if $(1+\tau_t) p_t - p_0 \approx 0$ or if $f$ is close to fixed proportions.

Welfare:

$$c'(p_t) = \tau p_t m'((1+\tau) p_t) - m(p_t(1+\tau)) p_t)$$

$$c'(\tau_t) = \tau_t p_t m'((1+\tau_t) p_t)$$

## Alternative income measures

U.S. NIPA: command-basis GDP

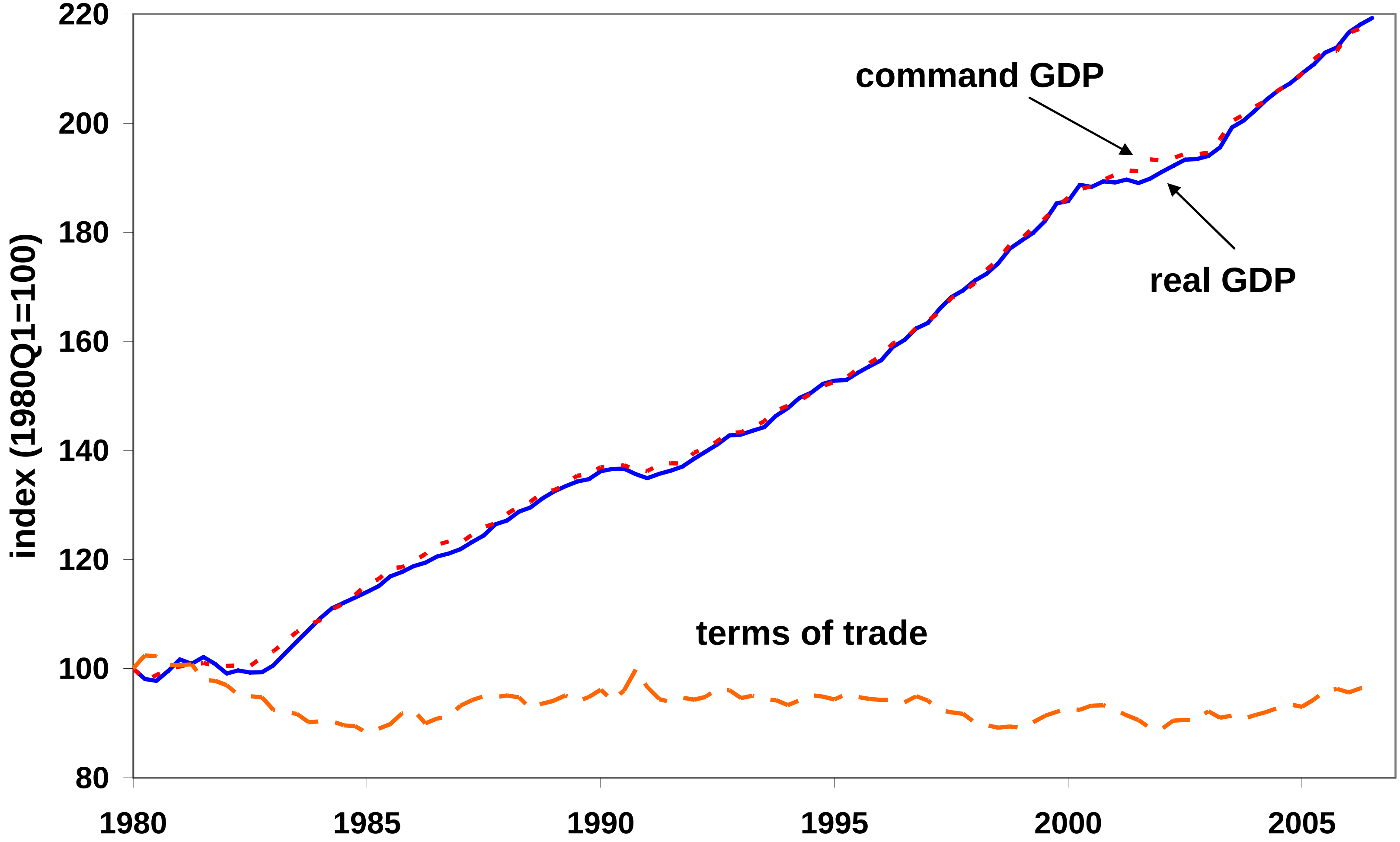
U.N. SNA: Gross Domestic Income

$$GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}$$

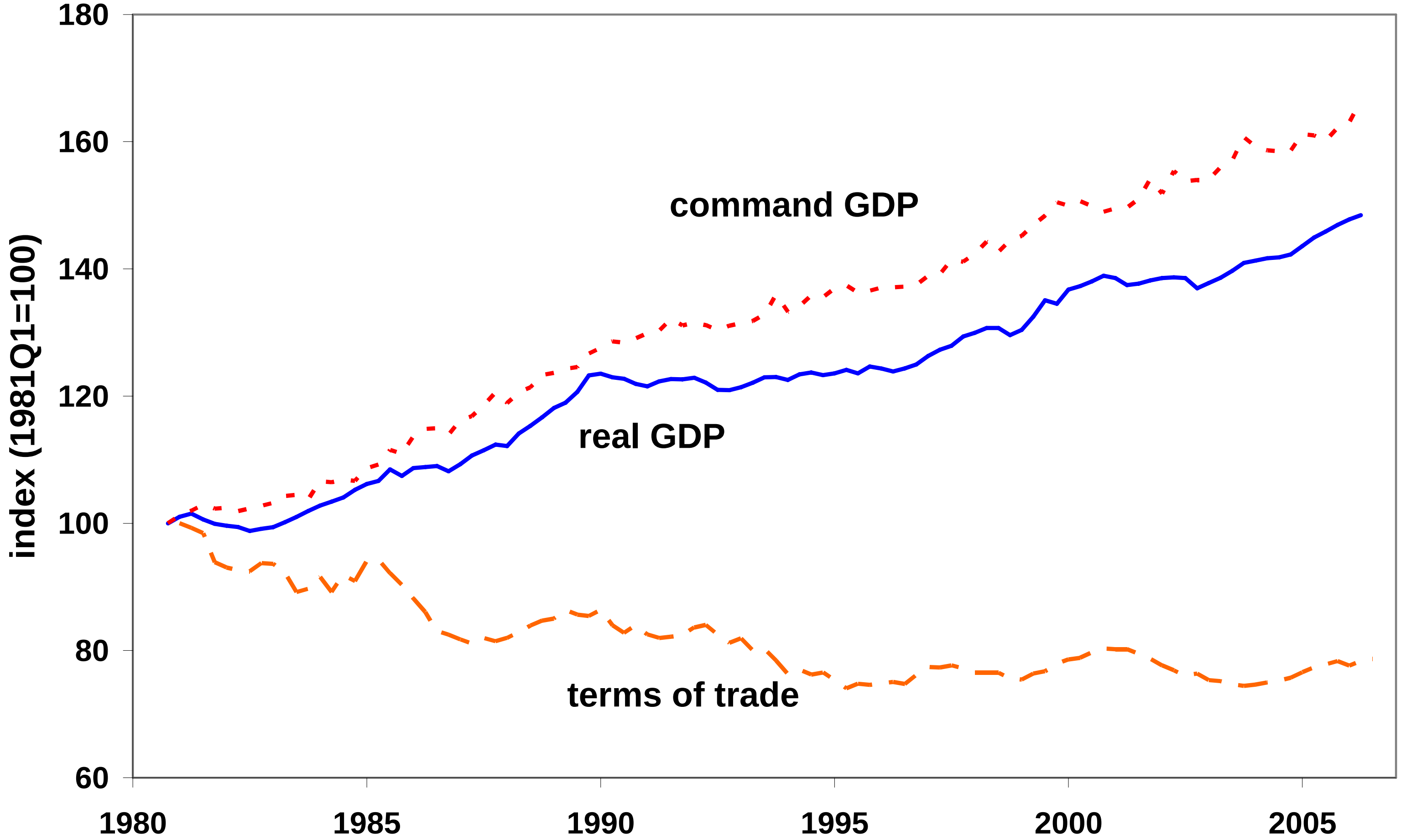
$$GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}.$$

