Suppose that $c_t, x_t, k_t$ solve the one-sector growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t. } c_t + x_t = f(k_t, 1)$$
$$k_{t+1} = (1-\delta)k_t + x_t$$
$$k_0 = \overline{k}_0$$
$$c_t, x_t, k_t \geq 0.$$

Then $c_{1t}, c_{2t}, x_{1t}, x_{1t}, k_t$ solve the two-sector growth problem

$$\max \sum_{t=0}^{\infty} \beta^t \log f(c_{1t}, c_{2t})$$
$$\text{s.t. } c_{1t} + x_{1t} = k_t$$
$$c_{2t} + x_{2t} = 1$$
$$k_{t+1} = (1-\delta)k_t + f(x_{1t}, x_{2t})$$
$$k_0 = \overline{k}_0$$
$$c_{1t}, c_{2t}, x_{1t}, x_{1t}, k_t \geq 0$$

where

$$c_{1t} = \frac{k_t}{f(k_t, 1)} c_t, \; c_{2t} = \frac{1}{f(k_t, 1)} c_t$$

$$x_{1t} = \frac{k_t}{f(k_t, 1)} x_t, \; x_{2t} = \frac{1}{f(k_t, 1)} x_t.$$

Suppose $c_{1t}, c_{2t}, x_{1t}, x_{1t}, k_t$ solve the two-sector growth problem

$$\begin{gathered}\max \sum_{t=0}^{\infty} \beta^t \log f(c_{1t}, c_{2t}) \\ \text{s.t. } c_{1t} + x_{1t} = k_t \\ c_{2t} + x_{2t} = 1 \\ k_{t+1} = (1-\delta)k_t + f(x_{1t}, x_{2t}) \\ k_0 = \bar{k}_0 \\ c_{1t}, c_{2t}, x_{1t}, x_{1t}, k_t \geq 0.\end{gathered}$$

Then $c_t, x_t, k_t$ solve the one-sector growth problem

$$\begin{gathered}\max \sum_{t=0}^{\infty} \beta^t \log c_t \\ \text{s.t. } c_t + x_t = f(k_t, 1) \\ k_{t+1} = (1-\delta)k_t + x_t \\ k_0 = \bar{k}_0 \\ c_t, x_t, k_t \geq 0\end{gathered}$$

where

$$c_t = f(c_{1t}, c_{2t})$$
$$x_t = f(x_{1t}, x_{2t}).$$