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Topics in Macroeconomics Spring 2004

Problem Set #3

1. Consider a reduced-form model of a small open economy in which the government that runs a constant deficit, $G-T$, at every date in continuous time. Domestic agents can either hold domestic money M or foreign money. These agents' demand for real balances of domestic money depends on the inflation rate,

$$\frac{M}{P}=L\left(\frac{\dot{P}}{P}\right)=\frac{\bar{M}}{1+\dot{P}/P}.$$

There are two possibilities:

(1) The nominal exchange rate is fixed. There is no inflation in the foreign country, and we normalize both the foreign price level and the exchange rate to be 1. The law of one price holds so

$$P=eP^{*}=e=1.$$

The government sells reserves, which are initially fixed at the level $R(0)=R_0$, to maintain the real exchange rate.

(2) The government lets the exchange rate float, does not sell reserves and prints money to finance the deficit.

a) Suppose that that are no reserves and that the exchange rate is floating. Calculate the equilibrium level of inflation

$$\pi=\frac{\dot{P}}{P}.$$

b) Suppose now that the exchange rate is initially fixed. Suppose too that the government cannot borrow, so that the lower level on reserves is

$$R(t)\geq 0.$$

How high does the initial level of reserves R_0 need to be for the fixed exchange rate regime not to be vulnerable to a speculative attack? Explain.

c) Suppose that the initial level of reserves is high enough so that the government can maintain the fixed exchange rate. Suppose too that it maintains the exchange rate until it has no reserves. Characterize the equilibrium of the model. In particular, find an expression for the date T at which a speculative attack exhausts all reserves and the government is forced to float the exchange rate. Explain carefully why it is not a

rational expectations equilibrium for there to be a speculative attack either before or after T.

d) Suppose now that the government can borrow foreign currency at a fixed interest rate r^* up to a fixed borrowing constraint

$$B(t) \leq \overline{B}.$$

Suppose too that the government borrows to maintain the fixed exchange rate as long as possible. Repeat the analysis in parts a, b, and c. Show that the collapse of the fixed exchange rate regime occurs later if $\overline{B} > 0$ than it does in the previous case, where $\overline{B} = 0$, but that the inflation after the collapse is higher.

2. Consider a reduced form model of a n economy in which the government chooses the nominal exchange rate in every period t to minimize the loss function

$$\left[a(e_t^* - e_t) + b(e_t^E - e_t)\right]^2 + \delta C.$$

Here $\left[a(e_t^* - e_t)\right]^2$ is the cost of deviating from the optimal exchange rate e_t^*, $\left[b(e_t^E - e_t)\right]^2$ is the cost of having private agents make decisions based on incorrect expectations e_t^E, and δC is the cost of breaking the commitment to maintain a fixed exchange rate $e_t = \overline{e}$. ($\delta = 1$ if devaluation, $\delta = 0$ otherwise.)

a) Calculate the crisis zone, the set of values of e_t^* for which the government will choose to devalue if private agents expect a devaluation but where it will not devalue if the government does not expect a devaluation.

b) Demonstrate that increasing the value of C, this is, increasing the default penalty, shifts the crisis zone but does not eliminate it.

3. Consider a model of a small open economy with three types of agents: Domestic consumer/ producers, a domestic government, and foreign lenders.

There are a continuum of measure 1 of domestic consumer/producers who choose the sequences c_t, k_t to solve the problem

$$\begin{gathered}
\max \sum_{t=0}^{\infty} \beta^t \left(c_t + \log g_t\right) \\
\text{s.t. } c_t + k_{t+1} = (1-\theta)(w_t + r_t k_t) \\
c_t, k_t \geq 0 \\
k_0 = \overline{k}_0.
\end{gathered}$$

Here θ is a constant income tax rate. These agents also operate the constant returns production technology $A_t k_t^{\alpha} \ell_t^{1-\alpha}$ where A_t is productivity term that depends on the government's default decision.

If the government has never defaulted, $A_t = 1$ and the government can borrow from or lend to the foreign lenders. If the government defaults, $A_t = A < 1$ forever, and the government is excluded from credit markets. The government chooses whether or not to defaults, along with spending levels g_t and borrowing levels B_t, to maximize the utility of the domestic consumer/producers. The government's budget constraint is

$$g_t + z_t B_t = \theta(w_t + r_t k_t) + q_t B_{t+1}.$$

Here z_t is the default decision ($z_t = 0$ in the case of default, and $z_t = 1$ otherwise), and q_t is the price of bonds sold in period t to be paid off in the case of no default in period $t+1$.

The international lenders are relatively passive. They are risk neutral with a discount factor of β and sufficient income to purchase all of the bonds offered by the government in period t if there will be no default in period $t+1$.

The sequence of events within a period is

(1) The government offers bonds B_t for sale and the price q_t is determined.

(2) The government chooses to default or not.

(3) Production and consumption occur.

The feasibility constraints for this economy are

$$c_t + k_{t+1} + g_t + \left(z_t B_t - q_t B_{t+1}\right) = A_t k_t^{\alpha} \ell_t^{1-\alpha}$$
$$\ell_t = 1.$$

a) Define an equilibrium of this economy.

b) Suppose that foreign lenders expect that the government will repay it debt unless they know that the government will default even if $q_t = \beta$. Demonstrate that there are two types of equilibria of this economy, depending on the initial values of government debt, B_0, and private capital, $\overline{k}_0$: In one equilibrium the government defaults in period $t = 0$, and in the other it never defaults. Demonstrate that in each of these equilibria, all of the variables are constant in period $t = 1$ and afterwards.

c) Assume now that $\bar{k}_0$ satisfies

$$\beta(1-\theta)\alpha\bar{k}_0^{\alpha-1} = 1.$$

Calculate the maximum value of B_0 for which the government will not default in period $t = 0$.

d) Suppose that we are in the equilibrium where the government does not default in period $t = 0$. Suppose too that, at some date, $t \geq 0$, the foreign lenders expect that the government will default if $q_t = 0$. (This is not an equilibrium outcome.) Find conditions under which it is possible for this expectation to be part of an equilibrium for period t on? In particular, characterize the crisis zone – the set of B_t that are less than the crucial value in part c but high enough for the unanticipated change in expectations in period t to e self-fulfilling.

e) Demonstrate that decreasing the value of A, this is, increasing the default penalty, shifts the crisis zone but does not eliminate it. Interpret this result.