or deflate $X_t - M_t$ by $P_t^Y$ or deflate $X_t - M_t$ by $P_t^X$, or…

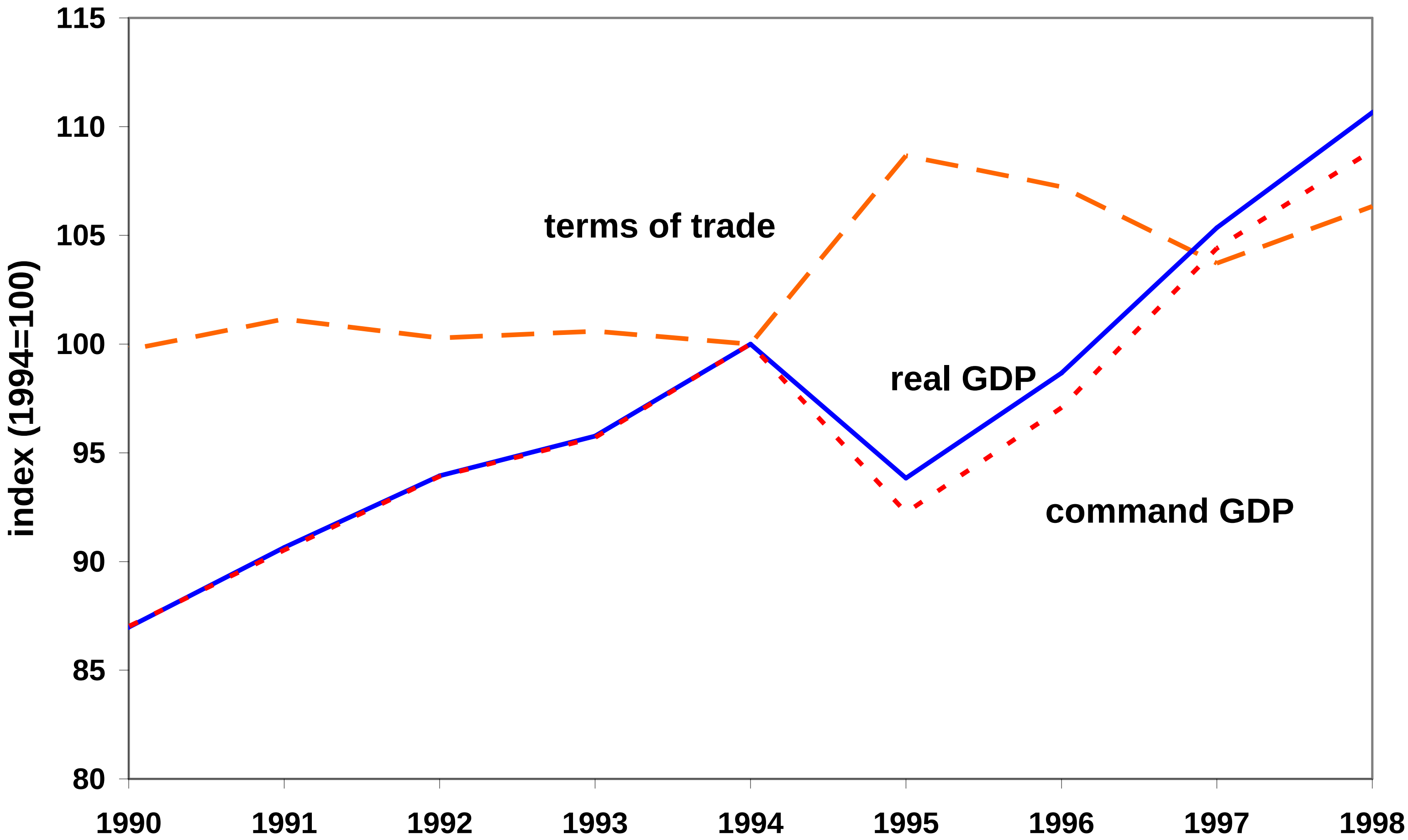
# United States



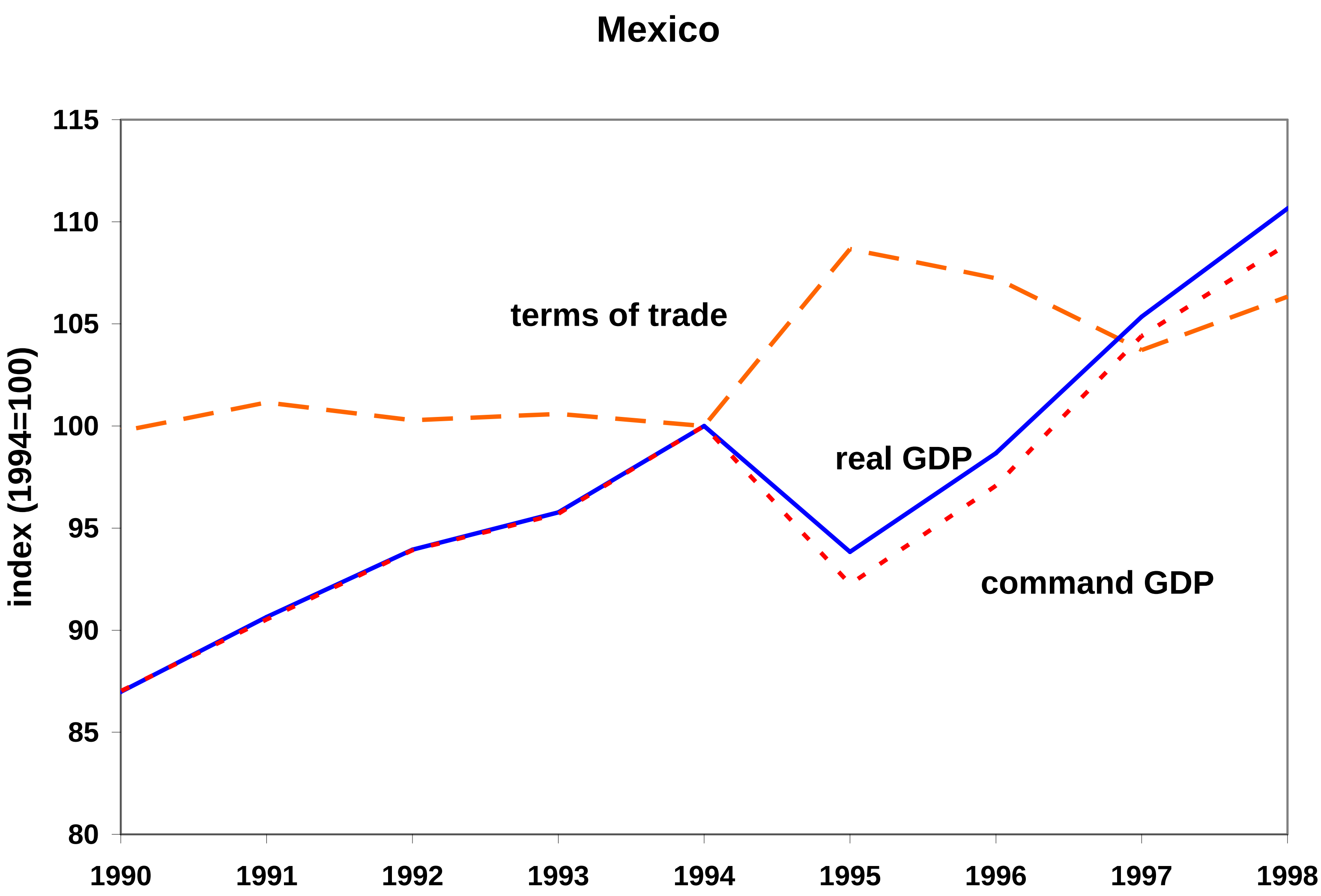
## Switzerland



# Mexico



index (1994=100)
115
110
105
100
95
90
85
80
1990
1991
1992
1993
1994
1995
1996
1997
1998
terms of trade
real GDP
command GDP




# Terms of trade shocks are worse than you thought!

# Quantitative Example

# Open Economy Model

Two kinds of goods:

- Imports ($m-$goods)
- Domestically produced goods ($d-$goods)

Domestic good is the numeraire

- The terms of trade, $p_m$, is exogenous

Add 3 exogenous variables

- Terms of trade, $p_{m,t}$
- Productivity – not TFP!!
- Investment-consumption good productivity, $D_t$

# Open Economy Model

Households

$$\max \sum_{t=T_0}^{\infty} \beta^t \left(\gamma \log\left(C_t\right) + \left(1-\gamma\right) \log(\bar{h}N_t - L_t)\right)$$

$$\text{s.t. } q_t C_t + q_t (K_{t+1} - (1-\delta) K_t) = w_t L_t + r_t K_t$$

Domestic Good Technology

$$Z_t + X_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Feasibility

$$C_t + K_{t+1} - (1-\delta) K_t = D_t \left(\omega Z_t^{\rho} + \left(1-\omega\right) M_t^{\rho}\right)^{\frac{1}{\rho}}$$

The firm's problem

$$\min_{Z_t, M_t} \quad Z_t + p_{m,t} M_t$$

$$\text{s.t.} \quad \overline{Y}_t \le D_t \left( \omega Z_t^{\rho} + (1-\omega) M_t^{\rho} \right)^{\frac{1}{\rho}}$$

Investment-consumption good price

$$q_t = D_t^{-1} \left( \omega^{\frac{1}{1-\rho}} + (1-\omega)^{\frac{1}{1-\rho}} \, p_{m,t}^{\frac{-\rho}{1-\rho}} \right)^{\frac{1-\rho}{-\rho}}$$

# Open Economy Model Calibration

Exogenous processes

- Terms of trade, $p_{m,t}$, from data
- Productivity in investment-consumption sector, $D_t$, from data
- Productivity in the domestic sector, $A_t$

