The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$.

$$a(x,t) = e^{x-2z(t)}$$

$$\dot{a}(x,t) = -2\dot{z}(t)e^{x-2z(t)}$$

$$\frac{\dot{a}(x,t)}{a(x,t)} = \frac{-2\dot{z}(t)e^{x-2z(t)}}{e^{x-2z(t)}} = -2\dot{z}(t)$$

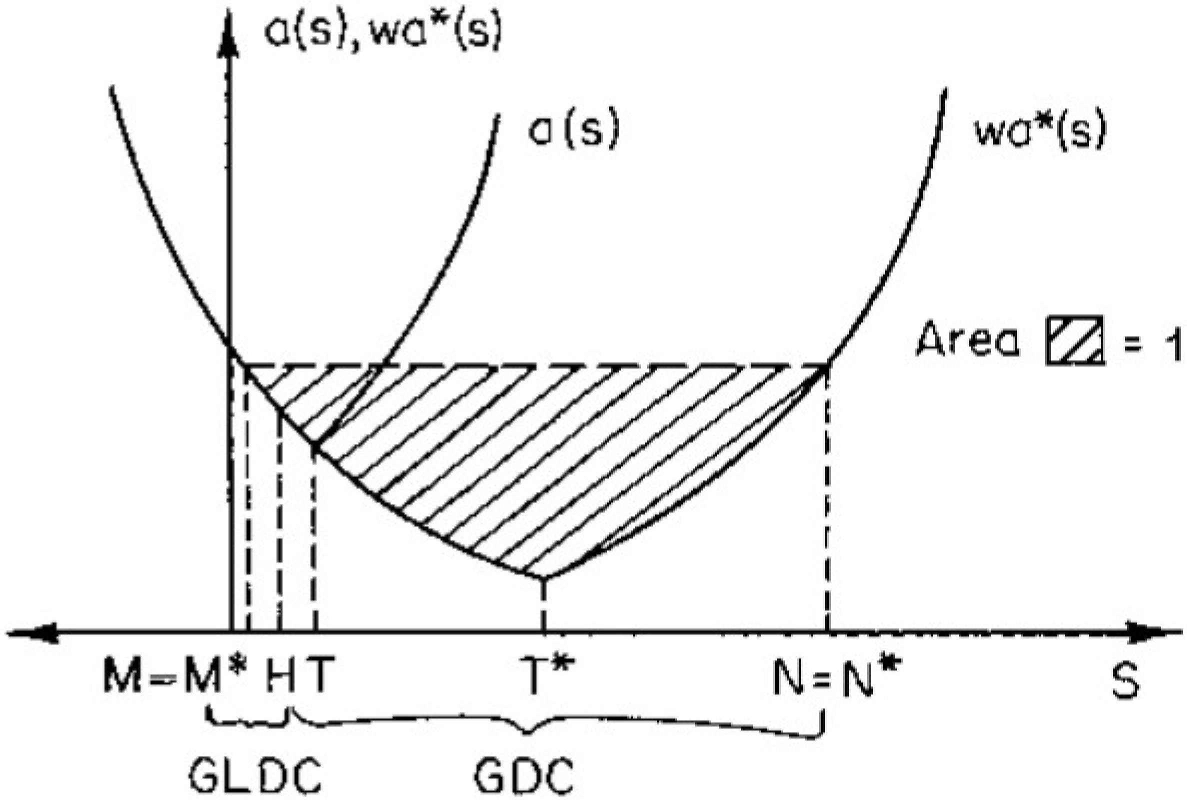
$$\dot{z}(t) = -\frac{1}{2}\frac{\dot{a}(x,t)}{a(x,t)} = \frac{1}{2}\int_0^{\infty} b(v,t)\ell(v,t)dv$$

$$\int_0^{\infty} b(v,t)\ell(v,t)dv = \int_{z(t)}^{\infty} b(v,t)\ell(v,t)dv = \frac{b}{2}\int_{z(t)}^{\infty} \ell(v,t)dv.$$

