Are Shocks to the Terms of Trade Shocks to Productivity?

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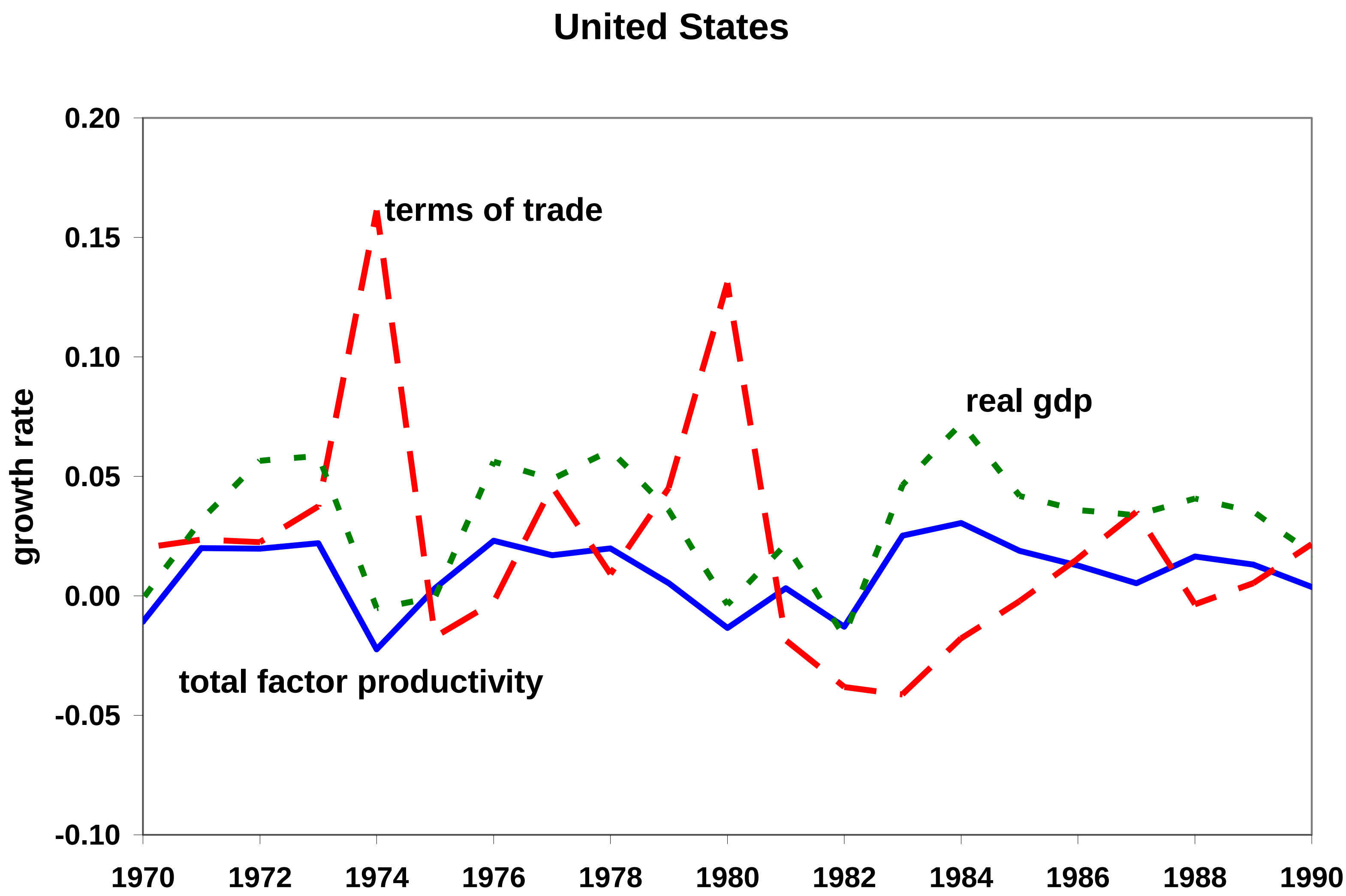
Evidence on terms of trade, GDP, and TFP



