TOPICS IN INTERNATIONAL ECONOMICS **T.J. KEHOE**
ECON 698K **FALL 2003**

EXAMINATION

You have 24 hours to answer **two** of the three questions. You can consult your notes, books, or articles. You should not discuss the exam with anyone else. Notice that each question includes both easy parts and difficult parts.

1. **[A Ricardian model with a continuum of goods]** Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$a_1(z) = e^{\alpha z}$$
$$a_2(z) = e^{\alpha(1-z)}.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz.$$

This consumer is endowed with $\overline{\ell}_j$ unites of labor where $\overline{\ell}_1 = \overline{\ell}_2 = \overline{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose that each country imposes an ad valorem tariff τ on imports from the other country. Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

c) Calculate gross domestic product and exports for each country as functions of α, $\overline{\ell}$, and τ. Calculate the ratio of trade to total output in this world economy.

d) Suppose now that goods are produced using both capital and labor:

$$y_j(z) = k_j(z)^{\alpha(z)} \ell_j(z)^{1-\alpha(z)},$$

where $\alpha(z) = z$, $z \in [0,1]$. Notice that production technologies are now identical across countries. Endowments, however, are different. Specifically,

$$\bar{\ell}_1 = \bar{k}_2 > \bar{\ell}_2 = \bar{k}_1.$$

Define an equilibrium of the economy. Derive a condition under which factor price equalization occurs. Supposing that this condition does not hold, draw a graph that illustrates the pattern of specialization in production and trade.

2. **[A trade model with monopolistic competition]** Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum_{j=1}^{n} c_j^{\rho}$$

where $\rho > 0$. There is an endowment of $\bar{\ell}$ units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 1$, $f = 3$, $\rho = 1/2$, and $\bar{\ell} = 50$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 200$. Calculate the equilibrium. [If you do not have a calculator, find a simple equation that you could solve to find the numerical value of n and explain carefully how you would find the values of the other variables.]

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 50$ and the second with $\bar{\ell}^2 = 150$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility? Illustrate the efficiency gains using an average cost curve diagram.

e) Suppose now that firms follow Bertrand pricing rules and there is free entry and exit. Define a monopolistically competitive equilibrium for this economy (in autarky).

f) If you were to do the trade liberalization exercise of parts c and d in the world where firms follow Bertrand pricing rules, would you expect to find that the gains from trade differ form those in the world where they follow Cournot pricing rules? Explain why or why not.

3. **[A dynamic Heckscher-Ohlin model]** Consider a two-sector growth model in which the representative consumer has the
utility function

$$\int_0^\infty e^{-\rho t} \log(c_1^b + c_2^b)^{1/b}\, dt.$$

and in which investment is produced according to

$$\dot{k} + \delta k = (x_1^b + x_2^b)^{1/b}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_1 + x_1 &= f_1(k_1, \ell_1) = k_1 \\ c_2 + x_2 &= f_2(k_2, \ell_2) = \ell_2 \ . \end{aligned}$$

The initial value of $k(t)$ is $k(0)$; $\ell(t)$ is fixed at 1 .

a) Define a competitive equilibrium.

b) Reduce the equilibrium conditions to two differential equations in k and $z = c/k$ and a transversality condition. Here $c = (c_1^b + c_2^b)^{1/b}$ is aggregate consumption.

c) Draw a phase diagram in (k, z) space illustrating the equilibrium path for any given $k(0)$. (You will have to consider different cases depending on the value of b; you can do one carefully for a case where $b < 0$ and just sketch out the other(s).)

d) Consider the case where $b < 0$ and suppose that there is a world of many such countries, each with a different population $\overline{\ell}^j$ and a different initial capital-labor ratio $k^j(0)$. Define a competitive equilibrium for the world economy.

e) Consider a country whose initial capital-labor ratio $k^j(0)$ is much lower than the initial worldwide capital-labor ratio $k(0)$, which is in turn less than the steady state capital-labor ratio. Explain how the development path for this economy depends crucially on whether it is open or closed to trade. (You can assume that the relation

$$\frac{k^j(t)}{k(t)} - 1 = \frac{z(t)}{z(0)}\left(\frac{k_j(0)}{k(0)} - 1\right)$$

holds in the trade equilibrium without proving that it does.)

f) How could you modify the analysis to be able to analyze the impact of trade on convergence of income per worker, rather than that of capital per worker,

$$\frac{y^j(t)}{y(t)} - 1,$$

where $y^j = p_1 c_1^j + p_2 c_2^j + p_1 x_1^j + p_2 x_2^j = w + rk^j$?