MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2017**

MIDTERM EXAMINATION

Answer **two** of the following three questions.

1. Consider an economy with two types of infinitely lived consumers, each of measure 1. There is one good in each period. Consumer i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers has an endowment of a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (2,1,2,1,...)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (1,2,1,2,...).$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establishes the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium. (You are **not** asked to prove this proposition or propositions.)

(d) Calculate the sequential markets equilibrium for this economy. (This equilibrium is unique, but you do not have to prove this fact.)

(e) Suppose now that there is a storage technology that takes one unit of the good in period t and transforms it into one unit of the good in period $t+1$:

$$y_{t+1} = x_t,$$

where x_t is the input of the good in period t and, y_{t+1} is the output of the good in period $t+1$. Define a sequential markets equilibrium for this economy. Will the storage technology be employed in equilibrium? Explain why or why not.

2. Consider an overlapping generations economy in which the representative consumer born in period $t, t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = 2\log c_t^t + c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u_0(c_1^0) = c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Define an Arrow-Debreu equilibrium for this economy.

(b) Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $(w_1, w_2) = (1,1)$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two consumer types of measure 1 in each generation. Consumer $1t$ has utility function

$$u_1(c_t^{1t}, c_{t+1}^{1t}) = 2\log c_t^{1t} + c_{t+1}^{1t},$$

while consumer $2t$ has the utility function

$$u_2(c_t^{2t}, c_{t+1}^{2t}) = \log c_t^{2t} + 2c_{t+1}^{2t}.$$

The initial old consumers have utility functions

$$u_{10}(c_1^{10}) = c_1^{10}$$

$$u_{20}(c_1^{20}) = 2c_1^{20}.$$

The consumers have endowments of goods $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i)$ and $w_1^{i0} = w_2^i$, $i = 1,2$, and the initial old consumers have endowments of fiat money m^i, $i = 1,2$. Define a sequential markets equilibrium for this economy.

3. Consider an economy with two types of measure 1 of consumers with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i ,$$

$i = 1, 2$, where $0 < \beta < 1$. The consumers have sequence of endowments of labor

$$(\overline{\ell}_0^1, \overline{\ell}_0^1, \overline{\ell}_0^1, \overline{\ell}_0^1, \ldots) = (2, 1, 2, 1, \ldots)$$
$$(\overline{\ell}_0^2, \overline{\ell}_0^2, \overline{\ell}_0^2, \overline{\ell}_0^2, \ldots) = (1, 2, 1, 2, \ldots)$$

and endowments of $\overline{k}_0^i$, $i = 1, 2$, units of capital in period 0. There is a production function

$$F(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha} ,$$

and a fraction δ of capital depreciates every period.

(a) Define an Arrow-Debreu equilibrium for this economy.

(b) Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium. (You are **not** asked to prove this proposition or propositions.)

(d) Suppose now that each consumer type has the utility function

$$\sum_{t=0}^{\infty} \beta^t \left(\gamma \log c_t^i + (1-\gamma) \log(\overline{\ell}_t^i - \ell_t^i) \right)$$

where $1 \geq \gamma > 0$. Define a sequential markets equilibrium.