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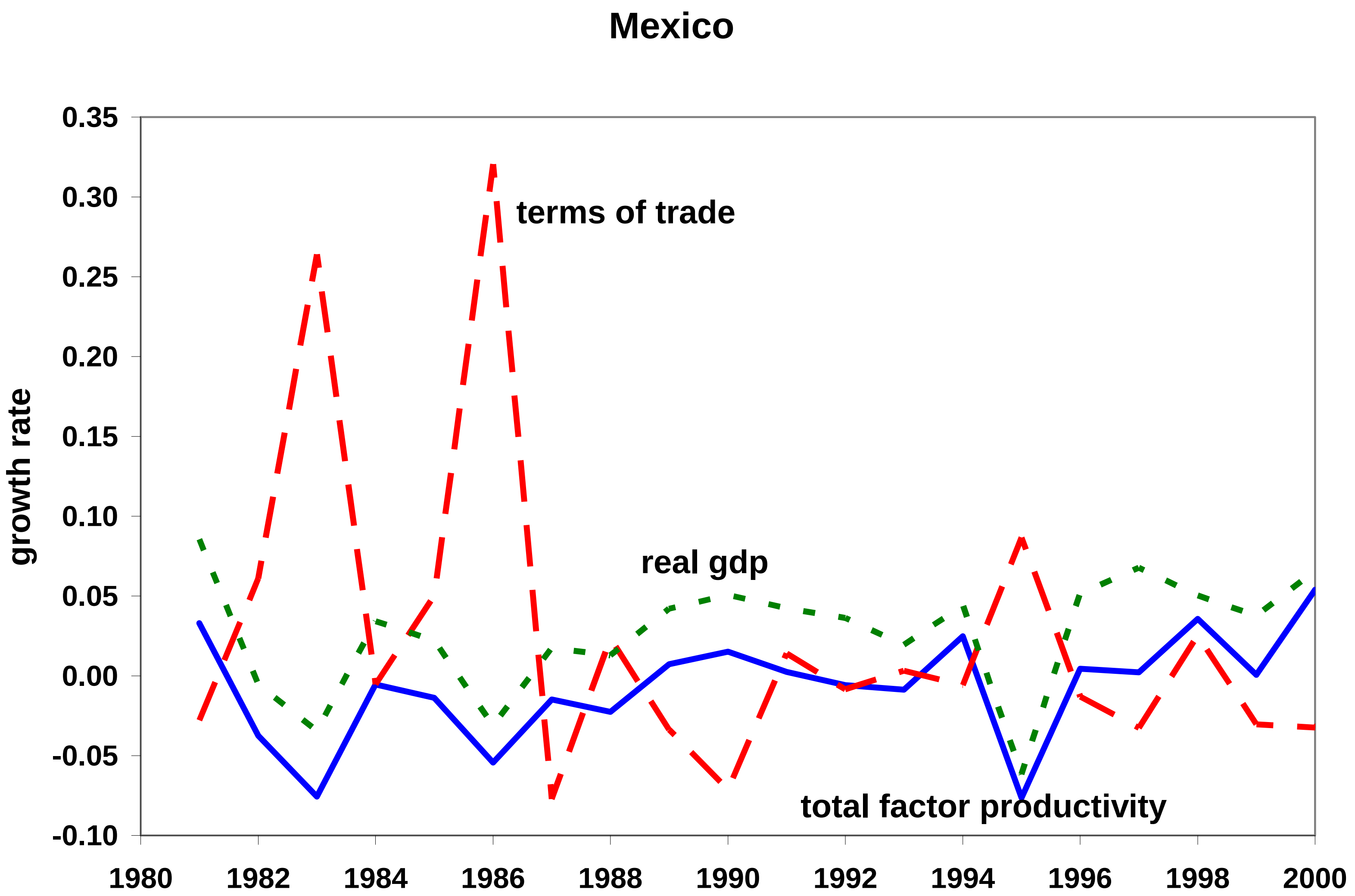

International trade is often thought of as a production technology.

Inputs are exports and outputs are imports.

$$p_t M_t = X_t \Rightarrow M_t = \frac{1}{p_t} X_t$$

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Or does it?

GDP in current prices

$$Y_t = (C_t + I_t + G_t + X_t) - p_t M_t$$

$$p_t \uparrow \;\Rightarrow\; M_t \downarrow \;\Rightarrow\; (C_t + I_t + G_t + X_t) \downarrow$$

Real GDP in base period prices

$$Y_t = (C_t + I_t + G_t + X_t) - p_0 M_t$$

$$p_t \uparrow \;\Rightarrow\; M_t \downarrow \;\Rightarrow\; (C_t + I_t + G_t + X_t) \downarrow \;\Rightarrow\; Y_t\,?$$

In a simple model, changes in p_t have no first order effect on chain weighted GDP or measured productivity.

With fixed proportions production, result is exact even for large shocks. (Forget about calculus!)

Without chain weighting, effect involves $p_t - p_0$. (Effect goes either way!)

With elastically supplied factors of production, effect goes either way.

Results generalize to changes in tariffs and other trade barriers.

What drives the correlation between p_t and real GDP and TFP?

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These ideas are well understood by economists interested in index numbers and national income accounting.

Diewert and Morrison (1986)

Kohli (1983, 2004)

Simple model: Closed economy

$$\ell_t = \bar{\ell}$$

$$y_t = f(\ell_t, m_t)$$

$$m_t = \frac{x_t}{a_t}$$

$$c_t + x_t = y_t$$

Normalize the price of the y good to be 1.

$$p_t = a_t$$

Real GDP:

expenditure side

$$Y_t = c_t = y_t - x_t$$

output side

$$Y_t = \left(y_t + p_0 m_t \right) - \left(p_0 m_t + x_t \right) = y_t - x_t$$

where $p_0 = a_0$

Firms solve

$$\max\; f(\bar{\ell}, m_t) - a_t m_t$$

$$f_m(\bar{\ell}, m_t) = a_t$$

$$m'(a_t) = \frac{1}{f_{mm}(\bar{\ell}, m(a_t))} < 0$$

With fixed proportions, $y_t = \min\left[\ell_t, m_t / b\right]$,

$$m'(a_t) = 0$$

How does real GDP change?

$$Y(a_{t+1}) - Y(a_t) \approx Y'(a_t)(a_{t+1} - a_t)$$

where

$$Y(a_t) = f(\overline{\ell}, m(a_t)) - a_t m(a_t)$$

$$Y'(a_t) = f_m(\overline{\ell}, m(a_t))m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0.$$

With fixed proportions, $y_t = \min\left[\ell_t, m_t / b\right]$,

$$Y(a_t) = \overline{\ell} - a_t b \overline{\ell}$$

$$Y'(a_t) = -b\overline{\ell} = -m_t.$$

Real GDP and productivity decline.

Simple model: Open economy

m_t is an imported intermediate input

x_t are exports of the y good

p_t is the terms of trade

we assume balanced trade,

$$p_t m_t = x_t$$

Real GDP

$$Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\overline{\ell}, m_t) - p_0 m_t$$

An increase in p_t has the identical impact on consumption and welfare as the decline in productivity in the closed economy.

