MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2017**

FINAL EXAMINATION

Answer ***two*** of the following four questions.

1. Consider an overlapping generations economy in which the representative consumer born in period $t, t=1,2,\ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \gamma \log c_t^t + (1-\gamma)\log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \gamma \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Suppose that $m=0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $\gamma = 2/3$ and $(w_1, w_2) = (5,4)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two consumption goods in each period. The representative consumer born in period $t, t=1,2,\ldots,$ has the utility function over consumption of the two goods in periods t and $t+1$

$$u(c_{1t}^t, c_{2t}^t, c_{1t+1}^t, c_{2t+1}^t) = \gamma_{11}\log c_{1t}^t + \gamma_{21}\log c_{2t}^t + \gamma_{12}\log c_{1t+1}^t + \gamma_{22}\log c_{2t+1}^t$$

and endowments $(w_{1t}^t, w_{2t}^t, w_{1t+1}^t, w_{2t+1}^t) = (w_{11}, w_{21}, w_{12}, w_{22})$. The representative consumer in the initial old generation has the utility function

$$u^0(c_{11}^0, c_{21}^0) = \gamma_{12}\log c_{11}^0 + \gamma_{22}\log c_{21}^0$$

and endowments $(w_{11}^0, w_{21}^0) = (w_{12}, w_{22})$ of the goods in period 1 and endowment m of fiat money. Define a sequential markets equilibrium for this economy. Suppose that $m=0$. Do you expect that $\hat{c}_{1t}^t = w_{11}$? Explain why or why not.

2. Consider an economy with a representative, infinitely lived consumer who has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. The consumer owns one unit of labor in each period and $\bar{k}_0$ units of capital in period 0. Suppose that feasible consumption/investment plans satisfy

$$c_t + k_{t+1} - (1-\delta)k_t \leq \theta k_t^{\alpha} \ell_t^{1-\alpha}.$$

(a) Suppose that the consumer borrows b_{t+1} bonds in period t to be paid off in period $t+1$. The consumer's initial endowment of bonds is $\bar{b}_0 = 0$, the wage rate in period t is w_t, the rental rate on capital is r_t^k, and the interest rate on bonds is r_t^b. Define a sequential markets equilibrium for this economy. Explain why you need to include a constraint to rule out Ponzi schemes. Write down the Euler conditions and the transversality conditions for this problem. Prove that in equilibrium $r_t^k - \delta = r_t^b$ if $k_t > 0$.

(b) Suppose that the consumer sells his endowment of capital to the firm in period 0. Thereafter, firms buy and sell capital from and to each other. Describe the production set for the Arrow-Debreu economy, the set of feasible $k_0, k_1, \ldots, \ell_0, \ell_1, \ldots, c_0, c_1, \ldots$. Define the Arrow-Debreu equilibrium for this economy.

(c) Carefully state theorems that relate the equilibrium allocation in part (a) to that in part (b).

(d) Suppose that now there are two types of consumers. The representative consumer of type 1 has the endowment of labor $(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, \ldots) = (2,1,2,1,\ldots)$ and the endowment of capital $\bar{k}_0^1$. The representative consumer of type 2 has the endowment of labor $(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, \ldots) = (1,2,1,2,\ldots)$ and the endowment of capital $\bar{k}_0^2$. Define a sequential markets equilibrium for this economy.

(e) Does the equilibrium allocation/production plan in part (d) solve a dynamic programming problem? If it does, write down the Bellman's equation for this problem. If it does not, explain why it does not.

4. Consider an economy in which the representative consumer lives forever. There is a good in each period that can be consumed or saved as capital. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t u(c_t) .$$

Feasible allocations satisfy

$$c_t + k_{t+1} \leq f(k_t)$$
$$c_t, k_t \geq 0$$
$$k_0 = \bar{k}_0 .$$

(a) Formulate the problem of maximizing the representative consumer's utility subject to feasibility conditions as a dynamic programming problem by carefully writing down the appropriate Bellman's equation.

(b) Provide assumptions on u, f, and β that are sufficient to guarantee that there is a unique bounded continuous function satisfying the functional equation you wrote in part (a). Explain why you need each assumption. [You do not need to prove anything.]

(c) Characterize the optimal policy function through the first-order condition and the envelope condition for the dynamic programming problem in part (a).

(d) Suppose that the policy function $k' = g(k)$ is the solution to the dynamic programming problem in (a). Use the first-order condition and envelope condition from part (c) to show that $g(k)$ is strictly increasing. Make sure you state all assumptions you use for u, f, and β.

(e) Specify an economic environment for which the solution to the social planner's problem in part (a) is a Pareto efficient allocation/production plan. Define a sequential markets equilibrium for this environment. Explain how you could use the policy function in parts (c) and (d) to calculate the unique sequential markets equilibrium. Make sure you state all assumptions you use for u, f, and β. [You do not have to prove that this equilibrium is unique.]

5. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

(a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

(b) Using Bellman's equation from part (a), characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \ge \bar{w}$ and rejects any wage offer $w < \bar{w}$.

(c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

(d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Define what it means for F_2 to be a mean preserving spread of F_1.

(e) Suppose that F_2 is a mean preserving spread of F_1 and that the inequality that defines a mean preserving spread holds strictly in the case of F_2 and F_1. Let $\bar{w}_1$ be the reservation wage in the economy with wage distribution F_1 and $\bar{w}_2$ be the reservation wage in economy with wage distribution F_2. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.