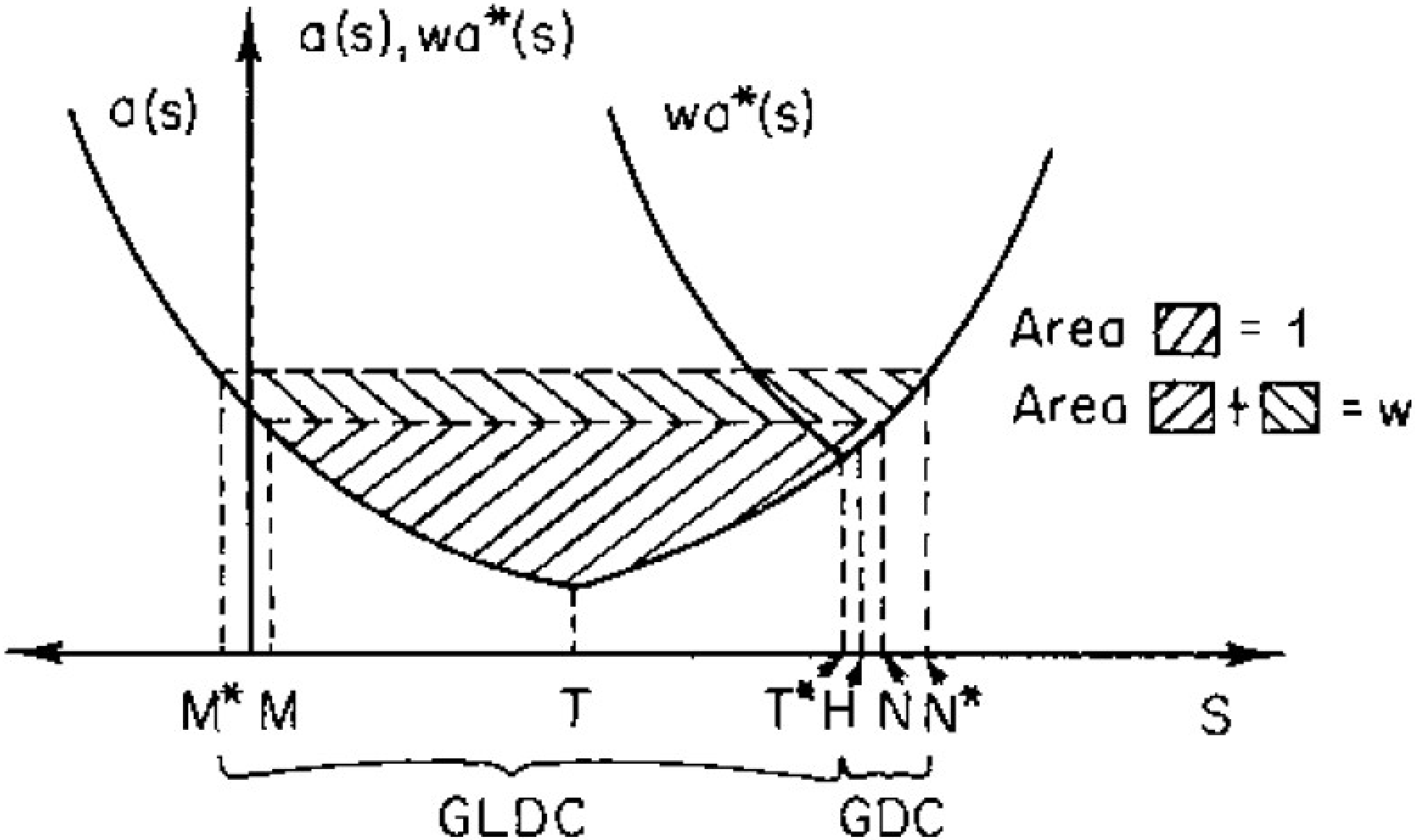
In a closed economy with symmetric unit labor requirements $a(x,t) = e^{-x}$ for all $x < z(t)$ and that $a(x,t) = e^{x-2z(t)}$ for all $x \geq z(t)$,

$$\dot{z}(t) = \frac{b}{2}\int_{z(t)}^{\infty} \ell(v,t)dv = \frac{b}{2}\left(\frac{\overline{\ell}}{2}\right) = \frac{b\overline{\ell}}{4}.$$