But what happens to real GDP and productivity?

$$Y(p_t) = f(\overline{\ell}, m(p_t)) - p_0 m(p_t)$$

$$Y'(p_t) = f_m(\overline{\ell}, m(p_t)) m'(p_t) - p_0 m'(p_t) = (p_t - p_0) m'(p_t)$$

With fixed proportions,

$$Y(p_t) = \overline{\ell} - p_0 b \overline{\ell}$$

$$Y'(p_t) = 0,$$

but

$$c(p_t) = (1 - p_t b)\overline{\ell}.$$

This is the case where consumption, and therefore welfare, falls the most in response to a deterioration in the terms of trade.

Extensions to the simple model

Variable labor supply

$$\max u(c_t, \bar{\ell} - \ell_t)$$

$$\text{s.t. } c_t = w_t \bar{\ell}$$

where $w_t = f_\ell(\ell_t, m_t)$.

$$w_t u_c(c_t, \bar{\ell} - \ell_t) = u_z(c_t, \bar{\ell} - \ell_t)$$

which implicitly defines the function $\ell(w)$:

$$w_t u_c(w_t \ell(w_t), \bar{\ell} - \ell(w_t)) = u_z(w_t \ell(w_t), \bar{\ell} - \ell(w_t))$$

$$\ell'(w_t) = -\frac{u_c(c_t, \bar{\ell} - \ell_t) + u_{cc}(c_t, \bar{\ell} - \ell_t) w_t \ell_t - u_{cz}(c_t, \bar{\ell} - \ell_t)\ell_t}{u_{cc}(c_t, \bar{\ell} - \ell v) w_t^2 - 2u_{cz}(c_t, \bar{\ell} - \ell_t) w_t + u_{zz}(c_t, \bar{\ell} - \ell_t)}.$$

C. E. S. case

$$u(c,z) = \begin{cases} \left(c^{\rho} + \gamma z^{\rho} - 1 - \gamma\right)/\rho & \text{for } \rho \le 1,\ \rho \ne 0 \\ \log c + \gamma \log z & \text{for } \rho = 0 \end{cases}$$

$$\ell'(w) = \frac{\rho c^{\rho-1}}{(1-\rho)\left(w^2 c^{\rho-2} + \gamma(\overline{\ell} - \ell)^{\rho-2}\right)}$$

$\ell'(w)$ has same sign as ρ.

How do w and m vary with p?

$$f_\ell(\ell(w(p)), m(p)) = w(p)$$

$$f_m(\ell(w(p)), m(p))) = p$$

$$f_{\ell\ell}(\ell, m)\ell'(w)w'(p) + f_{\ell m}(\ell, m)m'(p) = w'(p)$$

$$f_{\ell m}(\ell, m)\ell'(w)w'(p) + f_{mm}(\ell, m)m'(p) = 1$$

$$w'(p) = \frac{f_{\ell m}(\ell, m)}{f_{mm}(\ell, m) - \left(f_{mm}(\ell, m) f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2 \right)\ell'(w)}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell, m)\ell'(w)}{f_{mm}(\ell, m) - \left(f_{mm}(\ell, m) f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2 \right)\ell'(w)}$$

Consumer welfare:

$$c(p) = f(\ell(w(p)), m) - pm(p)$$

$$\frac{d}{dp}u(c(p_t), \bar{\ell} - \ell(w(p_t))) = -u_c(c_t, \bar{\ell} - \ell_t)m_t < 0.$$

Real GDP:

$$Y(p_t) = f(\ell(w(p_t)), m(p_t)) - p_0 m(p_t)$$

$$Y'(p_t) = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t)$$

Real GDP can either rise or fall with p_t

If $\ell'(w_t) > 0$, which implies that $w'(p_t) < 0$, and if $(p_t - p_0)m'(p_t)$ is small, real GDP falls.

Productivity:

$$Y(p_t)/\ell(w(p_t))$$

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))}=\frac{\ell(w_t)Y'(p_t)-Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}$$

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))}=\frac{(p_t-p_0)\left(\ell_t m'(p_t)-m_{t-1}\ell'(w_t)w'(p_t)\right)}{\ell_t^2}.$$

with fixed proportions case,

$$Y(p_t)=(1-p_0b)\ell(w(p_t))$$

Tariffs

$$\max f(\overline{\ell}, m_t) - (1+\tau_t) p_t m_t$$

Real GDP:

$$Y'(p_t) = ((1+\tau) p_t - p_0) p_t m'(p_t)$$

$$Y'(\tau_t) = ((1+\tau_t) p_t - p_0) a m'(\tau_t)$$

≈ 0 if $(1+\tau_t) p_t - p_0 \approx 0$ or if f is close to fixed proportions.

Welfare:

$$c'(p_t) = ((1+\tau) p_t - p_0) p_t m'(p_t) - m(p_t)$$

$$c'(\tau_t) = ((1+\tau_t) p_t - p_0) p_t m'(\tau_t)$$

Chain weighted real GDP

U.S. Bureau of Economic Analysis — National Income and Product Accounts (NIPA): Fisher chain weights (also Statistics Canada)

$$Y_t(p_t) = \frac{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}{P_t}$$

$$P_{t+1} = \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})} \right)^{\frac{1}{2}} \left(\frac{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} P_t$$

$$Y(p_{t+1}) = \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_{t+1} m(p_t)} \right)^{\frac{1}{2}} \left(\frac{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} Y(p_t)$$

U.N. Statistics Division — System of National Accounts (SNA): Laspeyres chain weights (although Fisher and Paasche are allowed)

$$Y_t(p_t) = \frac{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}{P_t}$$

$$P_{t+1} = \frac{f(\bar{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}$$

$$Y(p_{t+1}) = \frac{f(\bar{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\bar{\ell}, m(p_t)) - p_t m(p_t)} Y(p_t)$$

With any method of chaining, effect involving $p_t - p_0$ disappears.

