U.S.-CANADA FTA IN THE MONOPOLISTIC COMPETITION MODEL

Set $w = 1$ as numeraire.

We know that

$$p_0 = w = 1$$

$$y_0 = \frac{w\bar{\ell}}{2p_0} = \frac{\bar{\ell}}{2p_0}$$

$$y_i = \bar{y} = \frac{\rho(n-1)\bar{\ell}}{2n^2 b}$$

$$p_i = \bar{p} = \frac{\bar{\ell}\bar{y}^{\rho-1}}{2n\bar{y}^{\rho}} = \frac{\bar{\ell}}{2n\bar{y}} = \frac{bn}{\rho(n-1)}.$$

$$n = \frac{(1-\rho)\bar{\ell} + \sqrt{(1-\rho)^2\bar{\ell}^2 + 4(\rho\bar{\ell})(2f)}}{4f}.$$

Autarky equilibrium in Canada

$$b = 1,\ f = 2,\ \rho = 1/2,\ \bar{\ell} = 18$$

$$n = \frac{9 + \sqrt{(9)^2 + 4(9)(4)}}{8}$$

$$n = 3$$

$$\bar{y} = 1,\ \bar{p} = 3$$

$$p_0 = w = 1,\ y_0 = 9$$

Utility:

$$\log 9 + 2\log(3(1)^{1/2}) = 4.3944$$

Homogenous of degree one representation of utility (a real income index):

$$\exp[(1/2)(\log 9 + 2\log(3(1)^{1/2})] = \exp[(1/2)(4.3944)] = 9.00.$$

Autarky equilibrium in the United States

$$b=1,\ f=2,\ \rho=1/2,\ \bar{\ell}=180$$

$$n=\frac{90+\sqrt{(90)^2+4(90)(4)}}{8}$$

$$n=23.4591$$

$$\bar{y}=1.8365,\ \bar{p}=2.0891$$

$$p_0=w=1,\ y_0=90$$

Utility:

$$\log 90+2\log(23.4591(1.8365)^{1/2})=11.4182$$

Homogenous of degree one representation of utility (a real income index):

$$\exp[(1/2)(\log 90+2\log(23.4591(1.8365)^{1/2})]=\exp[(1/2)(13.2418)]=301.60\,.$$

Autarky equilibrium

	$\hat{n}^i$	$\hat{p}_0^i$	$\bar{p}$	$\hat{w}^i$	$\hat{c}_0^i$	$\bar{c}^i$	$\hat{y}_0^i$	$\hat{\ell}_0^i$	$\bar{y}^i$	$\bar{\ell}^i$	$e^{\hat{u}/2}$
Canada	3.0000	1.0	3.0000	1.0	9.0	1.0000	9.0	9.0	1.0000	3.0000	9.00
U.S.	23.4591	1.0	2.0891	1.0	90.0	1.8365	90.0	90.0	1.8365	3.8365	301.60

Free trade equilibrium

First calculate the equilibrium for the North American economy with $\bar{\ell}=\bar{\ell}^{CA}+\bar{\ell}^{US}=198$:

$$n=\frac{99+\sqrt{(99)^2+4(99)(4)}}{8}$$

$$n=25.7126$$

$$\bar{y}=1.8503,\ \bar{p}=2.0809$$

$$p_0=w=1,\ y_0=99$$

To calculate consumption of each variety in each country, we just divide the world production of the variety $\overline{y}$ proportionally. In Canada, for example,

$$\overline{c}^{CA} = \frac{\overline{\ell}^{CA}}{\overline{\ell}^{CA} + \overline{\ell}^{CA}} \overline{y} = \frac{18}{198} 1.8503 = 0.1682 .$$

We also divide the production and the consumption of the agricultural good proportionally.

Free trade equilibrium

	$\hat{n}^i$	$\hat{p}_0^i$	$\overline{p}$	$\hat{w}^i$	$\hat{c}_0^i$	$\overline{c}^i$	$\hat{y}_0^i$	$\hat{\ell}_0^i$	$\overline{y}^i$	$\overline{\ell}^i$	$e^{\hat{u}/2}$
Canada	2.3375	1.0	2.0809	1.0	9.0	0.1682	9.0	9.0	1.8503	3.8503	33.06
U.S.	23.3751	1.0	2.0809	1.0	90.0	1.6821	90.0	90.0	1.8503	3.8503	330.57

Notice that Canada stands to gain far more from trade than does the United States:

In Canada real income increases by 267 percent ($33.06/9.00 = 3.67$).

In the United States real income increases by 10 percent ($330.57/301.60 = 1.10$).