Exogenous productivity is

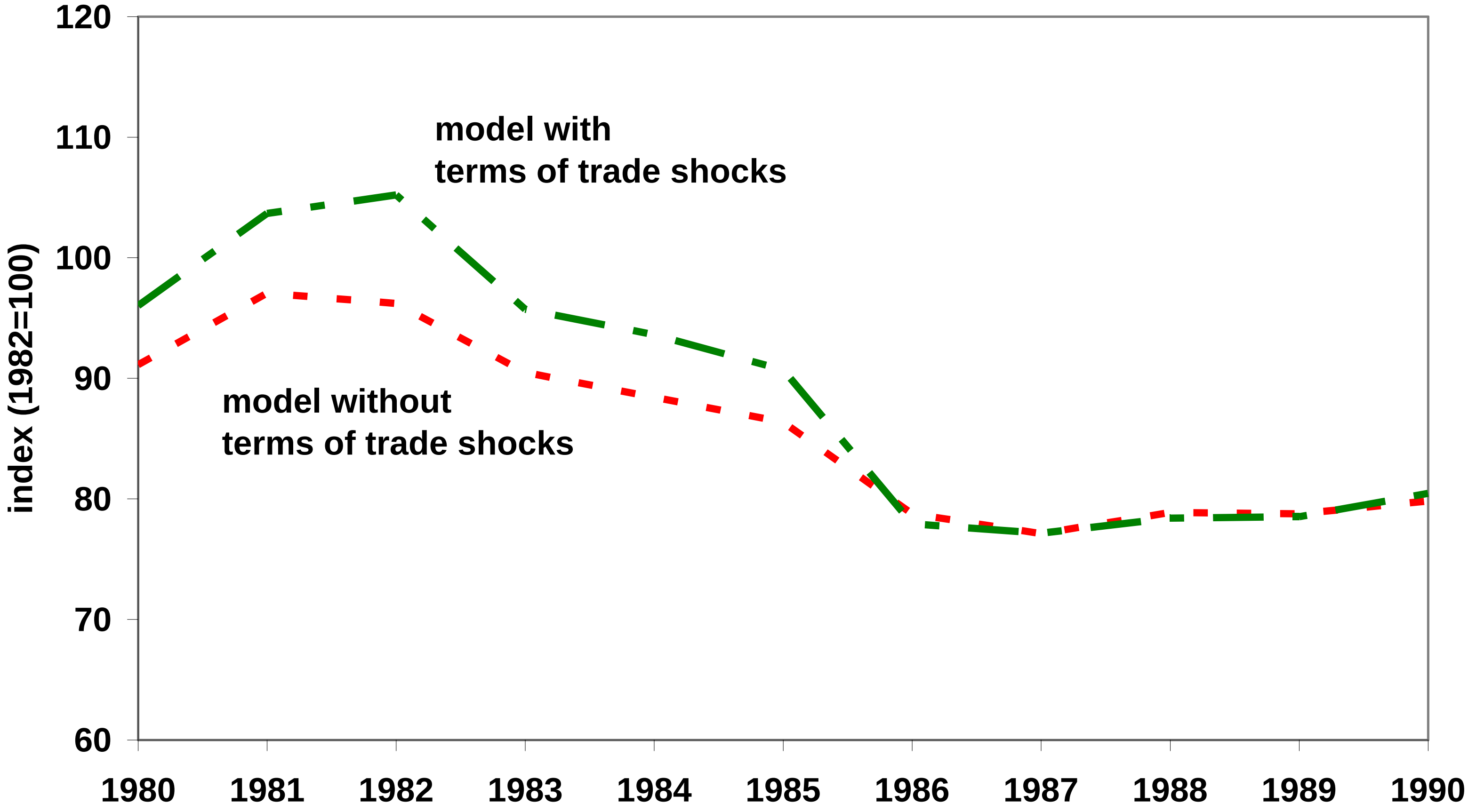
$$A_t = \frac{\omega^{-\frac{1}{\rho}}\left(\left(C_t + I_t\right)^{\rho} D_t^{-\rho} - (1-\omega) M_t^{\rho}\right)^{\frac{1}{\rho}} + X_t}{K_t^{\alpha} L_t^{1-\alpha}}$$

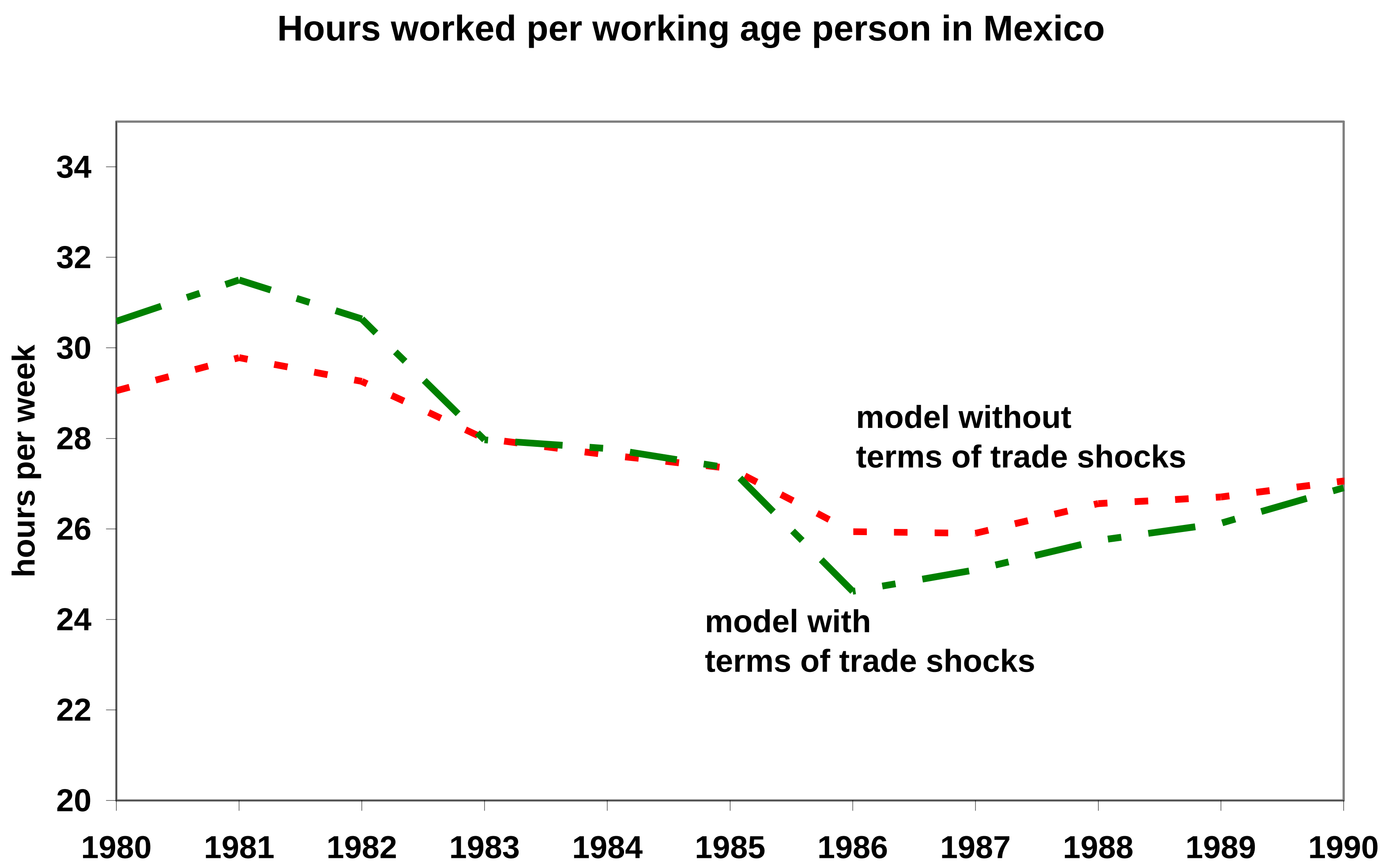
TFP is calculated with real GDP: $\hat{Y}_t = q_{\bar{T}}\left(C_t + I_t\right) + X_t - p_{m,\bar{T}} M_t$

