

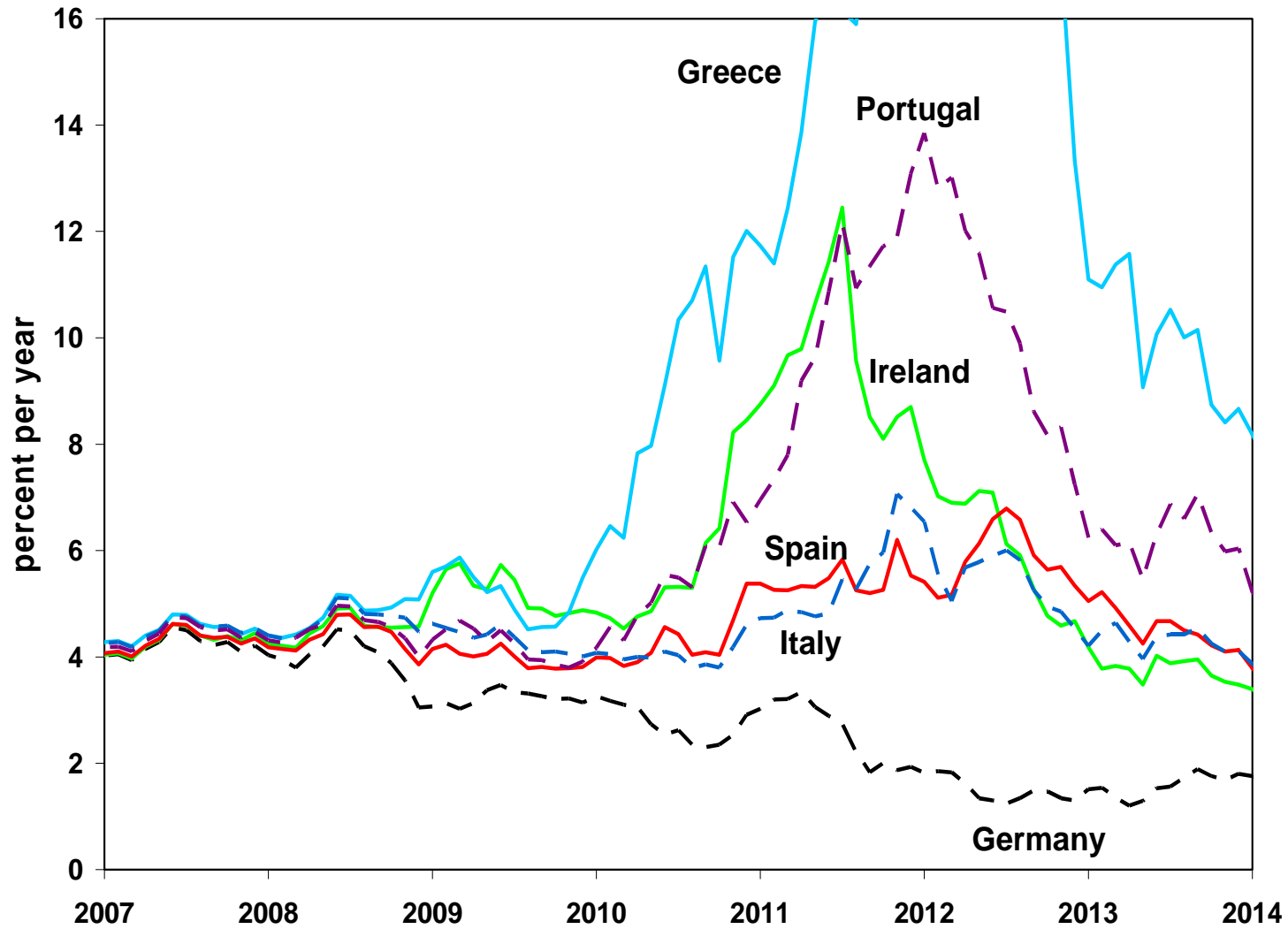
Gambling for Redemption and Self-Fulfilling Debt Crises

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Jumps in spreads on yields on bonds of PIIGS governments (over yields on German bonds)



Yields on 10-year government bonds

Theory of self-fulfilling debt crises (Cole-Kehoe)

