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PROBLEM SET #2

1. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b} .$$

The investment good is produced according to

$$k_{t+1} = d(a_1 x_{1t}^b + a_2 x_{2t}^b)^{1/b} .$$

In particular, the deprecation rate is $\delta = 1$. Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t} .$$

where

$$k_{1t} + k_{2t} = k_t$$
$$\ell_{1t} + \ell_{2t} = \ell_t .$$

The initial value of k_t is $\bar{k}_0$. ℓ_t is normalized to 1.

a) Define an equilibrium for this economy.

b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in k_t and c_t and a transversality condition. Here $c_t = d(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$ is aggregate consumption. (You do not need to go through all of the algebra, but you need to explain all of the logical steps carefully.)

c) Suppose now that there is a world made up of two different countries, each with the same technologies and preferences, but with different constant populations, $L_t^j = \bar{L}^j$, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_{1t} + p_{2t} = r_t k_t + w_t = d(a_1 k_t^b + a_2)^{1/b}$ is income per capita. Transform the two difference equation in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

where $y_t^i = p_{1t}k_t^i + p_{2t} = r_t k_t^i + w_t = d(a_1 k_t^{i\,b} + a_2)^{1/b}$ is income per capita in country i.

f) Discuss the economic significance of the result in part e.

2. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_t = rer_t^T + rer_t^N,$$

where rer_t is the natural logarithm of the bilateral real exchange rate and rer_t^T is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate ratio of the standard deviations of rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of rer_t in terms of rer_t^T and rer_t^N in levels, in 1 year differences, and in 4 year differences.

3. Consider a small open economy whose government borrows from international lenders. In every period, the value of output is

$$y(z) = Z^{1-z}\bar{y}$$

where $1 > Z > 0$ is a constant and $z = 0$ if the government defaults in that period or has defaulted in the past and $\bar{y}$ is a constant. The government's tax revenue is $\theta y(z)$ where the tax rate $1 > \theta > 0$ is constant. The consumers in the economy consume $c = \theta y(z)$. The government is benevolent and makes choices to maximize the expected discounted value of

$$u(c, g) = \log c + \gamma \log g$$

where $\gamma > 0$ and $1 > \beta > 0$ is the discount factor. At the beginning of every period, the state of the economy is $s = (B, z_{-1}, \zeta)$ where B is the level of government debt; $z_{-1} = 0$ if the government has defaulted in the past, and $z_{-1} = 1$ if not, and $\zeta \sim U[0,1]$ is the realization of a sunspot variable. The government first offers B' to international bankers. The intentional bankers have the same discount factor β as the government. They are also risk neutral and have deep pockets. These international bankers buy the bonds at a competitive auction that determines a price for B', $q(B',s)$. The government finally chooses to default or not, which determines private consumption c. Government spending g is determined by the government's budget constraint

$$g + zB = \theta y(z) + q(B',s)B'.$$

If the government defaults, setting $z = 0$, then assume that $z_{-1} = 0$ implies $z = 0$ thereafter; that is, the economy suffers from the default penalty $1 - Z$ forever. Furthermore, $z_{-1} = 0$ implies $q(B',s) = 0$; that is, the government is permanently excluded form credit markets.

a) Define a recursive equilibrium.

b) Assume that the bankers expect the government to default if $\zeta > 1 - \pi$ and if such an expectation would be self-fulfilling, where $1 > \pi \geq 0$ is an arbitrary constant. Find a level of debt $\bar{b}$ such that, if $B \leq \bar{b}$, no default occurs in equilibrium, but that, if $B > \bar{b}$, default occurs in equilibrium.

c) Suppose that $B_0 > \bar{b}$, and the government chooses to run down its debt to $B_T \leq \bar{b}$ in T periods. Prove that it cannot be optimal to set $B_T < \bar{b}$. Prove that it is optimal for the government to set g_t constant as long as $B_t > \bar{b}$ and no crisis occurs. Find expressions for g_t and B_t that depend on B_0 and T. Find an expression for the expected discounted value of the utility of running down the debt that starts at B_0 to $\bar{b}$ in T periods. Find the limit of these expressions when $T = \infty$.

d) Using the answers to part c, write down a formula that determines a value of debt $\bar{B}(\pi)$ such that the government would choose to default if $B > \bar{B}(\pi)$ even if international bankers do not expect a default.

e) Using the answers to parts a–d, construct a recursive equilibrium.

f) Use this model to interpret events of the Mexican financial crisis of December 1994 through January 1995.

g) Assume that $Z = 0.9$, $\bar{y} = 100$, $\theta = 0.4$, $\gamma = 0.5$, $\beta = 0.95$, and $\pi = 0.05$. Calculate $\bar{b}$. Calculate the expected discounted value of the utility of running down the debt that starts at B_0 to $\bar{b}$ in T periods for $T = 1,2,3,4,5,6$. Calculate $\bar{B}(0.05)$. Graph a policy function for government debt $B'(B)$. Graph a policy function for government spending $g(B)$.