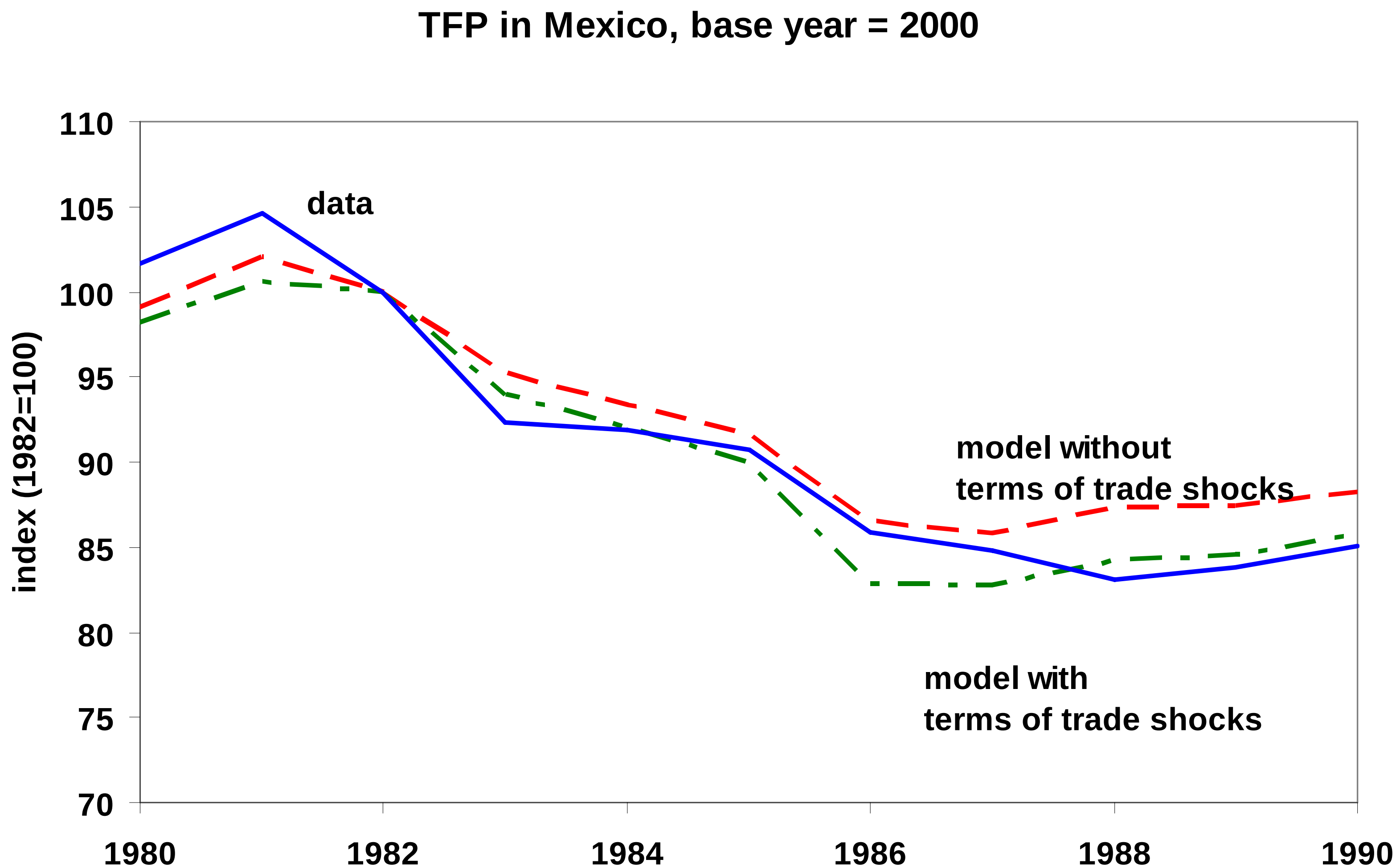
Solve the model two ways:

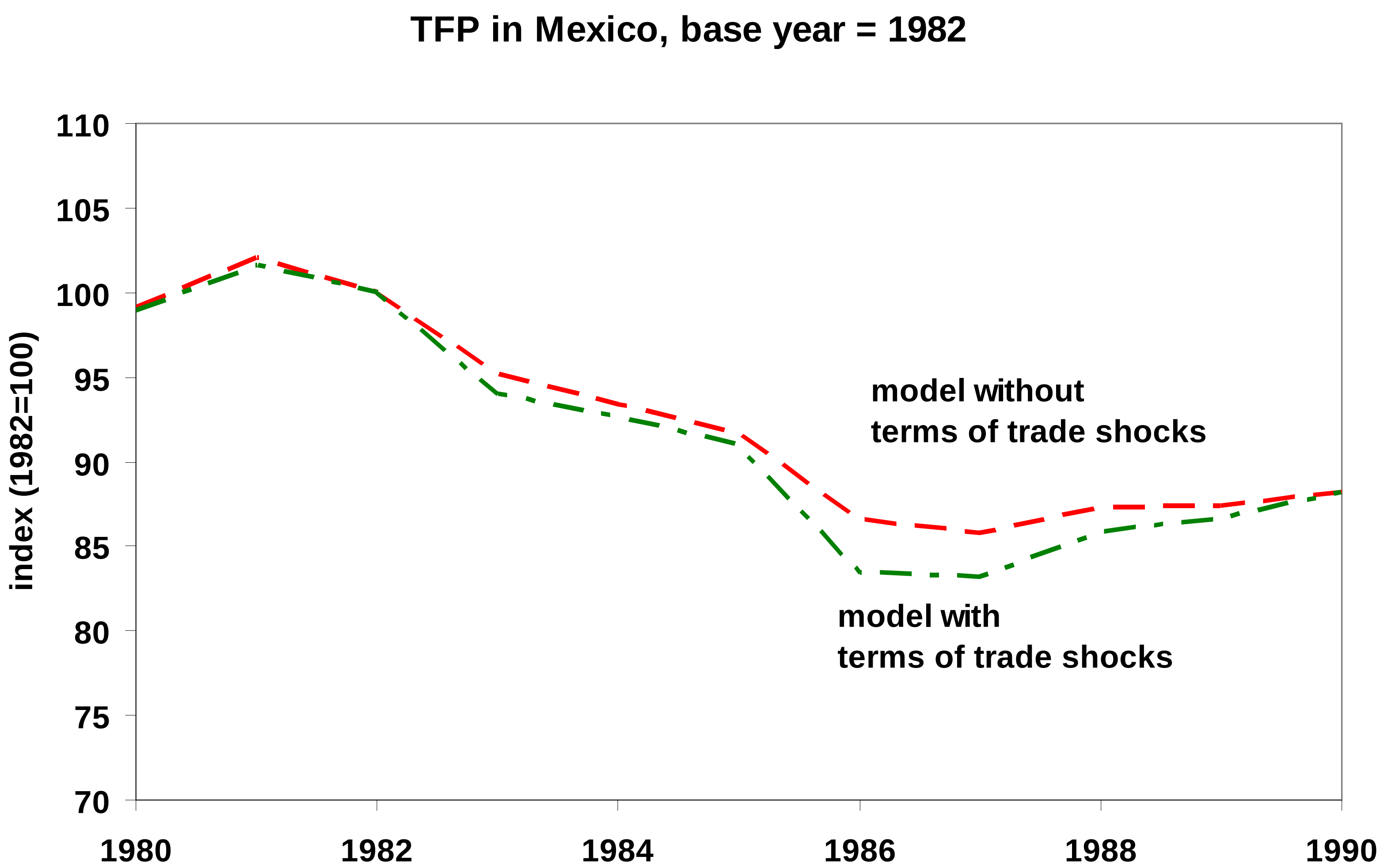
1. Model with terms of trade shocks
2. Model without terms of trade shocks (do not recalibrate)

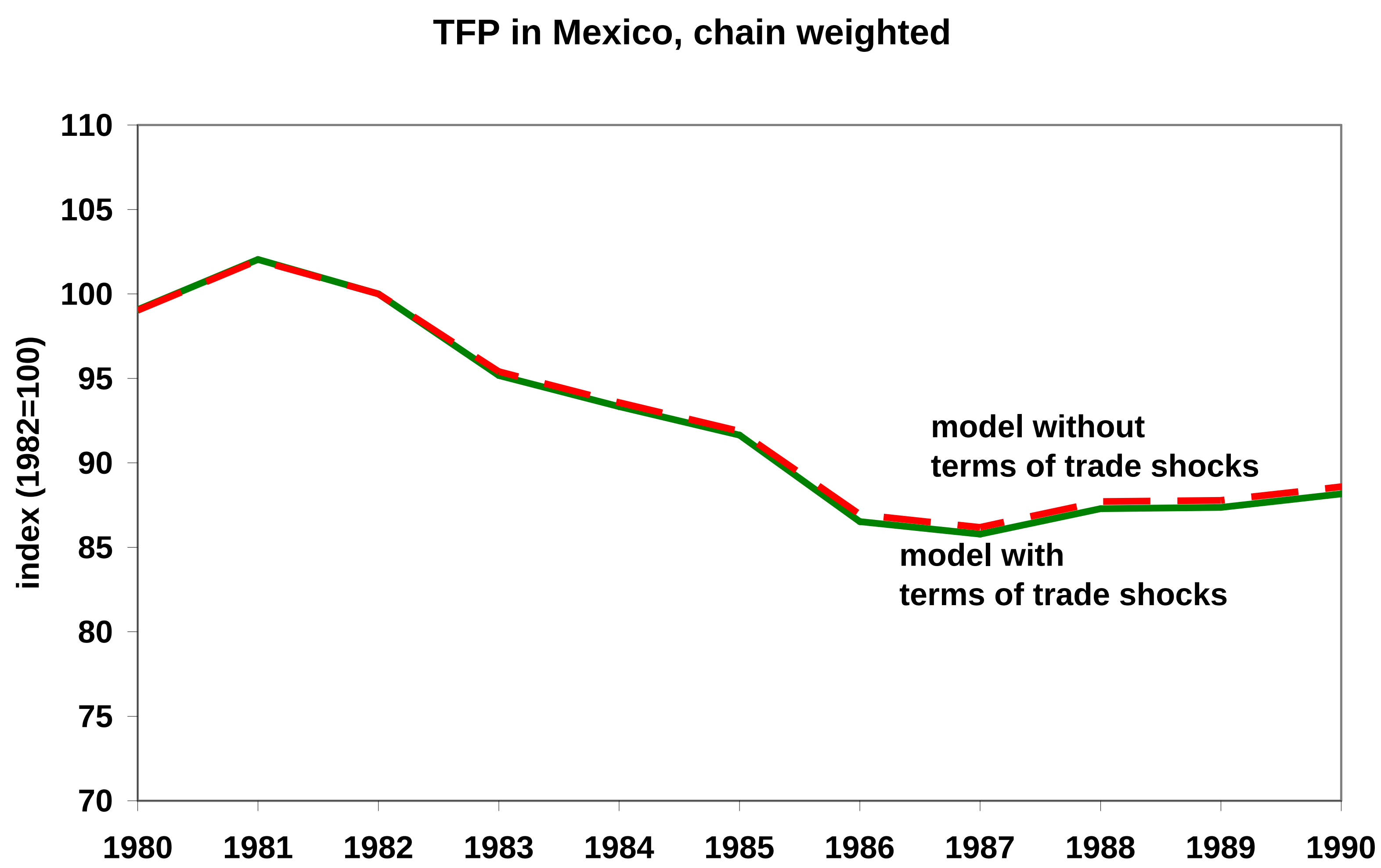
# Real GDP per working age person in Mexico




