

PROBLEM SET #1

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b) \max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho) \log \sum_{j=1}^n c_j^\rho,$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor.

- a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.
- b) Suppose that $b = 2$, $f = 4$, $\rho = 1/2$, and $\bar{\ell} = 36$. Calculate the autarky equilibrium.
- c) Suppose now that $\bar{\ell} = 180$. Calculate the equilibrium.
- d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 36$ and the second with $\bar{\ell}^2 = 144$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

e) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

f) Compare the gains in real income in parts e with those in part d.

2. Consider an economy in which there are two countries and a continuum of goods indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z) dz.$$

This consumer is endowed with $\bar{\ell}_j$ units of labor where $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$.

- a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.
- b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.
- c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.
- d) For the model in part c, calculate gross domestic product, exports, and the real income index

$$v_j = \exp \int_0^1 \log c_j(z) dz$$

as functions of τ . Suppose that in the base period $\tau = 0$ and calculate real GDP — that is, GDP in base period prices — as well as GDP in current prices.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1-\alpha)\log c_0 + (\alpha/\rho)\log \int_0^m c(v)^\rho dv \\ \text{s.t.} \quad & p_0 c_0 + \int_0^m p(v)c(v)dv = w\bar{\ell} + \pi \\ & c(v) \geq 0. \end{aligned}$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good v takes the price function $p(v)$ as given. Suppose too that this producer has the production function

$$y(v) = \max[z(v)(\ell(v) - f), 0].$$

Solve the consumer's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $z \geq 1$ according to the Pareto distribution with distribution function

$$F(z) = 1 - z^{-\gamma}.$$

Define an equilibrium for this economy.

c) Suppose that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Find an expression for the cutoff productivity level \bar{z} such that firms with productivity \bar{z} earn zero profits. Find an expression that relates the measure of goods consumed m to the measure of potential firms μ and the cutoff level \bar{z} . Find an expression for profits π .

d) Suppose now that there are two countries that engage in free trade. Each country i , $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i . Firms' productivities are again distributed according to the Pareto distribution, $F(z) = 1 - z^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j , $j \neq i$, of f_x where $f_x > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this economy.

e) Suppose now that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Explain now why there are two relevant cutoff levels of firm productivity, \bar{z}_d and \bar{z}_x . Find expressions for these cutoff productivity levels. Find an expression for profits π .

f) Discuss the strengths and weaknesses of this model. In particular: What economic phenomena can this sort of model help to account for? What sort of phenomena can it not account for? How can we modify the model to account for these phenomena?