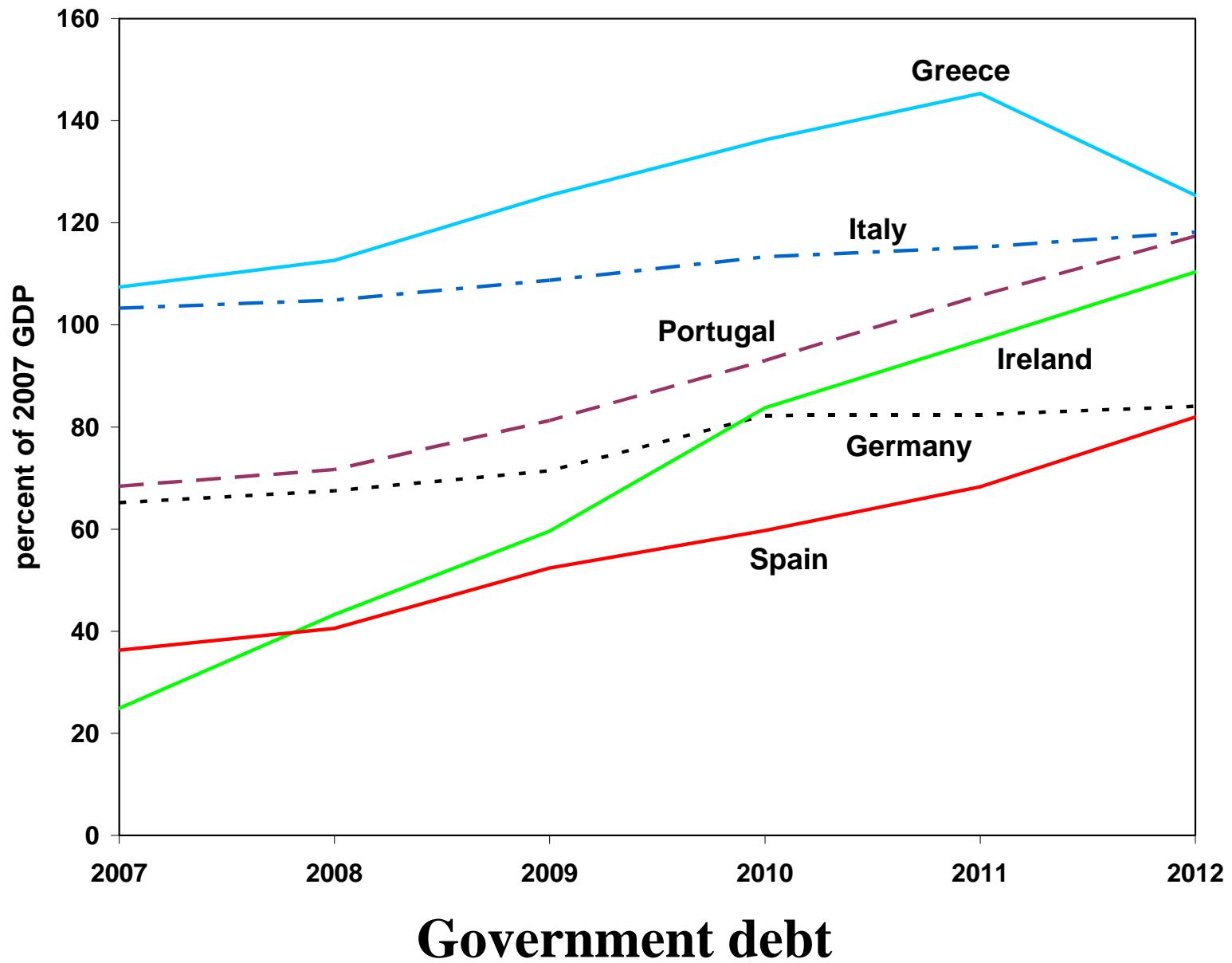
Spreads reflect probabilities of crises

For low enough levels of debt, no crisis is possible

For high enough levels of debt, default

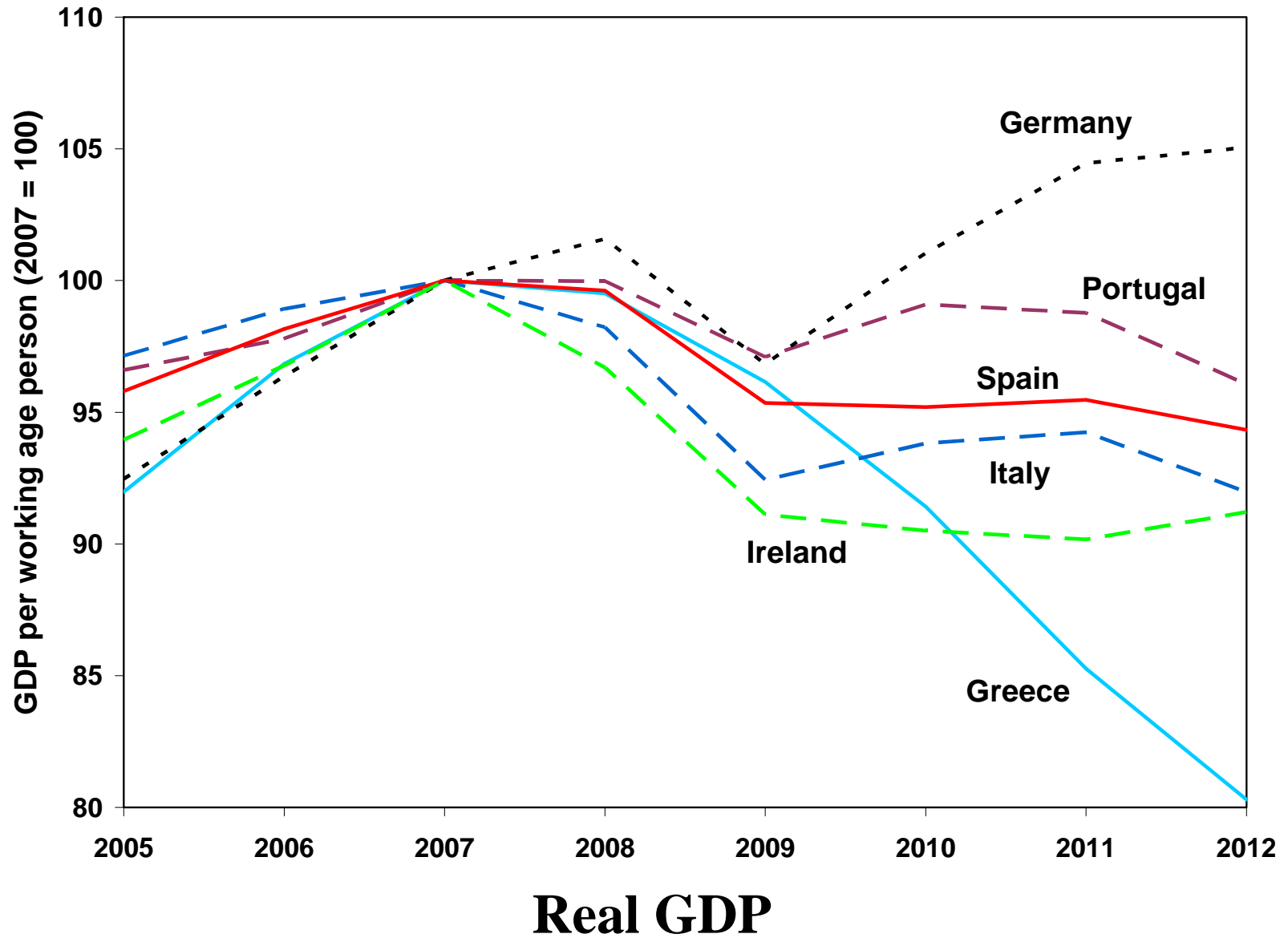
For intermediate levels of debt (crisis zone) optimal policy is to run down debt

...but PIIGS ran up debt.

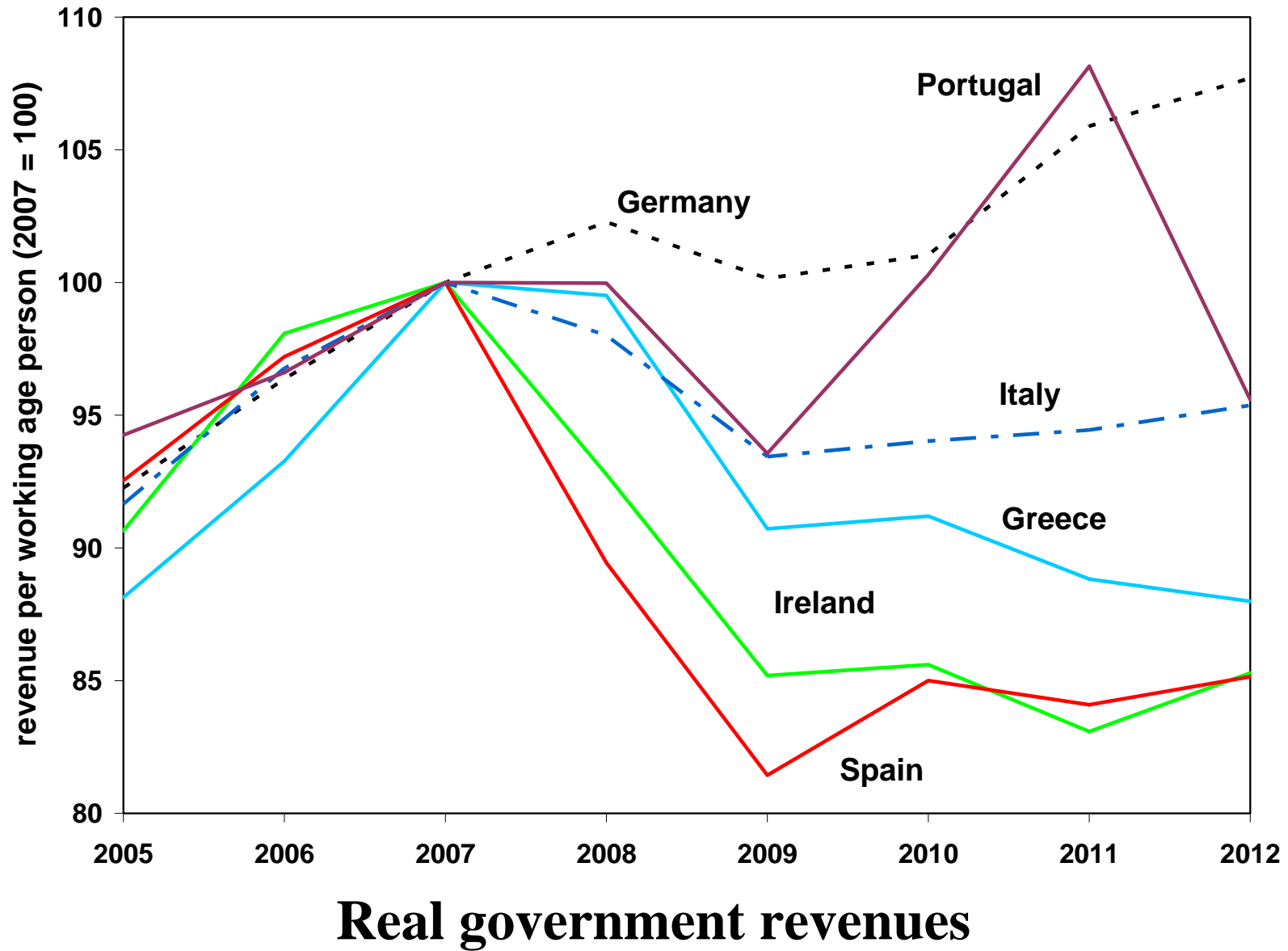


What is missing in Cole-Kehoe?

Severe recession in PIIGS, still ongoing



...government revenues also depressed.



This paper

Extends Cole-Kehoe to stochastic output.

Standard consumption smoothing argument in an incomplete markets model (as in Aiyagari, Huggett, Chatterjee et al, Arellano, Aguiar-Gopinath) can imply running up debt.

When running up debt is optimal, we call it “gambling for redemption.”

Use model to evaluate impact of Troika (EU-ECB-IMF) policy, compare with the Clinton (1995) bailout of Mexico.

Main mechanism of our theory

Model characterizes two forces in opposite directions:

1. Run down debt (as in Cole-Kehoe)
2. Run up debt (consumption smoothing)

Which one dominates depends on parameter values and Troika policies.

Run down debt

In crisis zone run down debt if:

- Interest rates are high.
- Costs of default are high.

Run up debt

In recession run up debt if:

- Interest rates are low.
- Costs of default are low.
- Recession is severe.
- Probability of recovery is high.

General model

Agents:

Government

International bankers, continuum $[0,1]$

Consumers, passive (no private capital)

Third party in policy experiments

General model

State of the economy: $s = (B, a, z_{-1}, \zeta)$

B : government debt

a : private sector, $a = 1$ normal, $a = 0$ recession

z_{-1} : previous default $z_{-1} = 1$ no, $z_{-1} = 0$ yes

ζ : realization of sunspot

GDP: $y(a, z) = A^{1-a} Z^{1-z} \bar{y}$

$1 > A > 0, 1 > Z > 0$ parameters.

Model with no recovery (Cole-Kehoe)

State of the economy: $s = (B, 1, z_{-1}, \zeta)$

B : government debt

z_{-1} : previous default $z_{-1} = 1$ no, $z_{-1} = 0$ yes

ζ : realization of sunspot

GDP: $y(1, z) = Z^{1-z} \bar{y}$

$1 > Z > 0$ parameter.