Elasticity of substitution

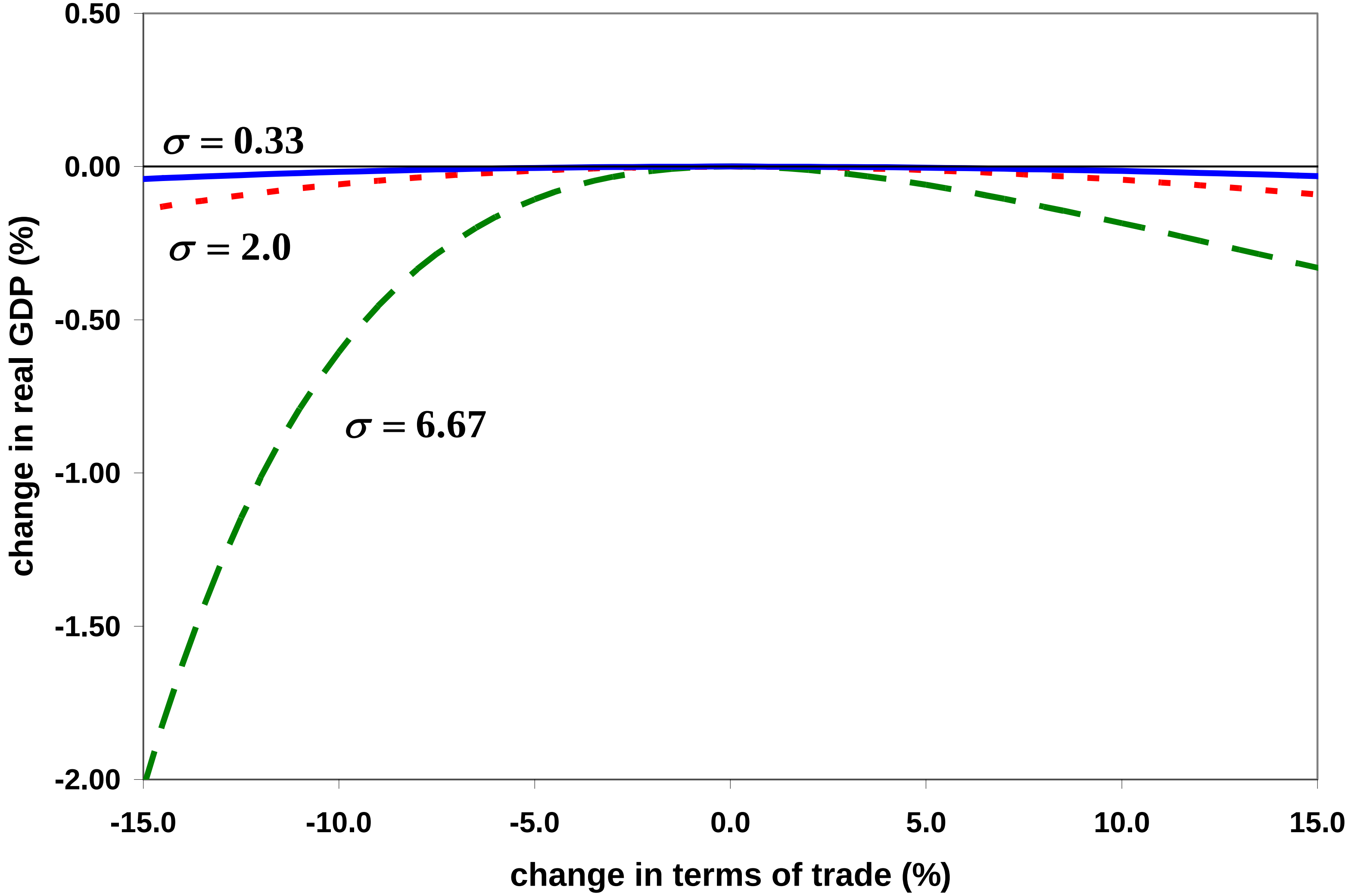
$$f\left(\ell_t, m_t\right) = \left(\left(1-\beta\right)\ell_t^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}$$

