

PROBLEM SET

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b) \max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho) \log \sum_{j=1}^n c_j^\rho$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor

- a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.
- b) Suppose that $b = 1$, $f = 3$, $\rho = 1/2$, and $\bar{\ell} = 50$. Calculate the autarky equilibrium.
- c) Suppose now that $\bar{\ell} = 200$. Calculate the equilibrium.
- d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 50$ and the second with $\bar{\ell}^2 = 150$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.
- e) Repeat the analysis of parts a-d for a variant of the model in which consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

2. Consider a world with three countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0,1]$ given by the utility function

$$\int_x \log c(x) dx.$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}_3 = \bar{\ell}$. Production functions are linear but differ across countries:

$$\begin{aligned} y_j^i(x) &= \ell_j^i(x) / a_j^i(x) \\ a_1^1(x) &= a_1^2(x) = a_1^3(x) = \alpha + \beta x \\ a_2^1(x) &= a_2^2(x) = a_2^3(x) = \alpha + \beta - \beta x \\ a_3^1(x) &= a_3^2(x) = a_3^3(x) = \gamma. \end{aligned}$$

Here, for example, $y_j^i(x)$ is the amount of good x produced in country j for consumption in country i . Initially, there are no transportation costs or tariffs.

- a) Define an equilibrium for this model.
- b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.
- c) Suppose now that there are 10 percent transportation costs between every pair of countries. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in parts a and b.
- d) Suppose now that the countries engage in a trade war in which each imposes a 10 percent tariff on imports from the other two. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in part c.

3. Download data on bilateral trade by sector at the 4 digit SITC level from the OECD web site, <http://oberon.sourceoecd.org>. Follow the methodology in Kehoe and Ruhl, “How Important is the New Goods Margin in International Trade?” to create a set of least traded goods and carry out one of the two following exercises:

a) Consider trade between two countries over time. Construct diagrams with fractions of trade at the end of the period by deciles of sets of goods at the beginning of the period. Graph of the fraction of trade accounted for by the least traded decile over time. Do imports and exports separately.

b) Consider exports of one country to a number of trading partners during one year. Compare the sets of least traded goods. Do you see any patterns?

4. Find data to calculate the bilateral real exchange rate between two countries that have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_t = rer_t^T + rer_t^N,$$

where rer_t is the natural logarithm of the bilateral real exchange rate and rer_t^T is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences.

Calculate ratio of the standard deviations of rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of rer_t in terms of rer_t^T and rer_t^N in levels, in 1 year differences, and in 4 year differences.