Model without crises

State of the economy: $s = (B, a, 1, \cdot)$

B : government debt

a : private sector, $a = 1$ normal, $a = 0$ recession

GDP: $y(a, 1) = A^{1-a} \bar{y}$

$1 > A > 0$ parameter.

General model

Before period 0, $a = 1$, $z = 1$.

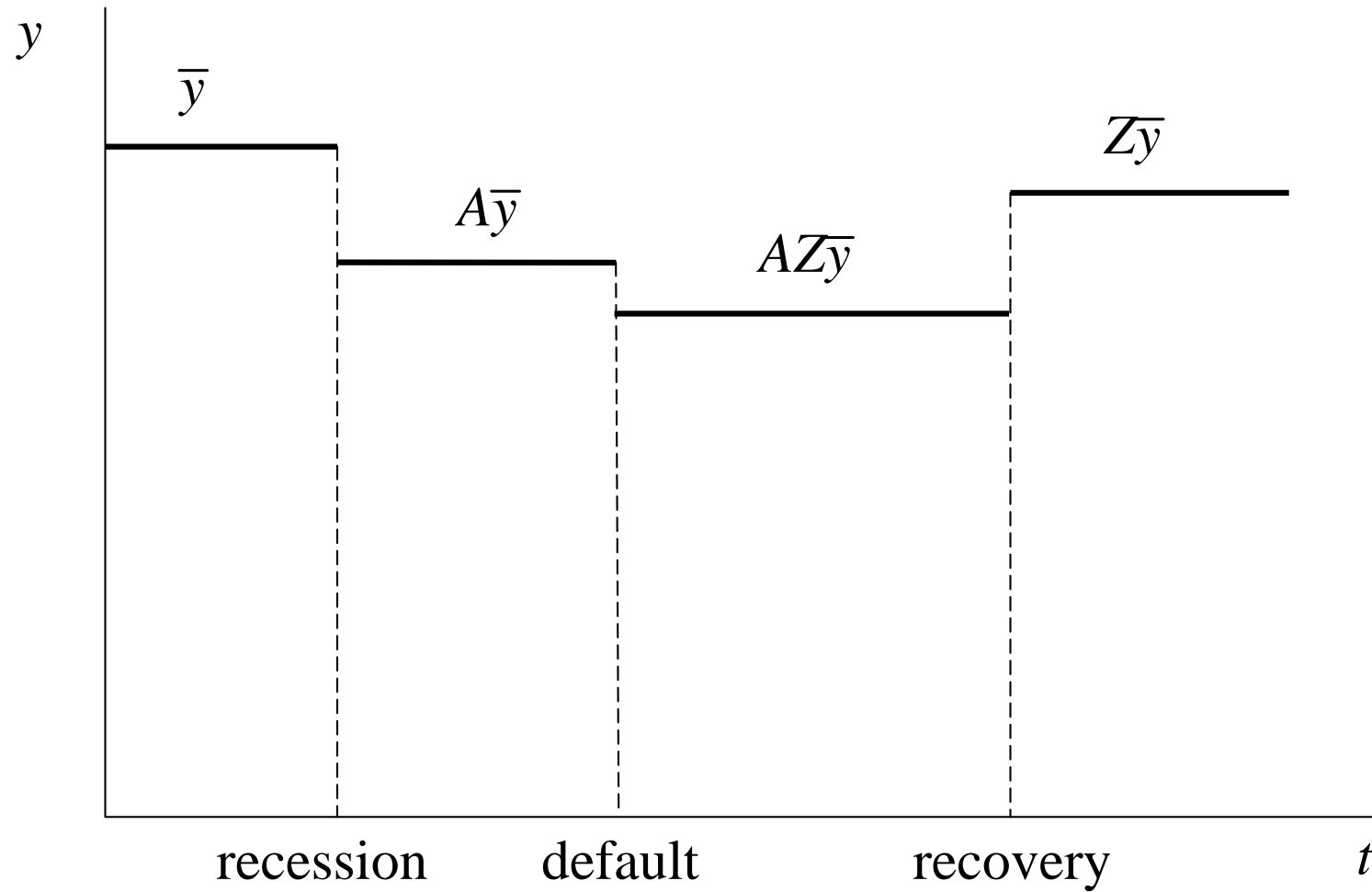
In $t = 0$, $a_0 = 0$ unexpectedly, GDP drops from \bar{y} to $A\bar{y} < \bar{y}$.

In $t = 1, 2, \dots$, a_t becomes 1 with probability p .

$1 - A$ is severity of recession. Once $a_t = 1$, it is 1 forever.

$1 - Z$ is default penalty. Once $z_t = 0$, it is 0 forever.

A possible time path for GDP



Sunspot

Coordination device for international bankers' expectations.

$$\zeta_t \sim U[0,1]$$

B_t outside crisis zone: if ζ_t is irrelevant

B_t inside crisis zone: if $\zeta_t \geq 1 - \pi$ bankers expect a crisis (π arbitrary)

Government's problem

Depends on timing, equilibrium conditions, to be described.

Government tax revenue is $\theta y(a, z)$, tax rate θ is fixed.

Choose c, g, B', z to solve:

$$V(s) = \max u(c, g) + \beta EV(s')$$

$$\text{s.t. } c = (1 - \theta)y(a, z)$$

$$g + zB = \theta y(a, z) + q(B', s)B'$$

$$z = 0 \text{ if } z_{-1} = 0.$$

International bankers

Continuum $[0,1]$ of risk-neutral agents with deep pockets

First order condition and perfect foresight condition:

$$q(B', s) = \beta \times Ez(B', s', q(B', s')).$$

bond price = risk-free price \times probability of repayment

Timing

a_t, ζ_t realized, $s_t = (B_t, a_t, z_{t-1}, \zeta_t)$



government offers B_{t+1}



bankers choose to buy B_{t+1} or not, q_t determined



government chooses z_t , which determines y_t, c_t , and g_t

Notes

Time-consistency problem: when offering B_{t+1} for sale, government cannot commit to repay B_t

Perfect foresight: bankers do not lend if they know the government will default.

Bond price depends on B_{t+1} ; crisis depends on B_t .

Recursive equilibrium

Value function for government $V(s)$ and policy functions

$B'(s)$ and $z(B', s, q)$ and $g(B', s, q)$,

and a bond price function $q(B', s)$

such that:

1. Beginning of period: Given $z(B', s, q)$, $g(B', s, q)$, $q(B', s)$ government chooses B' to solve:

$$V(s) = \max u(c, g) + \beta EV(s')$$

$$\text{s.t. } c = (1 - \theta)y(a, z(B', s, q(B', s)))$$

$$g(B', s, q(B', s)) + z(B', s, q(B', s))B = \theta y(a, z) + q(B', s)B'$$

2. Bond market equilibrium:

$$q(B'(s), s) = \beta Ez(B'(s'), s', q(B'(s'), s')).$$

3. End of period: Given $V(B', a', z, \zeta')$ and $B' = B'(s)$ and $q = q(B'(s), s)$, government chooses z and g to solve:

$$\max u(c, g) + \beta EV(B', a', z, \zeta')$$

$$\text{s.t. } c = (1 - \theta)y(a, z)$$

$$g + zB = \theta y(a, z) + qB'$$

$$z = 0 \text{ or } z = 1$$

$$z = 0 \text{ if } z_{-1} = 0.$$

Characterization of government's optimal debt policy

Four cutoff levels of debt: $\bar{b}(a)$, $\bar{B}(a)$, $a = 0, 1$:

- If $B \leq \bar{b}(a)$, repay
- If $\bar{b}(a) < B \leq \bar{B}(a)$, default if $\zeta > 1 - \pi$
- If $B > \bar{B}(a)$, default

We are interested in parameter values for which

$$\bar{b}(0) < \bar{b}(1), \bar{b}(0) < \bar{B}(0), \bar{b}(1) < \bar{B}(1), \text{ and } \bar{B}(0) < \bar{B}(1).$$

$$\bar{b}(1), \bar{B}(0)?$$

Most interesting case:

$$\bar{b}(0) < \bar{b}(1) < \bar{B}(0) < \bar{B}(1).$$

Other cases (catastrophic recessions):

$$\bar{b}(0) < \bar{B}(0) < \bar{b}(1) < \bar{B}(1)$$

$$\bar{b}(0) < \bar{b}(1) = \bar{B}(0) < \bar{B}(1).$$

Characterization of equilibrium prices

After default bankers do not lend: $q(B', (B, a, 0, \zeta)) = 0$.

During a crisis bankers do not lend: If $B > \bar{b}(a)$ and $\zeta \geq 1 - \pi$, $q(B', (B, a, 1, \zeta)) = 0$

Otherwise, q depends only on B' .