For each value of ρ, choose β so that

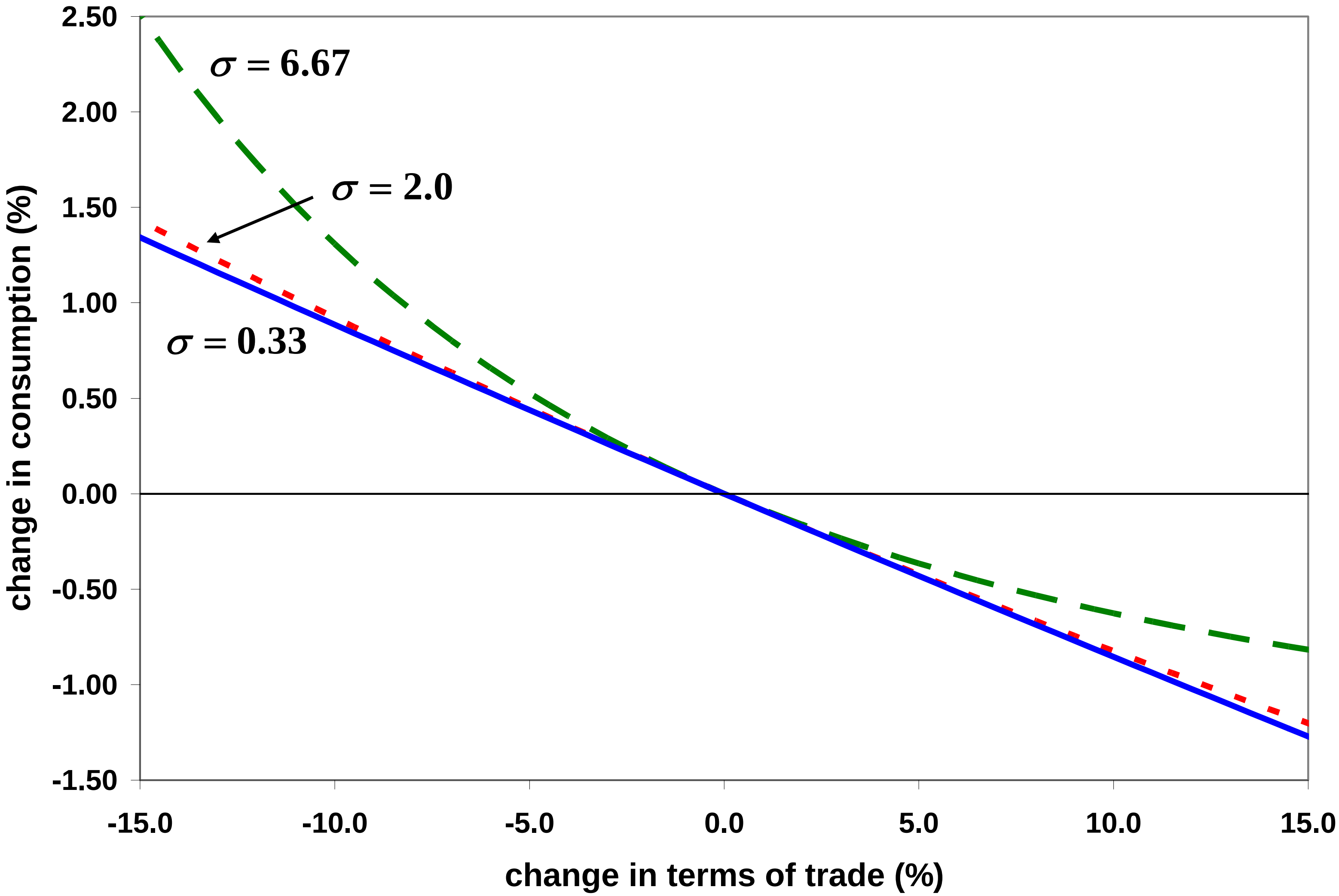
$$\frac{m_t}{\left(\left(1-\beta\right)\ell_t^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}} = 0.08$$

(U.S. data, 1998-2005).