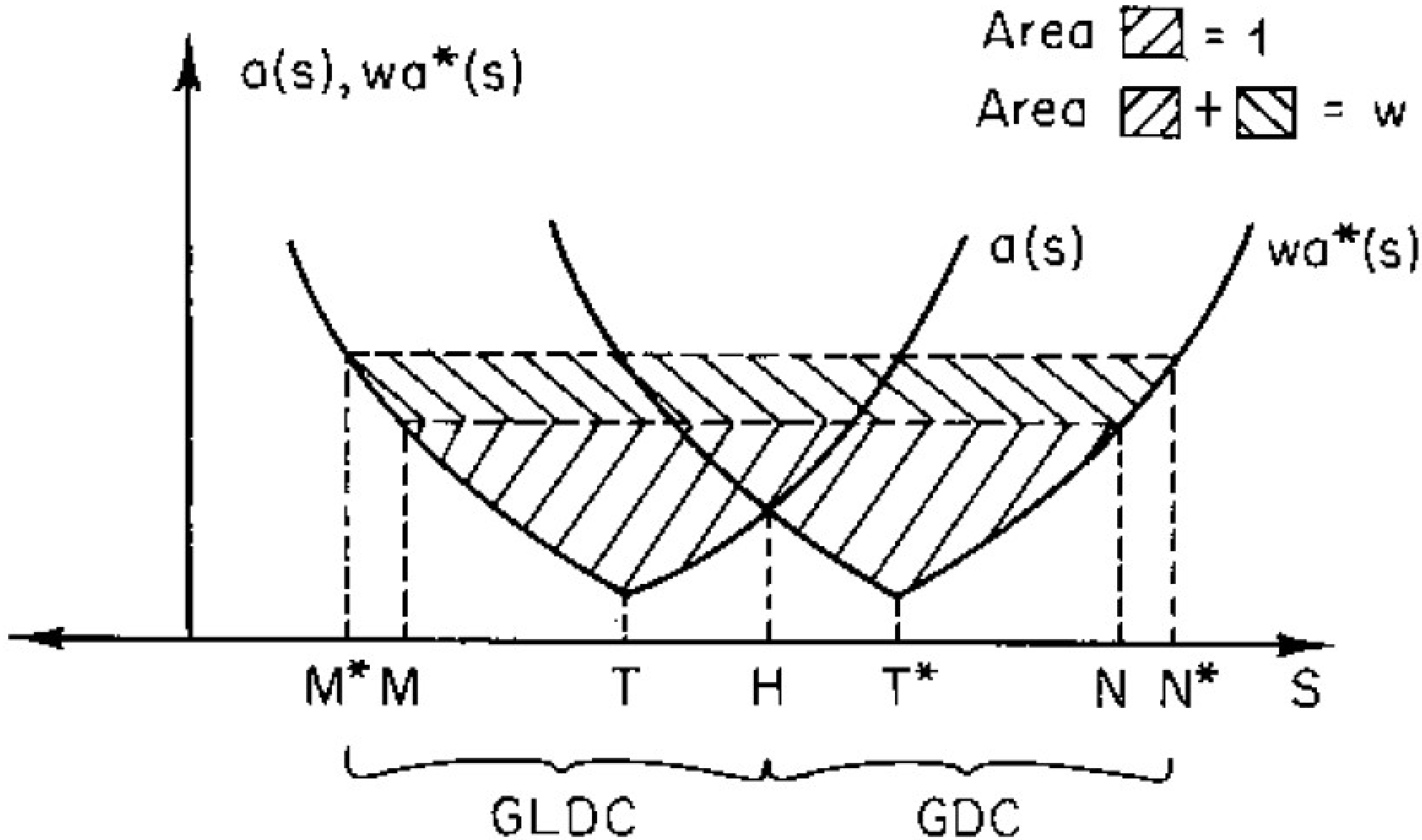
Equilibrium A



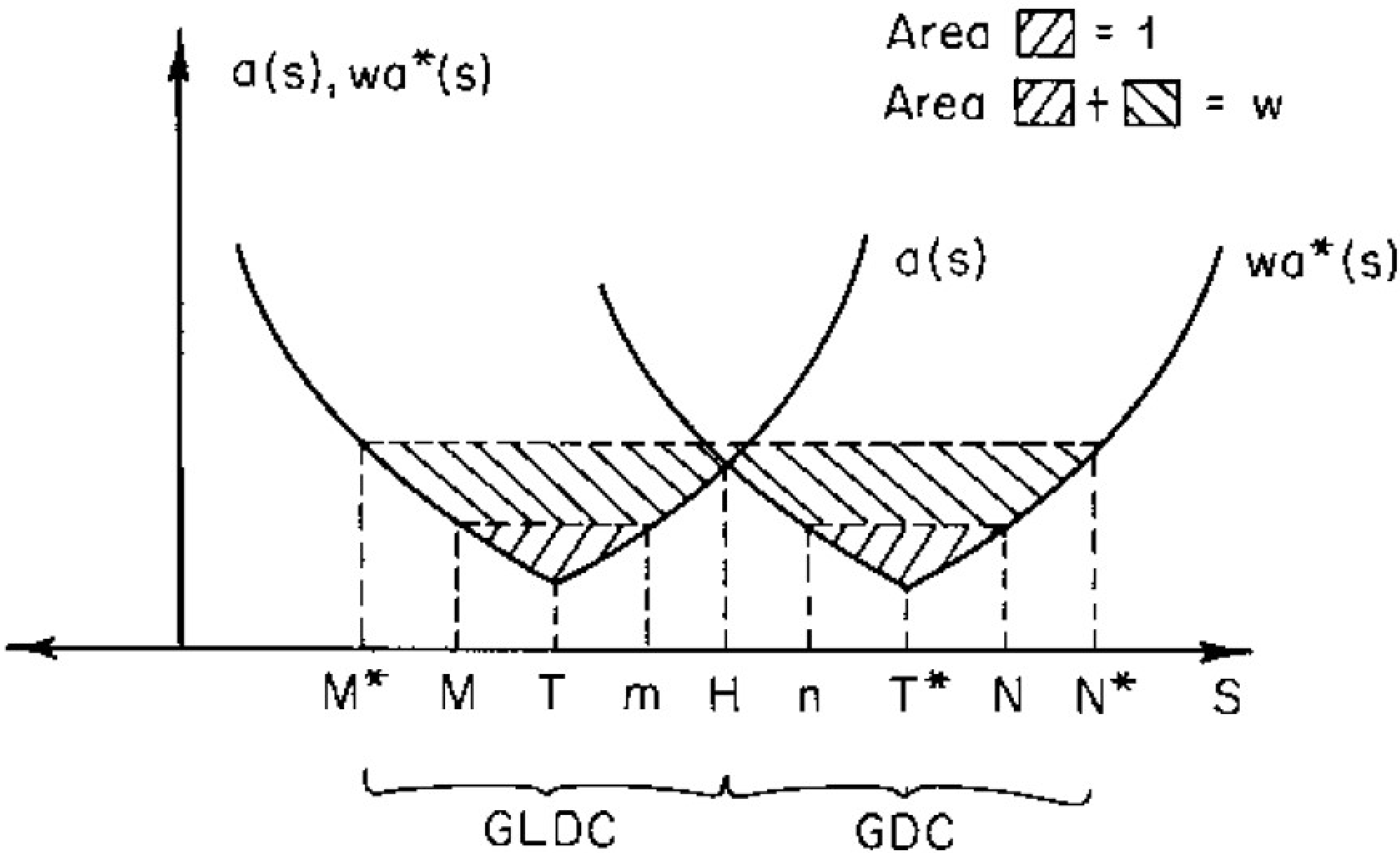
Equilibrium B



Equilibrium C



Equilibrium D



Equilibrium E