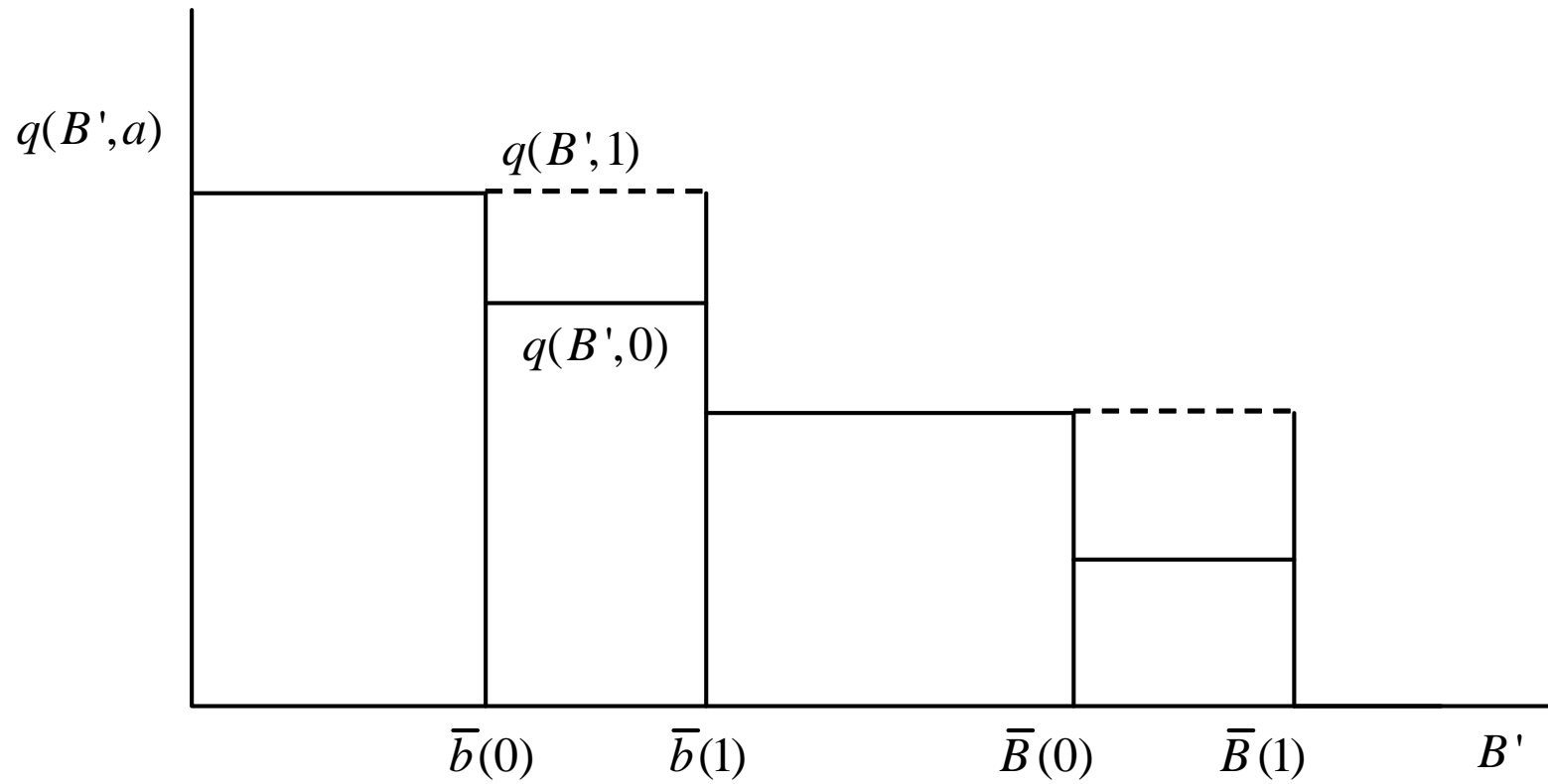
In normal times (as in Cole-Kehoe):

$$q(B', (B, 1, 1, \zeta)) = \begin{cases} \beta & \text{if } B' \leq \bar{b}(1) \\ \beta(1 - \pi) & \text{if } \bar{b}(1) < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

In a recession (for the most interesting case):

$$q(B', (B, 0, 1, \zeta)) = \begin{cases} \beta & \text{if } B' \leq \bar{b}(0) \\ \beta(p + (1 - p)(1 - \pi)) & \text{if } \bar{b}(0) < B' \leq \bar{b}(1) \\ \beta(1 - \pi) & \text{if } \bar{b}(1) < B' \leq \bar{B}(0) \\ \beta p(1 - \pi) & \text{if } \bar{B}(0) < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

Bond prices as function of debt and a



Characterization of optimal debt policy

Two special cases with analytical results:

- $p = 0$ (no gambling for redemption)
- $\pi = 0$ (no crises)

General model with numerical experiments:

- $V(s)$ has kinks and $B'(s)$ is discontinuous because of discontinuity of $q(B', s)$.
- $V(s)$ is discontinuous because government cannot commit not to default.

Self-fulfilling liquidity crises, no gambling

$p = 0$, also limiting case where $a = 0$ and $p = 0$: Replace \bar{y} with $A\bar{y}$.

Cole-Kehoe without private capital.

Start by assuming that $\pi = 0$.

When $s = (B, a, z_{-1}, \zeta) = (B, 1, 1, \zeta)$,

$$V(B, 1, 1, \zeta) = \frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B)}{1-\beta}.$$

When default has occurred, $s = (B, a, z_{-1}, \zeta) = (B, 1, 0, \zeta)$,

$$V(B, 1, 0, \zeta) = \frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}.$$

$\bar{b}(1)$:

Utility of repaying even if bankers do not lend:

$$u((1-\theta)\bar{y}, \theta\bar{y} - B) + \frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}$$

Utility of defaulting if bankers do not lend:

$$\frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}.$$

$\bar{b}(1)$ is determined by

$$u((1-\theta)\bar{y}, \theta\bar{y} - \bar{b}(1)) + \frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta} = \frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$

Determination of $\bar{B}(1)$ requires optimal policy.

If $B_0 > \bar{b}(1)$ and the government decides to reduce B to $\bar{b}(1)$ in T periods, $T = 1, 2, \dots, \infty$. First-order conditions imply

$$g_t = g^T(B_0).$$

$$g^T(B_0) = \theta \bar{y} - \frac{1 - \beta(1 - \pi)}{1 - (\beta(1 - \pi))^T} \left(B_0 - (\beta(1 - \pi))^{T-1} \beta \bar{b}(1) \right).$$

$$g^\infty(B_0) = \lim_{T \rightarrow \infty} g^T(B_0) = \theta \bar{y} - (1 - \beta(1 - \pi)) B_0.$$

Compute $V^T(B_0)$:

$$\begin{aligned}
 V^T(B_0) &= \frac{1 - (\beta(1 - \pi))^T}{1 + \beta(1 - \pi)} u((1 - \theta)\bar{y}, g^T(B_0)) \\
 &+ \frac{1 - (\beta(1 - \pi))^{T-1}}{1 + \beta(1 - \pi)} \frac{\beta\pi u((1 - \theta)Z\bar{y}, \theta Z\bar{y})}{1 - \beta} \\
 &+ (\beta(1 - \pi))^{T-2} \frac{\beta u((1 - \theta)\bar{y}, \theta\bar{y})}{1 - \beta}
 \end{aligned}$$

