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EXAMINATION

Please answer **two** of the three questions.

You can consult class notes, working papers, and articles while you are working on the exam, but you are asked not to discuss the exam with anyone until the exam period is finished for everyone.

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho},$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 2$, $f = 6$, $\rho = 1/2$, and $\bar{\ell} = 100$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 1000$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 100$ and the other with $\bar{\ell}^2 = 900$, but otherwise identical. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

e) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

f) Compare the gains in real income in parts e with those in part d.

2. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

The investment good is produced according to

$$k_{t+1} = d x_{1t}^{a_1} x_{2t}^{a_2},$$

where $\alpha_i \geq 0$ and $\alpha_1 + \alpha_2 = 1$. Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}, \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= 1. \end{aligned}$$

The initial value of k_t is $\bar{k}_0$. All of the variables specified above are in per capita terms. There is a measure L of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner's problem for this economy. Explain how solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\begin{gathered} \sum_{t=0}^{\infty} \beta^t \log c_t \\ \text{s.t. } c_t + k_{t+1} = d k_t^{a_1} \\ c_t, k_t \geq 0 \\ k_0 = \bar{k}_0. \end{gathered}$$

[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner's problem in part b. [Recall that the policy function for investment has the form $k_{t+1}(k_t) = Adk^{a_1}$ where A is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of n different countries, all with the same technologies and preferences, but with different constant populations, L^i, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_t + p_{2t} = dk_t^{a_1}$ is world GDP per capita. Transform the first-order conditions for the one-sector social planner's problem in part b into two difference equations in k_t and s_t. Use the first-order conditions for the consumer's problem of the equilibrium in part d to show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

f) Use the solution to the one-sector social planner's problem in part c to solve for s_t. Discuss the economic significance of the result that you obtain.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max\ (1-\alpha)\log c_0 + \frac{\alpha}{\rho}\log\int_0^m c(z)^\rho dz$$
$$\text{s.t. } p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$
$$c(z) \geq 0\,.$$

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Furthermore, $m > 0$ is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0\right].$$

where $x(z) > 0$ is the firm's productivity level and $f > 0$. Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$ Suppose that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where $\gamma > 2$ and $\gamma > \rho/(1-\rho)$. Also suppose that the measure of potential firms is fixed at μ. Define an equilibrium for this economy.

c) Suppose that, in equilibrium not all potential firms actually produce. Find an expression for the productivity of the least productive firm that produces. That is, find a productivity $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce. Relate the measure of firms that produce m to the measure of potential firms μ and the cutoff $\bar{x}$.

d) Suppose now that there are two countries that engage in trade. Each country i, $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$. Each country faces an iceberg transportation cost τ in exporting differentiated goods to the other country.

e) Suppose that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to calculate any cutoff values, but you to not actually need to calculate it.] Compare this value, or these values, with that in part c. Draw a graph depicting what happens when a closed economy opens to trade.

f) Discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.