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EXAMINATION

Please answer **two** (2) of the three questions.

1. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0,1]$ given by the utility function

$$\int_X \log\ c(x)\ dx.$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$. Production functions are linear but differ across countries:

$$y_j^i(x) = \ell^i_j(x) / a_j^i(x)$$
$$a_1^1(x) = a_1^2(x) = \alpha + \beta x$$
$$a_2^1(x) = a_2^2(x) = \alpha + \beta - \beta x\ .$$

Here, for example, $y_j^i(x)$ is the amount of good x produced in country j for consumption in country i. Initially, there are no transportation costs or tariffs.

a) Define an equilibrium for this model.

b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.

c) Suppose now that there are transportation costs,

$$a_j^i(x) = (1+\tau)a_j^j(x) \text{ when } j \neq i\ .$$

Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in parts a and b.

d) Suppose now that the countries engage in a trade war in which each imposes an *ad valorem* tariff of τ on imports of good x from the country. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in part c.

2. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}) .$$

The investment good is produced according to

$$k_{t+1} = d x_{1t}^{a_1} x_{2t}^{a_2} .$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t} , \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= 1 . \end{aligned}$$

The initial value of k_t is $\bar{k}_0$. All of the variables specified above are in per capita terms. There is a measure L of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner's problem for this economy. Explain how solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\begin{gathered} \sum_{t=0}^{\infty} \beta^t \log c_t \\ \text{s.t. } c_t + k_{t+1} = d k_t^{a_1} \\ c_t, k_t \geq 0 \\ k_0 = \bar{k}_0 . \end{gathered}$$

[You can write down a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner's problem in part b. [Recall that the policy function for investment has the form $k_{t+1}(k_t) = Adk^{a_1}$ where A is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of n different countries, all with the same technologies and preferences, but with different constant populations, L^i, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_t + p_{2t} = dk_t^{\alpha_1}$ is world GDP per capita. We can transform the first-order conditions for the one-sector social planner's problem in part b into two difference equations in k_t and s_t. We could use the first-order conditions for the consumer's problem of the equilibrium in part d to show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

You do not need to prove this, but use the solution to the one-sector social planner's problem in part c to solve for s_t. Discuss the economic significance of the result that you obtain.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \ (1-\alpha)\log c_0 + \frac{\alpha}{\rho}\log\int_0^m c(z)^\rho dz$$
$$\text{s.t. } p_0c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$
$$c(z) \geq 0 .$$

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Furthermore, $m > 0$ is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0\right].$$

where $x(z) > 0$ is the firm's productivity level and $f > 0$. Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$ Suppose that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma} ,$$

where $\gamma > 2$ and $\gamma > \rho/(1-\rho)$. Also suppose that the measure of potential firms is fixed at μ. Define an equilibrium for this economy.

c) Suppose that, in equilibrium not all potential firms actually produce. Explain how you could calculate the productivity of the least productive firm that produces. That is, explain how to find a productivity $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce. [You do not need to calculate this value.] Relate the measure of firms that produce m to the measure of potential firms μ and the cutoff $\bar{x}$.

d) Suppose now that there are two countries that engage in trade. Each country i, $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$. Each country also imposes an *ad valorem* tariff τ on imports of differentiated goods from the other country. The revenue from these tariffs is redistributed in lump-sum form to the consumer in that country. Define an equilibrium for this world economy.

e) Suppose that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to calculate any cutoff values, but you to not actually need to calculate it.] Compare this value, or these values, with that in part c. Draw a graph depicting what happens when a closed economy opens to trade.

f) Discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.