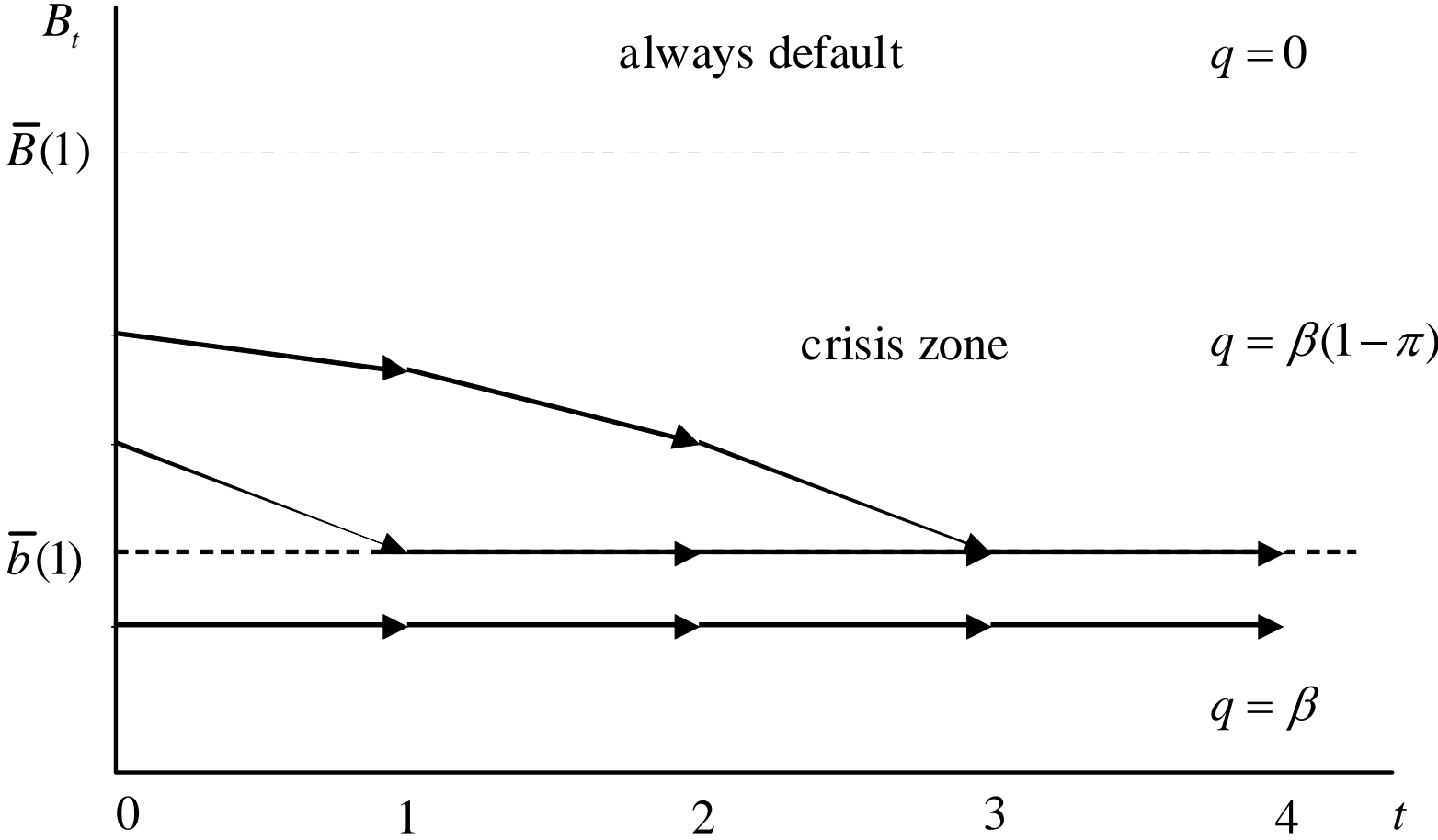
To find $\bar{B}(1)$, we solve

$$\begin{aligned} & \max \left[V^1(\bar{B}(1)), V^2(\bar{B}(1)), \dots, V^\infty(\bar{B}(1)) \right] \\ & = u((1-\theta)Z\bar{y}, \theta Z\bar{y} + \beta(1-\pi)\bar{B}(1)) + \frac{\beta u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} \end{aligned}$$

$V(B, 1, 1, \zeta) =$

$$\left\{ \begin{array}{ll} \frac{u((1-\theta)\bar{y}, Z\bar{y})}{1-\beta} & \text{if } B \leq \bar{b}(1) \\ \max \left[V^1(B), V^2(B), \dots, V^\infty(B) \right] & \text{if } \bar{b}(1) < B \leq \bar{B}(1), \zeta \leq 1-\pi \\ \frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} & \text{if } \bar{b}(1) < B \leq \bar{B}(1), 1-\pi < \zeta \\ \frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} & \text{if } \bar{B}(1) < B \end{array} \right.$$

Equilibrium with self-fulfilling crises, no crises

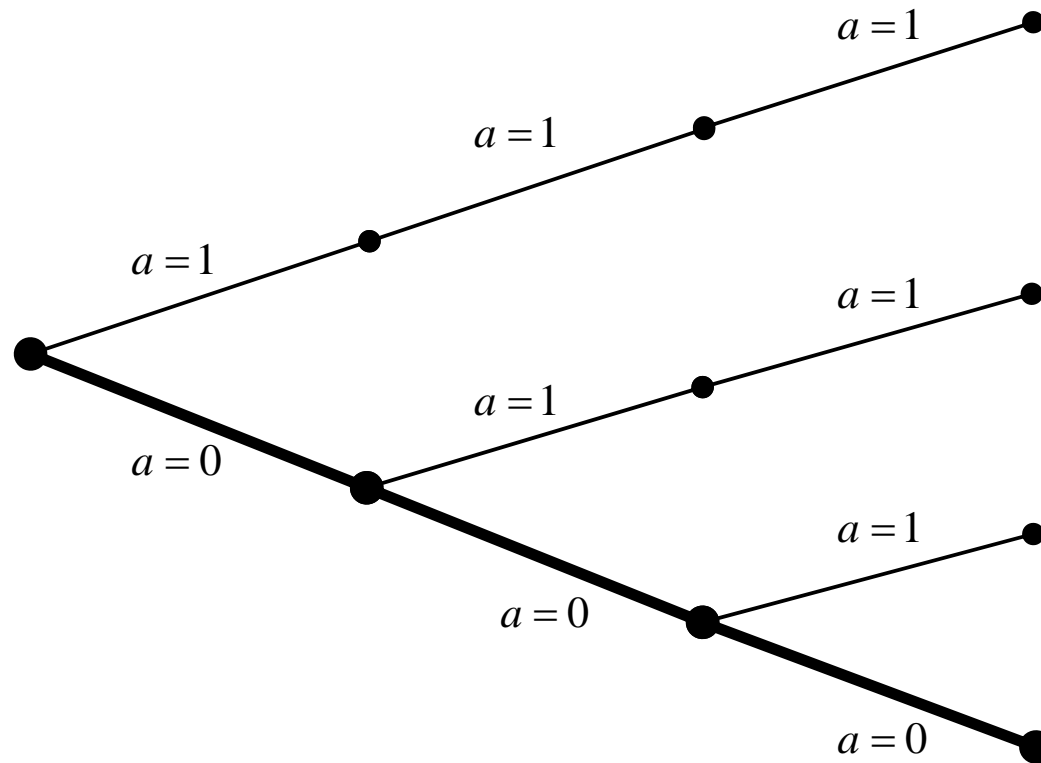


Consumption smoothing without self-fulfilling crises

$a = 0$ and $\pi = 0$.

Private sector is in a recession and faces the possibility p of recovering in every period.

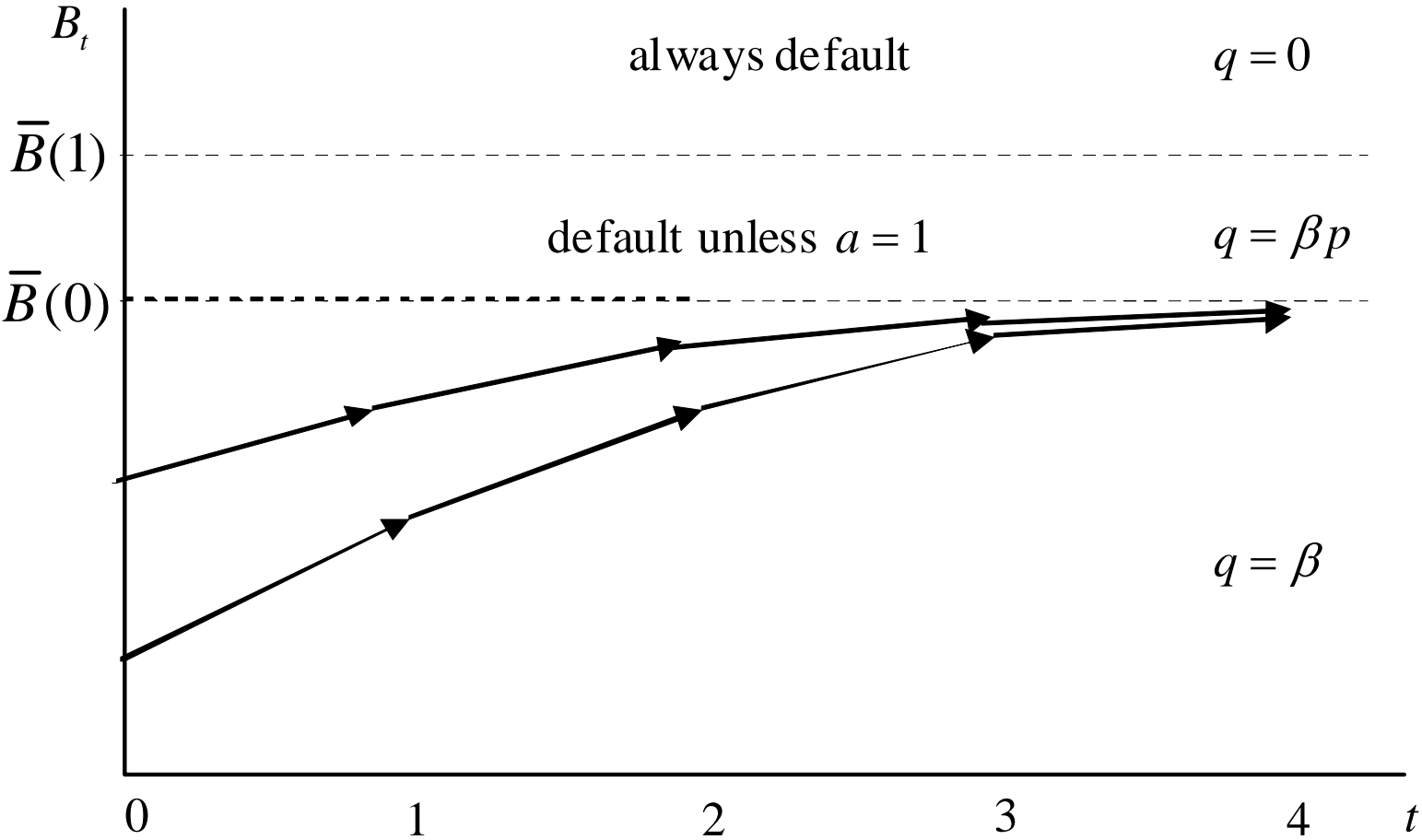
Uncertainty tree with recession path highlighted



