

PROBLEM SET

Answer three of the following six questions:

1. Download data on bilateral trade by sector at the 4 digit SITC level from the OECD web site, <http://oberon.sourceoecd.org>. Follow the methodology in Kehoe and Ruhl, “How Important is the New Goods Margin in International Trade?” to create a set of least traded goods and carry out one of the two following exercises:

a) Consider trade between two countries over time. Construct diagrams with fractions of trade at the end of the period by deciles of sets of goods at the beginning of the period. Graph of the fraction of trade accounted for by the least traded decile over time. Do imports and exports separately.

b) Consider exports of one country to a number of trading partners during one year. Compare the sets of least traded goods. Do you see any patterns?

2. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}.$$

The investment good is produced according to

$$k_{t+1} = d(a_1 x_{1t}^b + a_2 x_{2t}^b)^{1/b}.$$

In particular, the depreciation rate is $\delta = 1$. Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}. \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= \ell_t. \end{aligned}$$

The initial value of k_t is \bar{k}_0 . ℓ_t is normalized to 1.

a) Define an equilibrium for this economy.

b) Explain how you can reduce the equilibrium conditions of part a to two difference equations in k_t and c_t and a transversality condition. Here $c_t = d(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$ is

aggregate consumption. (You do not need to go through all of the algebra, but you need to explain all of the logical steps carefully.)

c) Suppose now that there is a world made up of two different countries, each with the same technologies and preferences, but with different constant populations, $L_t^j = \bar{L}^j$, and with different initial capital-labor ratios \bar{k}_0^i . Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_{1t} + p_{2t} = r_t k_t + w_t = d(a_1 k_t^b + a_2)^{1/b}$ is income per capita. Transform the two difference equation in part b into two difference equations in k_t and s_t . Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right).$$

where $y_t^i = p_{1t}k_t^i + p_{2t} = r_t k_t^i + w_t = d(a_1 k_t^{i,b} + a_2)^{1/b}$ is income per capita in country i .

f) Discuss the economic significance of the result in part e.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1-\alpha) \log c_0 + (\alpha / \rho) \log \int_0^\mu c(v)^\rho dv \\ \text{s.t.} \quad & p_0 c_0 + \int_0^\mu p(v) c(v) dv = w \bar{\ell} + \pi \\ & c(v) \geq 0. \end{aligned}$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good v takes the price function $p(v)$ as given. Suppose too that this producer has the production function

$$y(v) = \max \left[z(v) (\ell(v) - f), 0 \right].$$

Solve the consumer's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure μ of potential firms. Firm productivities are distributed on the interval $z \geq 1$ according to the Pareto distribution with distribution function

$$F(z) = 1 - z^{-\gamma}.$$

Define an equilibrium for this economy.

- c) Suppose that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Find an expression for the cutoff productivity level \bar{z} such that firms with productivity \bar{z} earn zero profits. Find an expression for profits π .
- d) Suppose now that there are two countries that engage in free trade. Each country i , $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of μ_i . Firms' productivities are again distributed according to the Pareto distribution, $F(z) = 1 - z^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j , $j \neq i$, of f_x where $f_x > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this economy.
- e) Suppose now that the two countries in part d are symmetric in the sense that $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$ and $\mu_1 = \mu_2 = \mu$. Suppose too that μ is large enough so that not all firms can earn nonnegative profits in equilibrium. Explain now why there are two relevant cutoff levels of firm productivity, \bar{z}_d and \bar{z}_x . Find expressions for these cutoff productivity levels. Find an expression for profits π .
- f) Discuss the strengths and weaknesses of this model. In particular: What economic phenomena can this sort of model help to account for? What sort of phenomena can it not account for? How can we modify the model to account for these phenomena?

4. Find annual time series data on real output, real investment, employment, working age population, and — if you can — hours worked for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate δ and a capital share α . Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

- a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose \bar{K}_{T_0} so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

- b) Repeat part a, but choose \bar{K}_{T_0} so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10.$$

- c) Compare the two series constructed in parts a and b.
- d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.

5. Choose two countries that are important (to each other) trading partners, one with higher output per worker than the other. Use Summers-Heston data or World Development Indicators data to try to answer Lucas's question: To what extent can differences in output per worker be explained by differences in capital per worker?

- a) Use IMF real interest rate data to try to make inferences about differences in capital per worker across countries. How much of the difference in output per worker be explained by differences in capital per worker? If capital flows equalized the rental rates on capital, what would happened to output per worker in the different countries?
- b) Use data on investment and the accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where $\delta = 0.05$ to determine capital per worker in some year. Answer the same questions as in part a and compare.

- c) Discuss what you have learned from this exercise and what more you think would be relevant in explaining differences in output per worker across countries.

6. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_t = rer_t^T + rer_t^N,$$

where rer_t is the natural logarithm of the bilateral real exchange rate and rer_t^T is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences.

Calculate ratio of the standard deviations of rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of rer_t in terms of rer_t^T and rer_t^N in levels, in 1 year differences, and in 4 year differences.