

Final Exam

Answer *two* of the following three questions.

1. Consider an economy with a representative infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

The set of feasible consumption and production plans satisfy

$$\begin{aligned} c_t + k_{t+1} - (1 - \delta)k_t &\leq g^t A k_t^\alpha \ell_t^{1-\alpha} \\ \ell_t &= 1 \\ k_0 &= \bar{k}_0. \end{aligned}$$

Here $1 > \beta > 0$, $1 \geq \delta \geq 0$, $g > 1$, $A > 0$, and $1 > \alpha > 0$.

- a) Define a competitive equilibrium for this economy.
- b) Suppose that both consumption and the capital stock grow at, possibly different, constant rates in equilibrium. Prove that they have to grow at the same rate. Derive the relationship between this rate of growth and g .
- c) Use your answer to part b to define a balanced growth path for this economy. Show that this balanced growth path satisfies Nicholas Kaldor's stylized facts of economic growth.
- d) Suppose now that there is an economy with roughly constant population. In 1980 its national income and product accounts were

Product		Income	
Consumption	80	Labor Income	70
Investment	20	Capital Income	20
		Depreciation	<u>10</u>
GDP	<u>100</u>	GDP	100

Between 1980 and 2000 all of these numbers grew at roughly 2 percent per year in real terms. Either calibrate the model economy to match this set of balanced growth observations or carefully specify a procedure to do so.

2. Consider a model with an infinitely-lived, representative consumer. The production function is $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$. The consumer solves the problem

$$\begin{aligned} \max \quad & \sum_{t=t_0}^{\infty} \beta^t [\gamma \log C_t + (1-\gamma) \log(N_t \bar{h} - L_t)] \\ \text{s.t.} \quad & (1 + \tau_t^c) C_t + K_{t+1} - K_t = (1 - \tau_t^l) w_t L_t + (1 - \tau_t^k)(r_t - \delta) K_t + T_t. \\ & C_t, N_t \bar{h} - L_t, L_t, K_{t+1} - (1 - \delta) K_t \geq 0 \\ & K_{t_0} = \bar{K}_{t_0}. \end{aligned}$$

- a) Define a competitive equilibrium for this economy.
- b) Suppose that you are given time series for GDP, gross investment, and capital consumption, all deflated by a GDP deflator. Explain carefully how you could calculate a series for capital. How would you calibrate the depreciation rate δ ? Discuss the limitations of this procedure.
- c) What sort of information would you need to calibrate the capital share α ?
- d) Suppose that you are given a time series for GDP at factor prices. How could you use this information to perform a growth accounting decomposition of economic growth for this economy? What can you hope to learn from this sort of exercise? Discuss the limitations of this sort of exercise. Explain briefly why you need information on GDP at factor prices if you are working with a model with distortionary taxes.
- e) What sort of information would you need to calibrate the parameters β and γ ? Specify a procedure for doing so.
- f) Explain how you could use this model to evaluate the impact of tax changes on economic growth in an economy for which you had the relevant data. Discuss the limitations of this sort of exercise.

3. Consider a reduced-form model of a small open economy in which the government that runs a constant deficit, $G - T$, at every date in continuous time. Domestic agents can either hold domestic money M or foreign money. These agents' demand for real balances of domestic money depends on the inflation rate,

$$\frac{M}{P} = L\left(\frac{\dot{P}}{P}\right) = \frac{\bar{M}}{1 + \dot{P}/P}.$$

There are two possibilities:

(1) The nominal exchange rate is fixed. There is no inflation in the foreign country, and we normalize both the foreign price level and the exchange rate to be 1. The law of one price holds so

$$P = eP^* = e = 1.$$

The government sells reserves, which are initially fixed at the level $R(0) = R_0$, to maintain the real exchange rate.

(2) The government lets the exchange rate float, does not sell reserves and prints money to finance the deficit.

a) Suppose that there are no reserves and that the exchange rate is floating. Calculate the equilibrium level of inflation

$$\pi = \frac{\dot{P}}{P}.$$

b) Suppose now that the exchange rate is initially fixed. Suppose too that the government cannot borrow, so that the lower level on reserves is

$$R(t) \geq 0.$$

How high does the initial level of reserves R_0 need to be for the fixed exchange rate regime not to be vulnerable to a speculative attack? Explain.

c) Suppose that the initial level of reserves is high enough so that the government can maintain the fixed exchange rate. Suppose too that it maintains the exchange rate until it has no reserves. Characterize the equilibrium of the model. In particular, find an expression for the date T at which a speculative attack exhausts all reserves and the government is forced to float the exchange rate. Explain carefully why it is not a rational expectations equilibrium for there to be a speculative attack either before or after T .

d) Suppose now that the government can borrow foreign currency at a fixed interest rate r^* up to a fixed borrowing constraint

$$B(t) \leq \bar{B}.$$

Suppose too that the government borrows to maintain the fixed exchange rate as long as possible. Repeat the analysis in parts a, b, and c. Show that the collapse of the fixed exchange rate regime occurs later if $\bar{B} > 0$ than it does in the previous case, where $\bar{B} = 0$, but that the inflation after the collapse is higher.

e) Explain the differences between this sort of model and the so-called “second generation” crisis models in which the government faces a choice of devaluing or not in every period. Discuss the sorts of empirical observations that motivated researchers to move away from the sort of model analyzed in this question to the second generation crisis models.