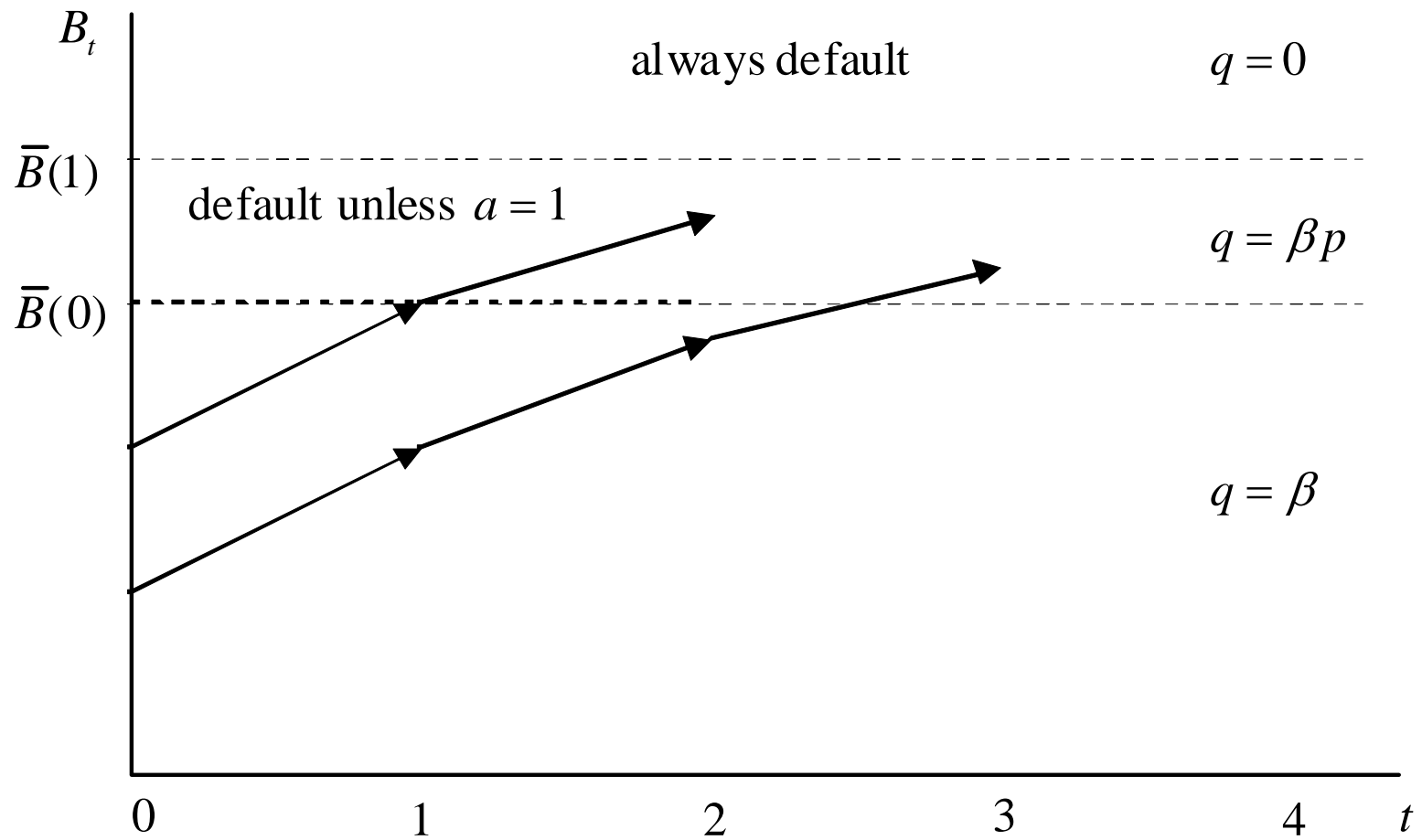
Two cases:

- Government chooses to never violate the constraint $B \leq \bar{B}(0)$ and debt converges to $\bar{B}(0)$ if $a = 0$ sufficiently long.
- Government chooses to default at T if $a = 0$ sufficiently long.

Equilibrium with no default



Equilibrium with eventual default



Quantitative analysis in a numerical model

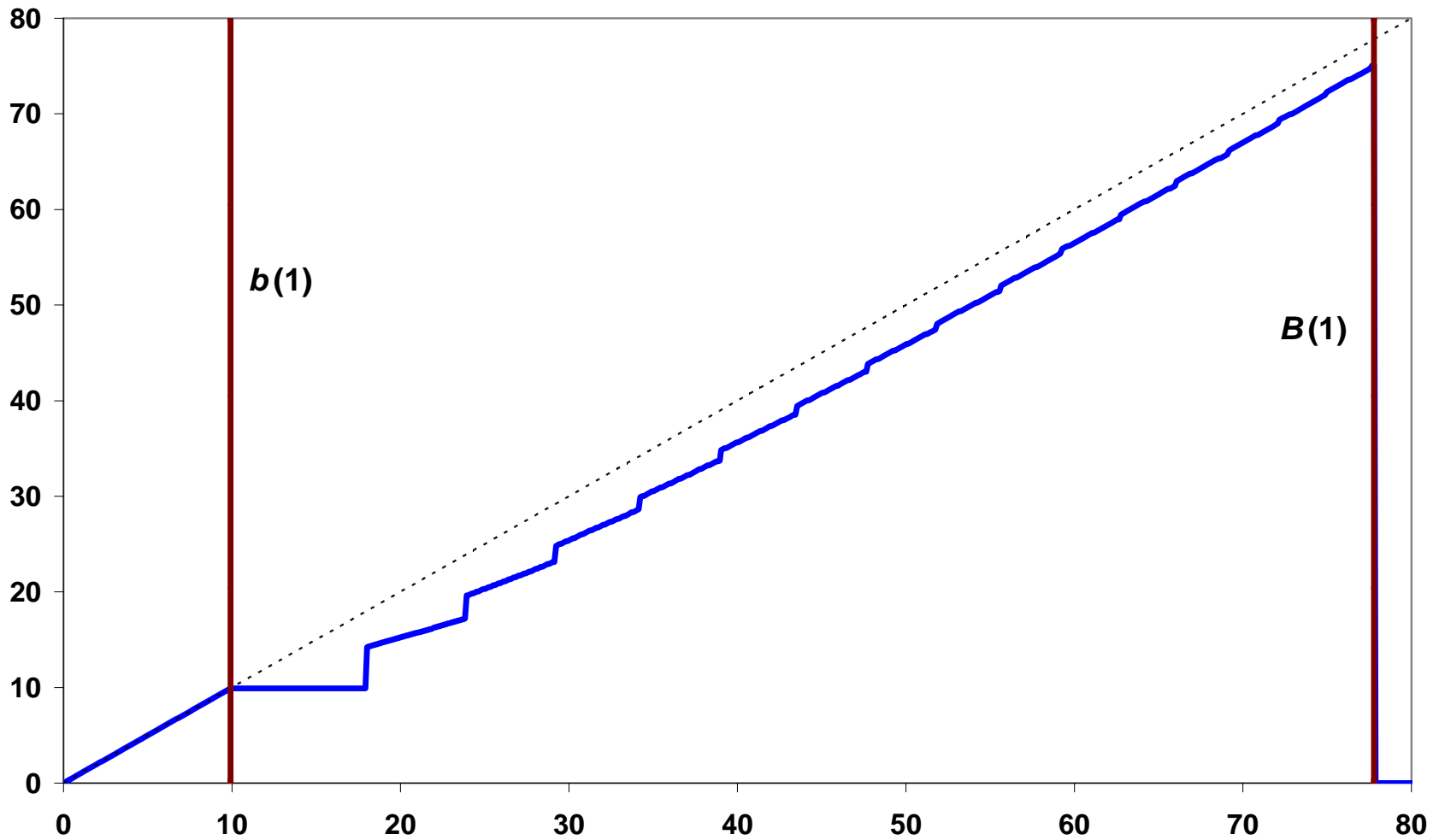
$$u(c, g) = \log(c) + \gamma \log(g - \bar{g})$$