Real GDP and the elasticity of substitution



Consumption and the elasticity of substitution



Alternative income measures

U.S. NIPA: command-basis GDP

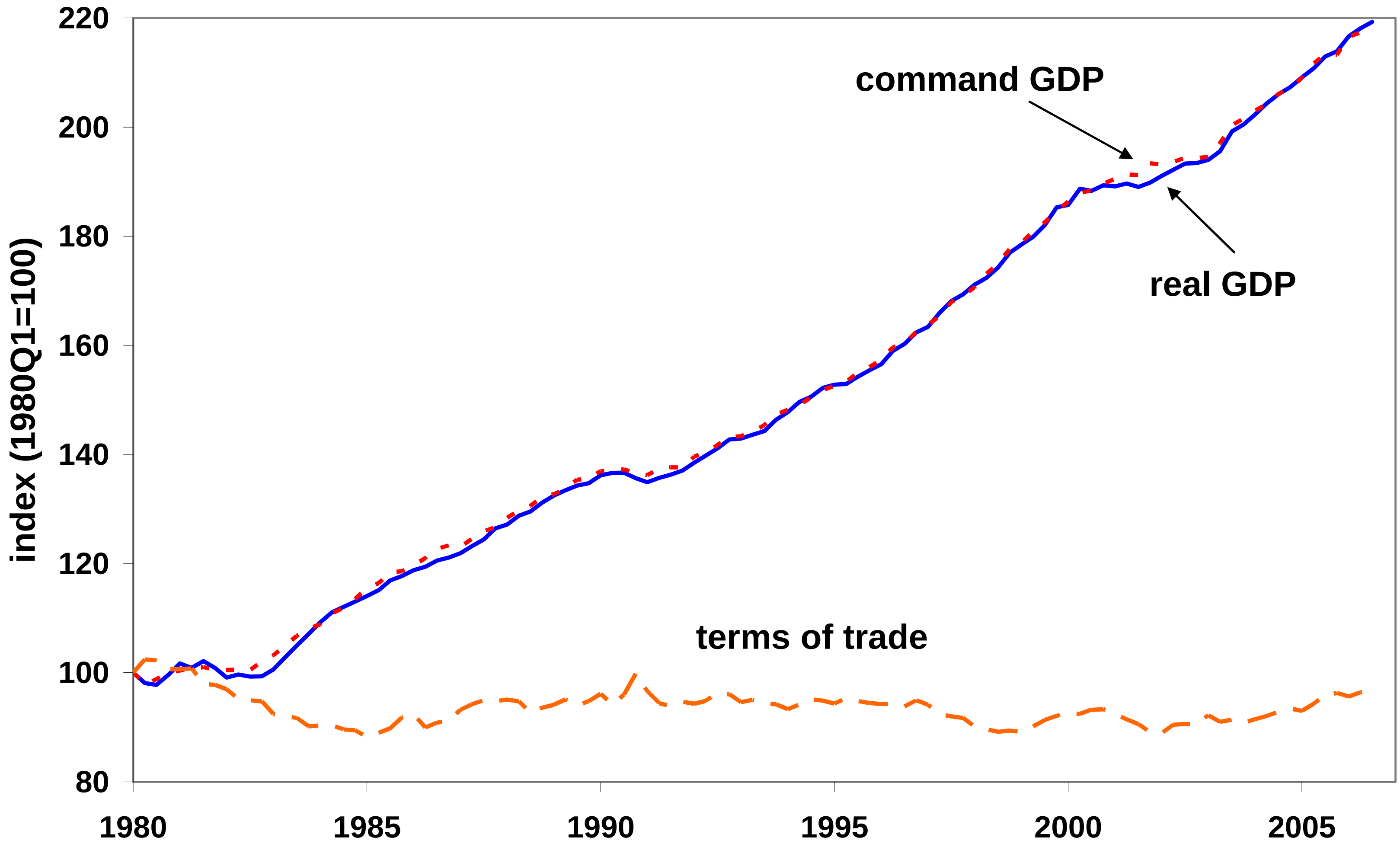
U.N. SNA: Gross Domestic Income

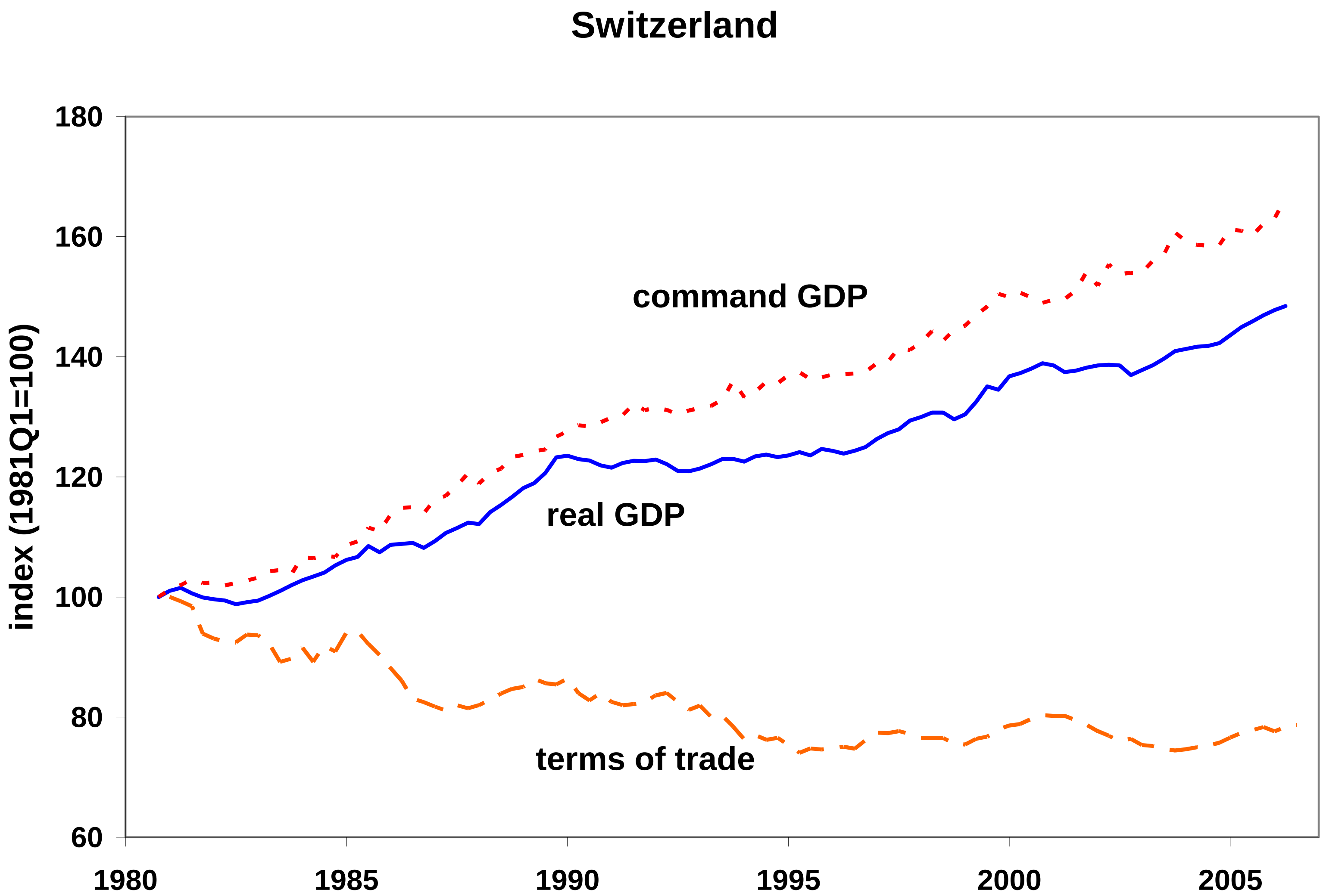
$$GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}$$

