

PROBLEM SET #4

1. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1-\alpha)\log c_0 + (\alpha/\rho)\log \int_0^m c(z)^\rho dz \\ \text{s.t.} \quad & p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi \\ & c(z) \geq 0. \end{aligned}$$

Here  $\pi$  are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good  $z$  takes the prices  $p(z')$ , for  $z' \neq z$ , as given. Suppose too that this producer has the production function

$$y(z) = \max[x(z)(\ell(z) - f), 0].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure  $\mu$  of potential firms. Firm productivities are distributed on the interval  $x \geq 1$  according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma}.$$

Define an equilibrium for this economy.

c) Characterize the equilibrium of this economy in two different cases: In the first,  $\mu$  is so small that all firms make nonnegative profits. In the second,  $\mu$  is large enough so that there is a cutoff level of productivity  $\bar{x} > 1$  such that all firms with productivity  $x(z) \geq \bar{x}$  produce and earn nonnegative profits but no firm with productivity  $x(z) < \bar{x}$  finds it profitable to produce. Find the relation between  $m$  and  $\mu$  and  $\bar{x}$ .

d) Suppose that  $\alpha = 0.5$ ,  $\rho = 0.5$ ,  $\bar{\ell} = 30$ ,  $\mu = 10$ ,  $\gamma = 4$ , and  $f = 2$ . Calculate the equilibrium of this economy.

2. Consider a world with two countries like that in question 1 that engage in free trade. Each country  $i$ ,  $i = 1, 2$ , has a population of  $\bar{\ell}_i$  and a measure of potential firms of  $\mu_i$ . Firms' productivities are again distributed according to the Pareto distribution,  $F(x) = 1 - x^{-\gamma}$ . A firm in country  $i$  faces a fixed cost of exporting to country  $j$ ,  $j \neq i$ , of  $f_e$  where  $f_e > f_d = f$  and an iceberg transportation cost of  $\tau_i^j - 1 = \tau - 1 \geq 0$ ,  $j \neq i$ .

- a) Define an equilibrium for this economy.
- b) Explain the different cases for cutoff productivity levels in the equilibrium of the economy with trade. Suppose that the two countries are symmetric in the sense that  $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$  and  $\mu_1 = \mu_2 = \mu$ . Suppose too that  $\mu$  is large enough so that not all firms can earn nonnegative profits in equilibrium. Characterize the equilibrium of this economy in the case where there are two cutoff levels,  $\bar{x}_d$  and  $\bar{x}_e$ , where  $\bar{x}_e > \bar{x}_d > 1$ . Firms with  $x(z) \geq \bar{x}_e$  produce for both the domestic and the export market; firms with  $\bar{x}_e > x(z) \geq \bar{x}_d$  produce only for the domestic market, and firms with  $\bar{x}_d > x(z)$  do not produce.
- c) Suppose that  $\mu = 0.5$ ,  $\rho = 0.5$ ,  $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$ ,  $\mu_1 = \mu_2 = \mu = 10$ ,  $\gamma = 4$ ,  $f_d = 2$ ,  $f_e = 3$ , and  $\tau_2^1 = \tau_1^2 = \tau = 1.2$ . Calculate the equilibrium of this economy.
- d) Suppose now that a free trade agreement sets  $\tau_2^1 = \tau_1^2 = \tau = 1$ . Recalculate the equilibrium in part c.
- e) Suppose now that a different reform sets  $f_e = 2$ . Again recalculate the equilibrium in part b. Contrast the results with those in part d. What sort of reform can lower  $f_e$ ?