Problem Set #2

1. Consider the data for Spain in the attached Excel file. Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{1954} = \bar{K}_{1954}.$$

Choose δ so that

$$\left(\sum_{t=1970}^{2000} \delta K_t / Y_t\right)/31 = \delta\left(\sum_{t=1970}^{2000} K_t / Y_t\right)/31 = 0.12895034.$$

and choose $\bar{K}_{1954}$ so that

$$K_{1954} / Y_{1954} = \left(\sum_{t=1955}^{1964} K_t / Y_t\right)/10.$$

2. Consider a model with an infinitely-lived, representative consumer. The production function is $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$. The consumer solves the problem

$$\max \ \sum_{t=t_0}^{\infty} \beta^t [\gamma \log C_t + (1-\gamma)\log(N_t \bar{h} - L_t)]$$
$$\text{s.t. } (1+\tau_t^c)C_t + K_{t+1} - K_t$$
$$= (1-\tau_t^\ell)w_t L_t + (1-\tau_t^k)(r_t - \delta)K_t + T_t.$$
$$K_{t_0} = \bar{K}_{t_0}$$

a) Using the series for the capital stock from question 1 and the data in the attached Excel file, calculate a series for total factor productivity A_t. Choose $\alpha = 0.31149775$. Be careful: In this model GDP is

$$GDP_t = (1+\tau_t^c)C_t + I_t = r_t K_t + w_t L_t + \tau_t^c C_t = A_t K_t^\alpha L_t^{1-\alpha} + \tau_t^c C_t.$$

b) Using the data in the attached Excel file for the period 1965-1974 and the first order conditions from the consumer's problem, estimate the values of the parameters β and γ.

c) Using the MATLAB programs in the attached zip file, calculate the equilibrium of this model for the period 1970-2030. Assume that the rate of population growth over the period 2000-2030 is constant at its average value 1970-2000, that the rate of increase of TFP over the period 2000-2030 is constant at its average value 1970-2000, and that tax

rates over the period 2000-2030 are constant at their 2000 values. Assume that by 2030, the equilibrium has converged to its balanced growth path.

3. Repeat the analysis of question 2 where all tax rates stay constant at their average values 1965-1975. Be careful: The parameters need to be recalibrated.

4. Repeat the analysis of question 2 where tax rates change but where the rate of increase of TFP stays constant at its average value 1965-1975.