$$GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}.$$

or deflate $X_t - M_t$ by P_t^Y or deflate $X_t - M_t$ by P_t^X, or…

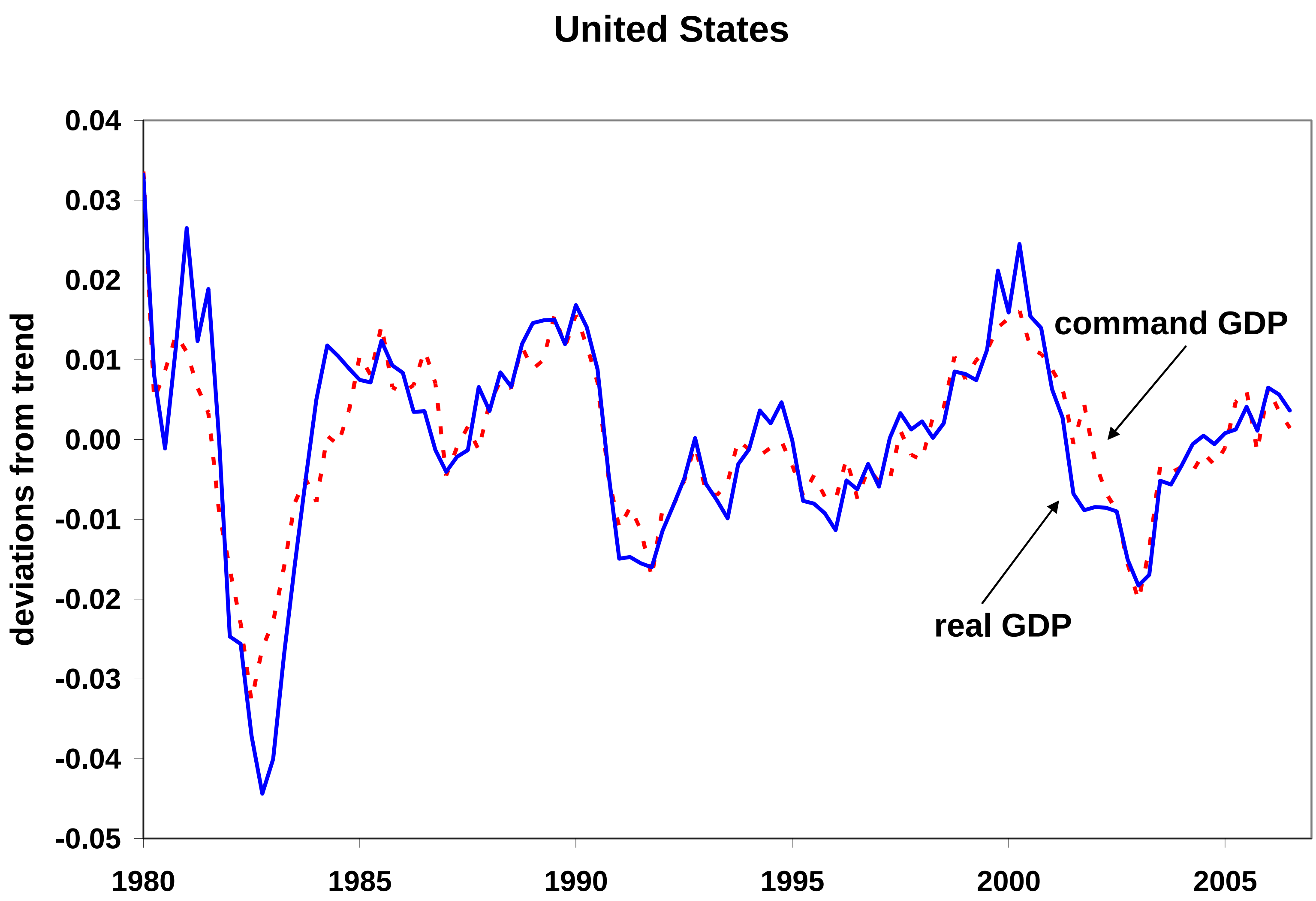
United States



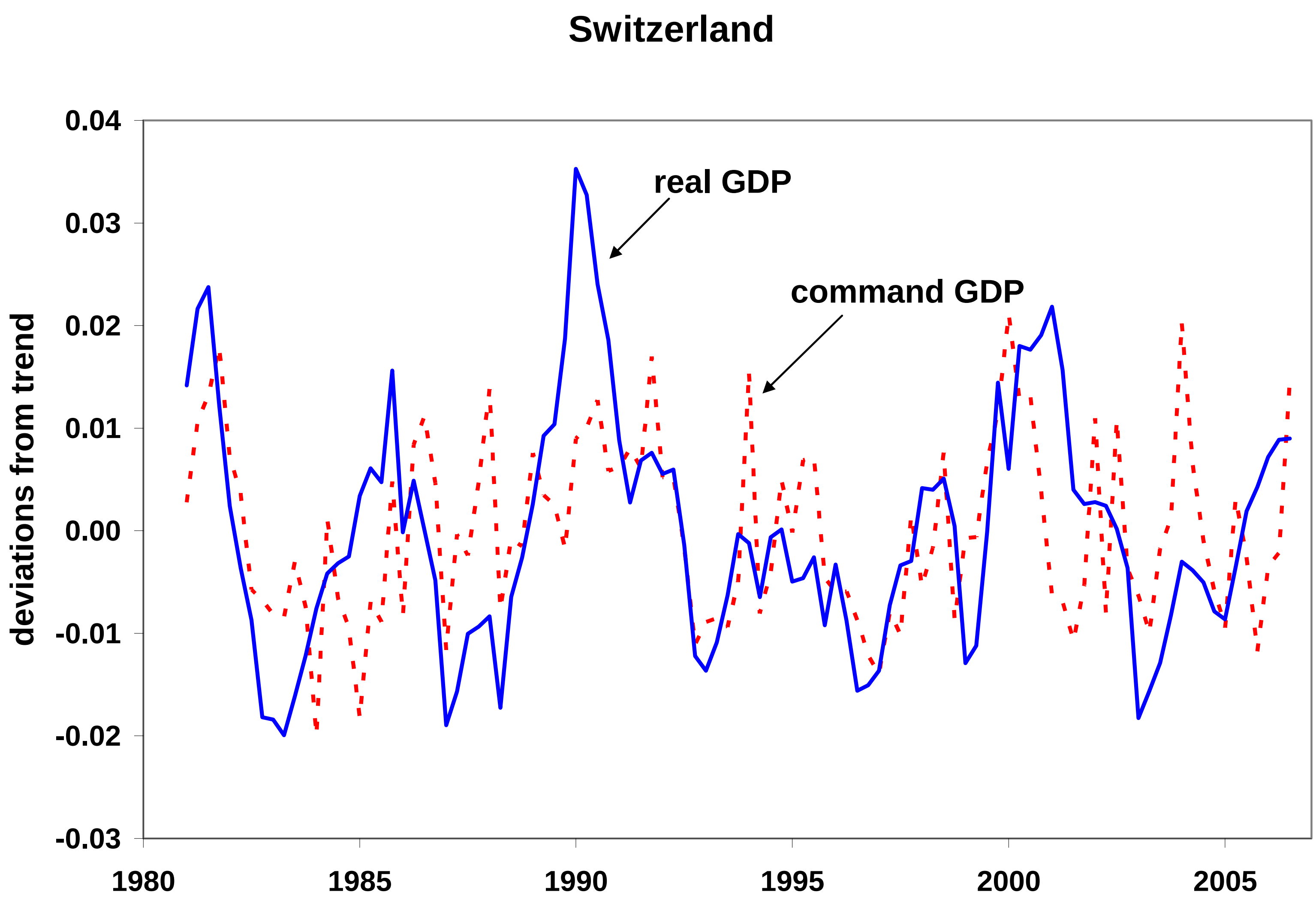


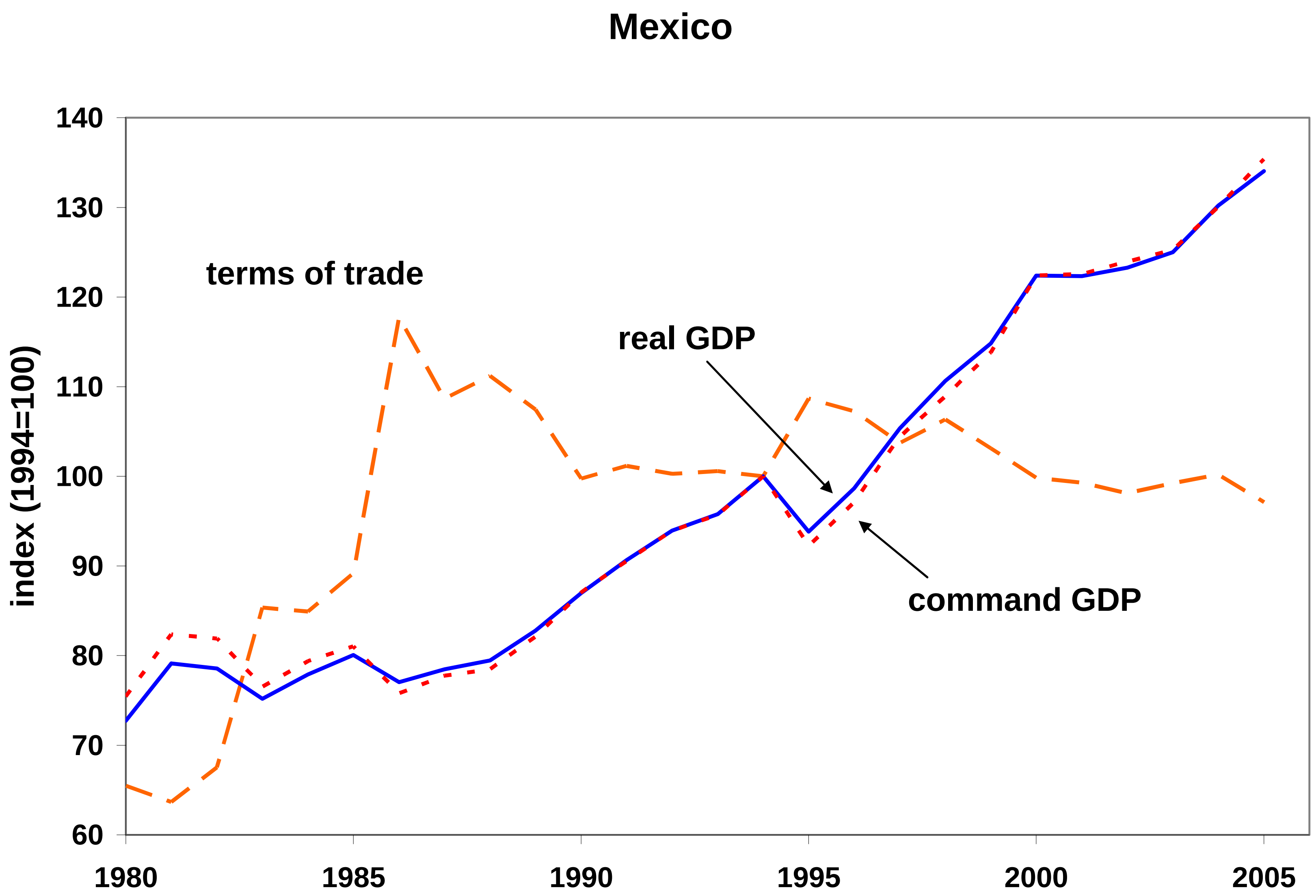
Switzerland
index (1981Q1=100)
180
160
140
120
100
80
60
1980
1985
1990
1995
2000
2005
command GDP
real GDP
terms of trade

Hodrick-Prescott filtered data



Hodrick-Prescott filtered data




Mexico
index (1994=100)
140
130
120
110
100
90
80
70
60
1980
1985
1990
1995
2000
2005
terms of trade
real GDP
command GDP

Conclusion

Terms of trade shocks are worse than you thought!