Hours worked per working age person in Mexico
hours per week
34
32
30
28
26
24
22
20
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
model without
terms of trade shocks
model with
terms of trade shocks


TFP in Mexico, base year = 2000
index (1982=100)
110
105
100
95
90
85
80
75
70
1980
1982
1984
1986
1988
1990
data
model without
terms of trade shocks
model with
terms of trade shocks


TFP in Mexico, base year = 1982
110
105
100
95
90
85
80
75
70
index (1982=100)
1980
1982
1984
1986
1988
1990
model without
terms of trade shocks
model with
terms of trade shocks



TFP in Mexico, chain weighted
index (1982=100)
110
105
100
95
90
85
80
75
70
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
model without
terms of trade shocks
model with
terms of trade shocks

# Conclusion

Base period prices: terms of trade have an ambiguous effect on TFP

Chain weighting: terms of trade have no effect on TFP

Terms of trade shocks can increase GDP volatility, but only by changing factor inputs, not productivity.

**In models with heterogeneous firms (for example, Melitz, Chaney), trade liberalization can cause resources to shift from less productive firms to more productive firms. Although this is often interpreted as an increase in productivity, it does not show up as such in productivity measures that use real GDP as a measure of output.**

M. J. Gibson, "Trade Liberalization, Reallocation, and Productivity," University of Minnesota, 2006.

http://www.econ.umn.edu/~tkehoe/papers/Gibson.pdf.

Some countries experience aggregate productivity increases following trade liberalization.

What is the economic mechanism through which this occurs?

Does trade liberalization increase aggregate productivity through reallocation toward more productive firms or through productivity increases at individual firms?

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**Not in the Melitz model!**

**Reallocation mechanism**

Technology of each firm is fixed

Trade liberalization results in a reallocation of resources:

- The least efficient firms exit
- Resources are moved toward more efficient firms, particularly exporters

**Main findings**

Reallocation following trade liberalization has no first-order effect on productivity, but it matters for welfare

Productivity gains must primarily come from firm-level productivity increases

Gibson studies a technology adoption mechanism in which firms can upgrade to a better technology, but it is costly to do so. Trade liberalization encourages technology adoption.

**Model**

$I$ symmetric countries, each with an *ad valorem* tariff on imports

Monopolistically competitive firms that are heterogeneous in technological efficiency

Sunk cost of entering export markets — only the most efficient firms export

Fixed cost of production — not all firms choose to operate

No aggregate uncertainty

## Consumer's problem

$$\max \sum_{t=0}^{\infty} \beta^t \log\left(\int_{z \in Z_t} c_t(z)^{\rho}\, dz\right)^{1/\rho}$$

$$\text{s.t.} \quad \int_{z \in Z_t^d} p_t(z) c_t(z)\, dz + (1+\tau_t) \int_{z \in Z_t^x} p_t(z) c_t(z)\, dz = \bar{N} + \Pi_t + \mathrm{T}_t$$

## Aggregation

Ideal real income index:

$$C_t = \left( \int_{z \in Z_t} c_t(z)^{\rho} \, dz \right)^{1/\rho}$$

Ideal price index:

$$P_t = \left( \int_{z \in Z_t^d} p_t(z)^{\frac{-\rho}{1-\rho}} \, dz + (1+\tau_t)^{\frac{-\rho}{1-\rho}} \int_{z \in Z_t^x} p_t(z)^{\frac{-\rho}{1-\rho}} \, dz \right)^{\frac{-(1-\rho)}{\rho}}$$

Budget constraint again:

$$P_t C_t = \bar{N} + \Pi_t + \mathrm{T}_t$$

## Demand functions

Firms take the consumer's demand functions as given

Demand for domestically produced goods:

$$\tilde{c}_t^d(p) = \left(\frac{P_t}{p}\right)^{\frac{1}{1-\rho}} C_t$$

Demand for imported goods:

$$\tilde{c}_t^x(p) = \left(\frac{P_t}{(1+\tau_t)p}\right)^{\frac{1}{1-\rho}} C_t$$

## Firms: Timing within a period

Entrants learn their efficiencies

Each firm decides whether to operate or exit — producing requires paying a fixed cost of $f^{p}$ units of labor

Non-exporters decide whether to pay the sunk cost of entering export markets, $f^{x}$ units of labor

After producing, each firm faces exogenous probability of death $\delta$

**Technologies**

A firm of type $a$ has the increasing-returns technology

$$y(n;a) = \max\left[a\left(n - f^{p}\right), 0\right]$$

$a \in [1,\infty)$ is the firm's technology draw from Pareto distribution

$F(a) = 1 - a^{-\eta}$

$f^{p}$ is the fixed cost, in units of labor, of producing

**Firm's static problem: Maximize period profits**

Non-exporters:

$$\pi_t^d(a) = \max_{p,n} p\tilde{c}_t^d(p) - n$$

$$\text{s.t. } a\left(n - f^p\right) = \tilde{c}_t^d(p)$$

Exporters:

$$\pi_t^x(a) = \max_{p,n} p\left(\tilde{c}_t^d(p) + (I-1)\tilde{c}_t^x(p)\right) - n$$

$$\text{s.t. } a\left(n - f^p\right) = \tilde{c}_t^d(p) + (I-1)\tilde{c}_t^x(p)$$

## Prices

The profit-maximizing price is a constant markup over marginal cost:

$$p(a) = \frac{1}{\rho a}$$

The price of a good is inversely related to the efficiency with which it is produced.

**Exporter's dynamic problem**

$$v_t^x(a) = \max\left[0,\ \pi_t^x(a) + \frac{1-\delta}{1+r_{t+1}} v_{t+1}^x(a)\right]$$

**Non-exporter's dynamic problem**

$$v_t^d(a) = \max\left[0,\ \pi_t^d(a) + \frac{1-\delta}{1+r_{t+1}} \max\left[v_{t+1}^d(a), v_{t+1}^x(a) - \frac{1+r_{t+1}}{1-\delta} f^x\right]\right]$$

Outer maximization: Whether to operate

Inner maximization: Whether to devote $f^x$ units of labor to enter export markets

**Firm entry**

There is free entry of firms, and firms enter as non-exporters.

