MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2020**

PROBLEM SET #5

Questions 1, 2, and 3 require you to find annual time series data on constant price GDP, current price GDP, current price investment, hours worked, and working age population for the some country not the United States. Instead of constant price GDP, you may need to use a chained weighted quantity index. If you cannot find data on hours worked, use data on employment and assume that all workers worked a fixed amount, like 40 hours per week.

1. Construct a real investment series by calculating

$$I_t = \frac{\tilde{I}_t}{P_t},$$

where $\tilde{I}_t$ is nominal (current price) investment and P_t is the GDP deflator

$$P_t = \frac{\tilde{Y}_t}{Y_t},$$

where $\tilde{Y}_t$ is nominal (current price) GDP and Y_t is real (constant price) GDP or the chain weighted quantity index for GDP.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$K_{T_0} = \bar{K}_{T_0}.$$

where T_0 is the first year for which you have data on output and investment. Choose $\bar{K}_{T_0}$ so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

If you have sufficient data, calibrate the depreciation rate δ. Otherwise, use $\delta = 0.05$.

b) Repeat part a, but choose $\bar{K}_{T_0}$ so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t\right)/10.$$

c) Compare the two series constructed in parts a and b.

2. Suppose that the aggregate production function for the country that you are studying takes the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

If you have sufficient data, calibrate the capital share α. Otherwise, use $\alpha = 0.35$.

a) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person:

$$\frac{Y_t}{N_t} = A_t^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}.$$

b) Discuss what happens during different time periods.

3. Consider an economy in which the equilibrium solves the optimal growth problem

$$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t \left[\theta \log C_t + (1-\theta) \log(N_t \bar{h} - L_t) \right] \\ &\text{s.t.} \quad C_t + K_{t+1} - (1-\delta) K_t \le (\gamma^{1-\alpha})^t A_0 K_t^{\alpha} L_t^{1-\alpha} \\ &\qquad C_t, K_t \ge 0 \\ &\qquad K_0 = \bar{K}_0 \\ &\qquad N_t = \eta^t N_0. \end{aligned}$$

a) Define a balanced growth path for this economy. Write down conditions that characterize this balanced growth path. Verify that the balanced growth path exhibits characteristics consistent with Kaldor's stylized facts on economic growth.

b) Calibrate the parameters of this economy — $\beta, \theta, \gamma, A_0$, and η and, if you have sufficient data, α and δ — so that that the behavior of this economy matches that in the data. Do the data for this country look like those of a balanced growth path? Discuss.

4. Consider the problem faced by an unemployed worker searching for a job. Every period that the worker searches, she receives a job offer with the wage w drawn independently from the time invariant probability distribution $F(v) = \text{prob}(w \le v)$, $v \in [0, B]$, $B > 0$. After receiving the wage offer w the worker faces the choice (1) to accept it or (2) to reject it, receive unemployment benefit b, and search again next period. That is,

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where $1 > \beta > 0$. Once a job offer has been accepted, there are no fires or quits.

a) Formulate the worker's problem as a dynamic programming problem by writing down Bellman's equation.

b) Using Bellman's equation from part a, characterize the value function $V(w)$ in a graph and argue that the worker's problem reduces to determining a reservation wage $\bar{w}$ such that she accepts any wage offer $w \geq \bar{w}$ and rejects any wage offer $w < \bar{w}$.

c) Consider two economies with different unemployment benefits b_1 and b_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Suppose that that $b_2 > b_1$. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result.

d) Consider two economies with different wage distributions F_1 and F_2 but otherwise identical. Let $\bar{w}_1$ and $\bar{w}_2$ be the reservation wages in these two economies. Define a mean preserving spread. Suppose that F_2 is a mean preserving spread of F_1 and that the inequality that defines a mean preserving spread holds strictly in the case of F_2 and F_1. Prove that $\bar{w}_2 > \bar{w}_1$. Provide some intuition for this result. Explain why expected utility is higher in the economy with wage distribution F_2 than it is in the economy with wage distribution F_1.