The cost of a technology draw from probability distribution $F$ is $f^e$ units of labor.

The measure of draws taken, $e_t$, is determined endogenously through a free-entry condition:

$$\frac{1}{1+r_{t+1}}\int v_{t+1}^d(a)F(da)-f^e\leq 0,\ \ =0 \text{ if } e_t>0.$$

The inequality reflects the constraint that $e_t \geq 0$.

**Distributions of firms by efficiency**

Suppose that at the beginning of period $t$ the distribution of non-exporters is $m_t^d$ and the distribution of exporters is $m_t^x$.

To obtain the distributions of firms that choose to operate, apply the decision rules:

$$\mu_t^x(a) = \int_1^a \chi_t^x(\alpha) m_t^x(d\alpha)$$

$$\mu_t^d(a) = \int_1^a \chi_t^d(\alpha) m_t^d(d\alpha).$$

Distributions evolve in response to firm entry, $e_t$ and changes in export status, $\chi_t^e$ .

## Labor market clearing

The supply of labor is fixed at $\bar{N}$ and is allocated among 3 activities: production, entering export markets, and entering the domestic market

$$\sum_{s}\int\left(n_t^d\left(a\right)\mu_t^d\left(da\right)+n_t^x\left(a\right)\mu_t^x\left(da\right)+f^x\chi_t^e\left(a\right)\mu_t^d\left(da\right)\right)+f^e e_t=\bar{N}.$$

## Measuring productivity

Labor productivity in the data is a measure of real value added per worker or per hour.

Standard way of calculating real value added is to use base-period prices.

## Measuring real value added per worker

Value added at current prices:

$$y_t = \int_{z \in Z_t^d} p_t(z) y_t(z) dz$$

Value added at base-period (period-0) prices:

$$Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) dz$$

Real value added per worker is $Y_t / \bar{N}$.

**What if a good was not produced in the base period?**

This is an issue in the data as well.

The standard recommendation for obtaining a proxy for the base-period price is to deflate the current price by the price index for a basket of goods that were produced in both periods, say $\tilde{Z}$:

$$\tilde{P}_t = \frac{\int_{\tilde{Z}} p_t(z) y_0(z) dz}{\int_{\tilde{Z}} p_0(z) y_0(z) dz}.$$

Proxy for the period-0 price of a good not produced in period $0$:

$$p_0(z) = \frac{p_t(z)}{\tilde{P}_t}.$$

## Measuring social welfare

Ideal real income index:

$$\frac{\bar{N} + \Pi_t + \mathrm{T}_t}{P_t} = C_t = \left( \int_{z \in Z_t} c_t(z)^{\rho} \, dz \right)^{1/\rho}$$

The ideal price index $P_t$ takes into account changes in variety and the consumer's elasticity of substitution — in contrast to price indices in the data.

**To what extent can reallocation following trade liberalization account for long-term productivity gains?**

To determine the long-term effects of trade liberalization, we compare stationary equilibria of the model

two versions of the model:

Static version with $\beta \rightarrow 1$ (similar to Melitz (2003)): analytical result

Dynamic version with $0 < \beta < 1$: illustrative numerical example

**Static model: An analytical finding**

Proposition: In a stationary equilibrium with $\beta \to 1$, real value added per worker does not depend on the level of the tariff.

To see why:

With $\beta \to 1$, $\Pi = 0$, so the budget constraint gives

$$\int_{z \in Z_t^d} p_t(z) c_t(z) dz + \int_{z \in Z_t^x} p_t(z) c_t(z) dz = \bar{N}.$$

The balanced trade condition is

$$\int_{z \in Z_t^d} p_t(z) \left( y_t(z) - c_t(z) \right) dz = \int_{z \in Z_t^x} p_t(z) c_t(z).$$

Add them together to get

$$\int_{z \in Z_t^d} p_t(z) y_t(z) dz = \bar{N}.$$

So value added at current prices is constant, does not depend on $\tau$

What about base-period prices? Without technology adoption, the price of each good in the economy is constant: $p(z;a) = 1/(\rho a)$

So base-period prices are equal to current prices and the prices of new goods do not get deflated

Result:

$$Y_t = \int_{z \in Z_t^d} p_0(z) y_t(z) dz = \int_{z \in Z_t^d} p_t(z) y_t(z) dz = \bar{N}.$$

**Intuition for the result**

Reallocation following trade liberalization has no long-term effect on measured productivity

Why?  Two factors:

Prices — they are inversely related to the efficiency with which a good is produced

General equilibrium effects — changes in the real wage (partial equilibrium analysis would predict a substantial increase in measured productivity)

**Parameterization for illustrative numerical experiment**

| | |
|---|---|
| $\bar{N}=1$ | normalization |
| $\rho=0.5$ | elasticity of substitution of 2 (Ruhl 2003) |
| $\eta=1.5$ | |
| $\delta=0.05$ | |
| $f^{e}=1$ | |
| $f^{x}$ | 20 percent of firms export initially |
| $f^{p}$ | efficiency cutoff for operating is 1 initially |

**Illustrative numerical experiment in the static model**

$\beta \to 1$

Policy experiment:  Eliminate a 10 percent tariff between 2 countries

Compare stationary equilibria to assess long-term effects of trade liberalization:

| | |
|---|---|
| Percent change in measured productivity | 0.0 |
| Percent change in welfare | 0.5 |

## A note on the welfare increase

The increase in welfare following trade liberalization is not due to an increase in variety — the measure of varieties available to the consumer decreases.

Reallocation toward more efficient firms drives the welfare increase.

This is in sharp contrast to trade models with homogeneous firms, in which the increase in welfare is driven by an increase in variety.

Main point: Reallocation matters for welfare but not for measured productivity.

**Illustrative numerical experiment in the dynamic model**

To what extent can the fully dynamic model account for measured productivity gains?

$\beta = 0.96$ Real interest rate of 4 percent

Same numerical experiment:

| | |
|---|---|
| Percent change in measured productivity | 0.7 |
| Percent change in welfare | 1.8 |