Parameter	Value
A	0.90
Z	0.95
p	0.20
β	0.98
π	0.03
γ	0.25
θ	0.40
\bar{y}	100
\bar{g}	25

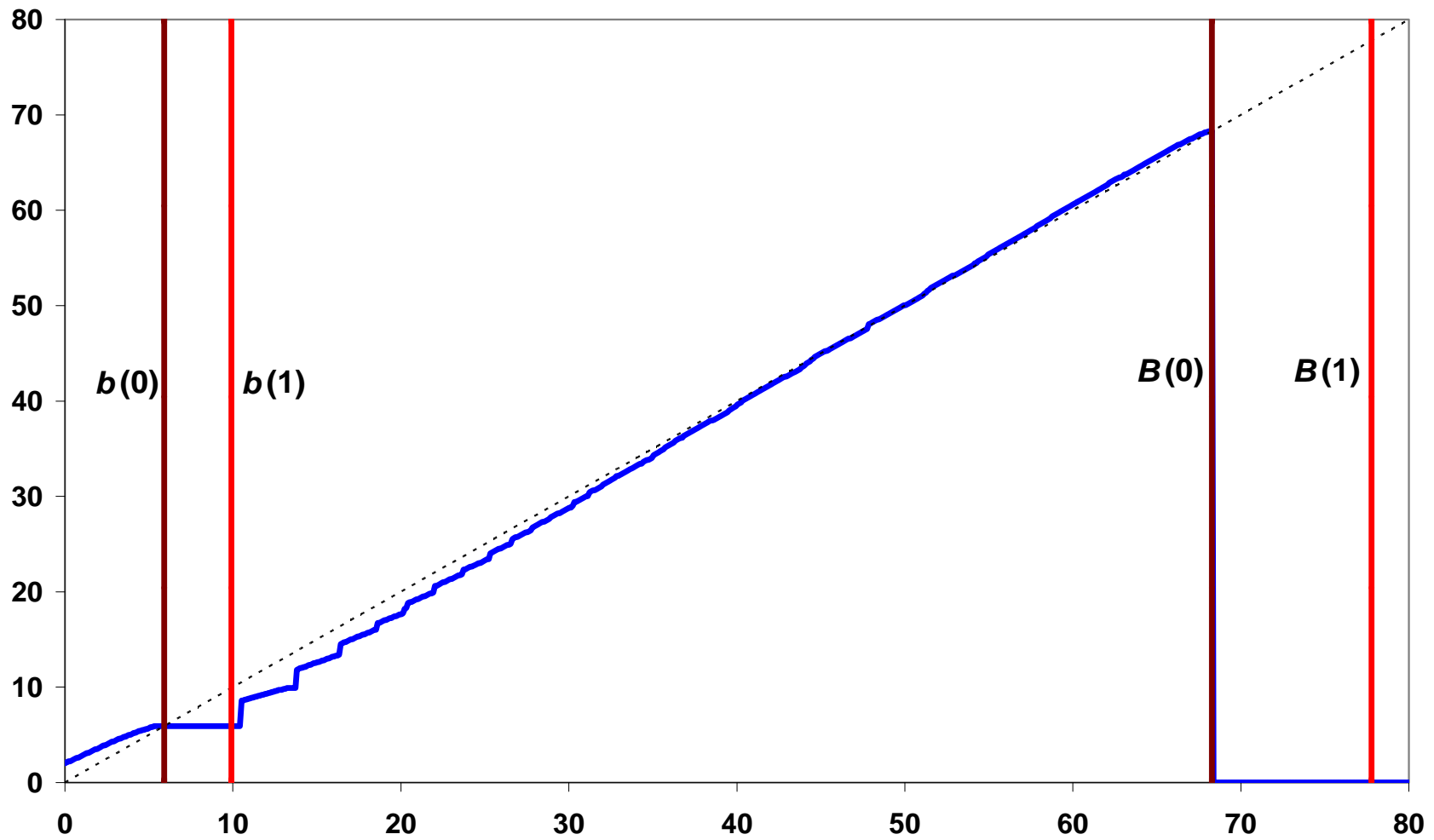
We work first with one-year bonds.

We then extend model to multi-year bonds.

Results: The benchmark economy in normal times



Then, a recession hits...



Maturity of debt in 2011

	Weighted average years until maturity
Germany	6.4
Greece	15.4
Ireland	4.5
Italy	6.5
Portugal	5.1
Spain	5.9

Think of results in terms of debt needing refinancing every year — say one-sixth, as in Spain.

The extended model

The government's problem is to choose c, g, B', z to solve

$$V(s) = \max u(c, g) + \beta EV(s')$$

$$\text{s.t. } c = (1 - \theta)y(a, z)$$

$$g + z\delta B = \theta y(a, z) + q(B', s)(B' - (1 - \delta)B)$$

$$z = 0 \text{ if } z_{-1} = 0.$$

Here $\delta \in [0, 1]$ is the fraction of the stock of debt due every period.

Debt is memoryless, as in Hatchondo-Martinez, Chatterjee-Eyigungor.

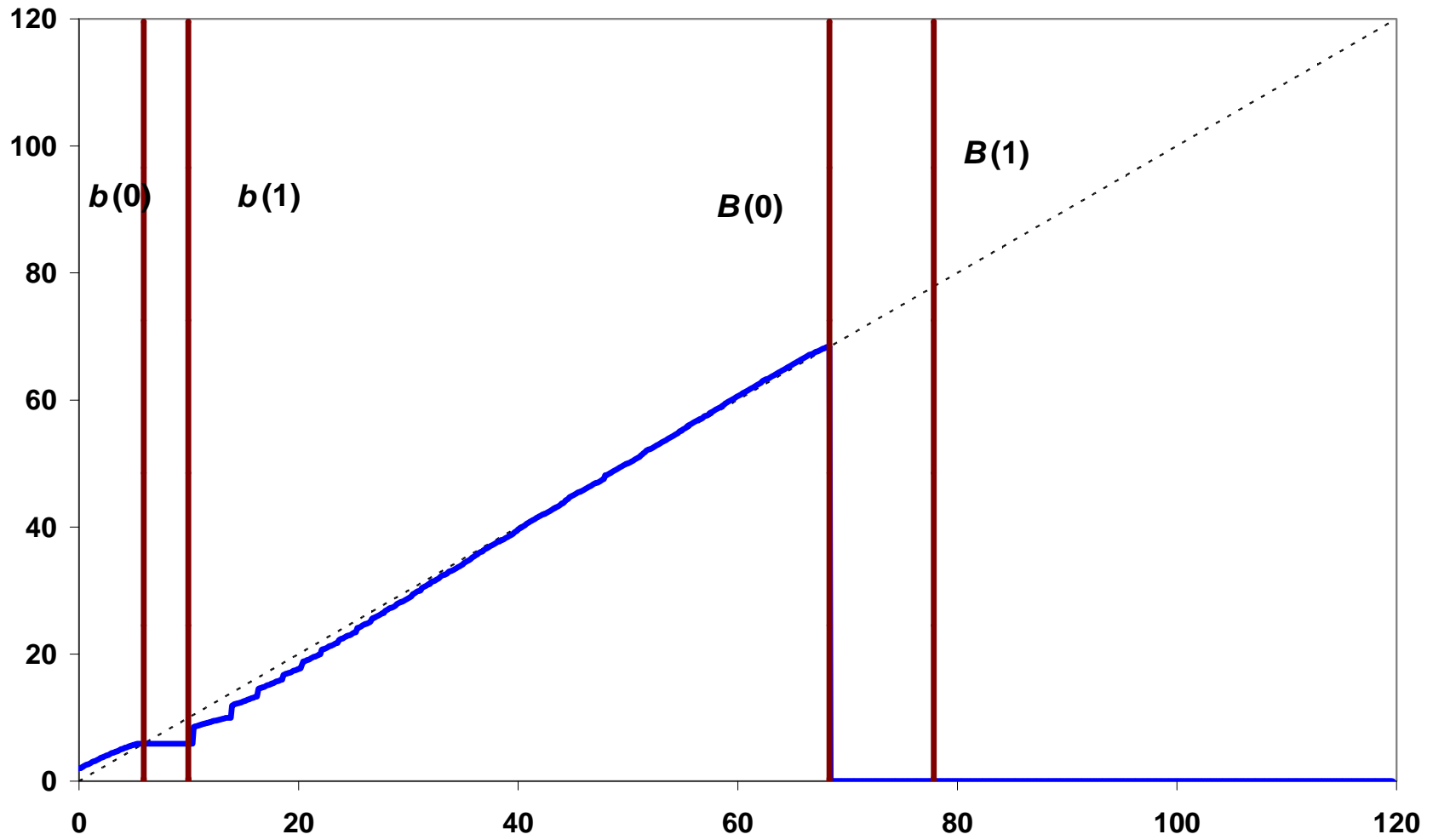
Prices are also adjusted

In the benchmark case $\bar{b}(0) < \bar{b}(1) < \bar{B}(0) < \bar{B}(1)$:

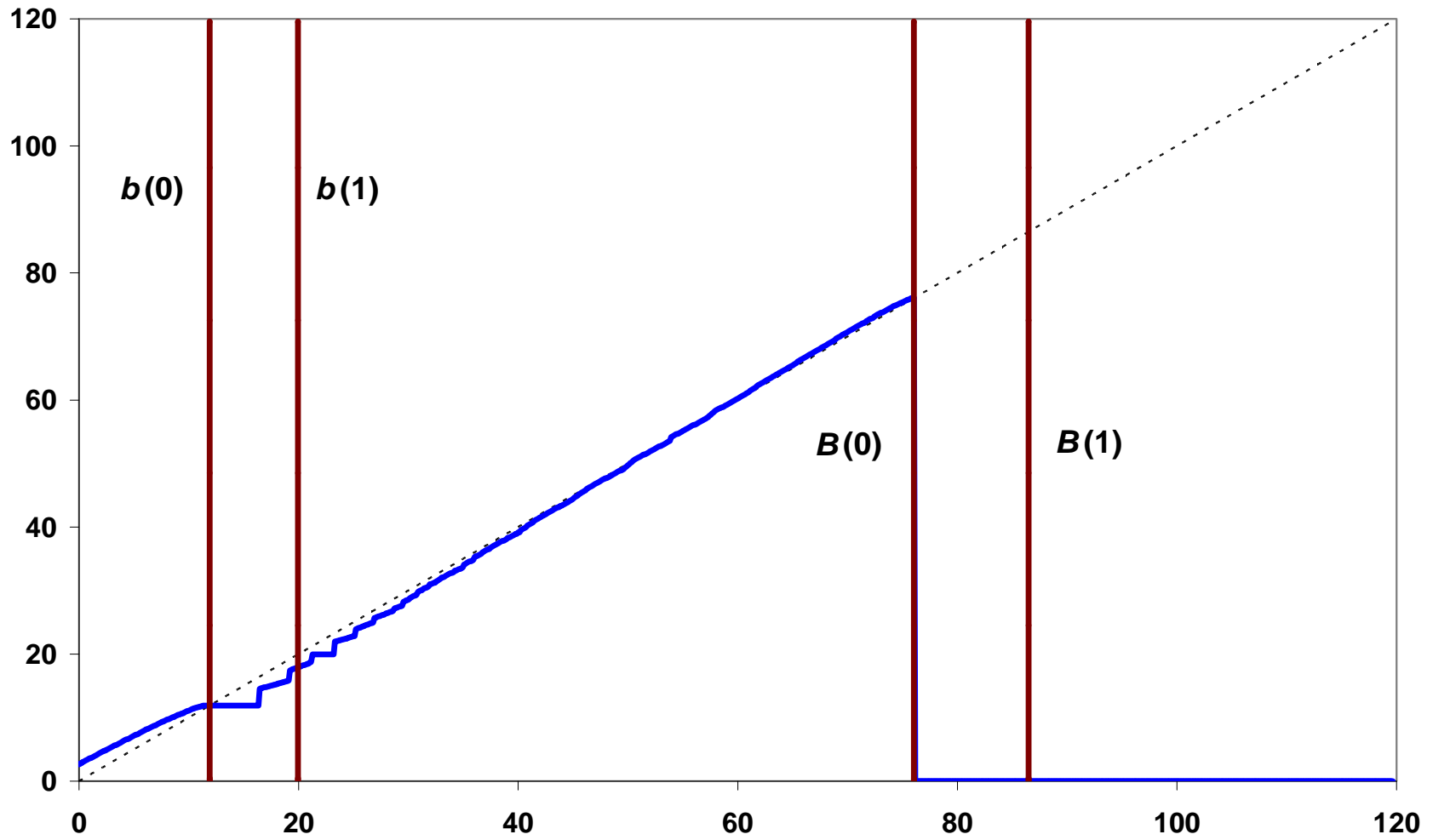
$$q(B', s) = \begin{cases} \beta[\delta + (1 - \delta)Eq'] & \text{if } B' \leq \bar{b}(0) \\ \beta(p + (1 - p)(1 - \pi))[\delta + (1 - \delta)Eq'] & \text{if } \bar{b}(0) < B' \leq \bar{b}(1) \\ \beta(1 - \pi)[\delta + (1 - \delta)Eq'] & \text{if } \bar{b}(1) < B' \leq \bar{B}(0) \\ \beta p(1 - \pi)[\delta + (1 - \delta)Eq'] & \text{if } \bar{B}(0) < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

where $Eq' = Eq(B'(B', s), s')$.

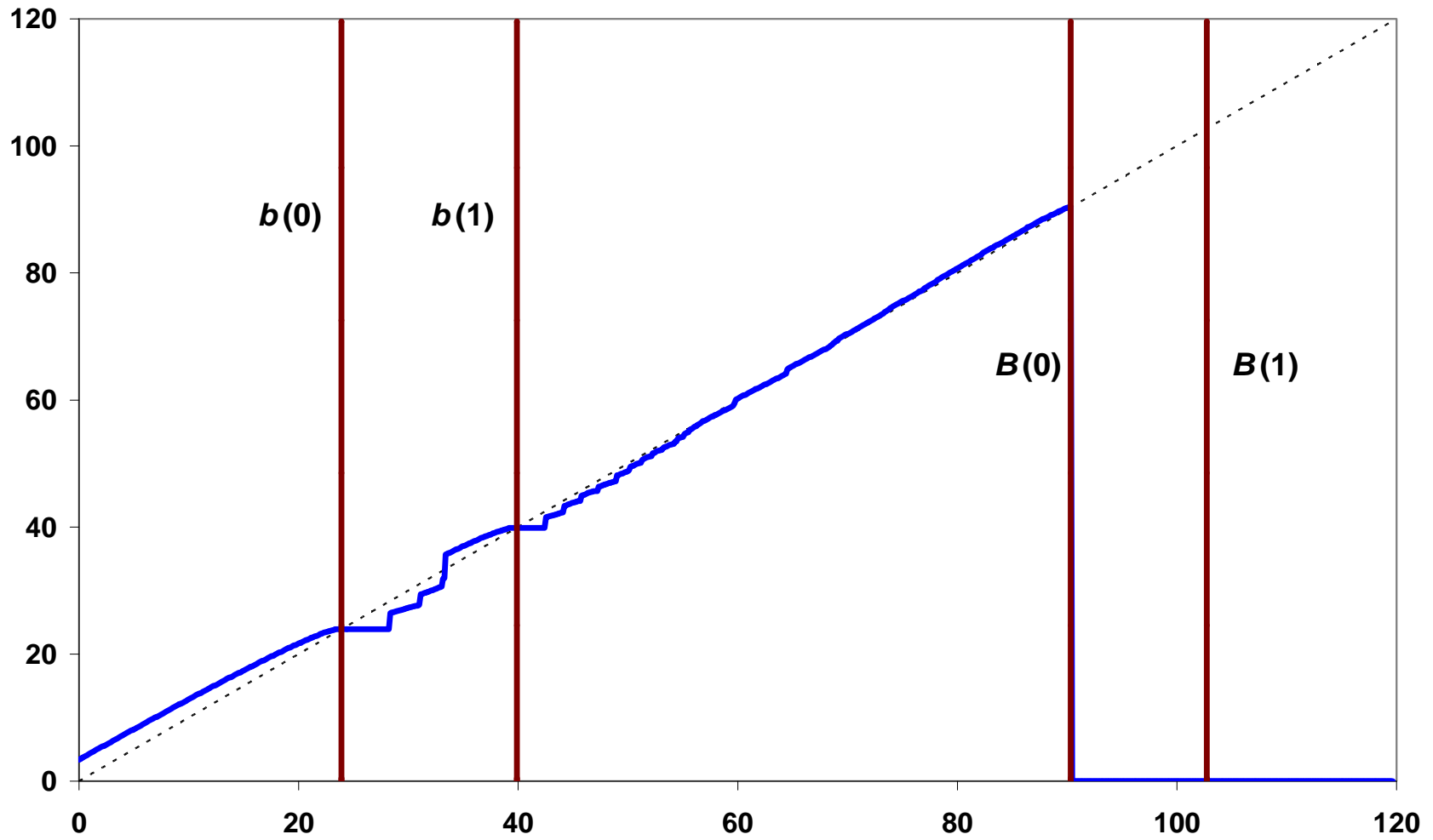
Benchmark is $\delta = 1$



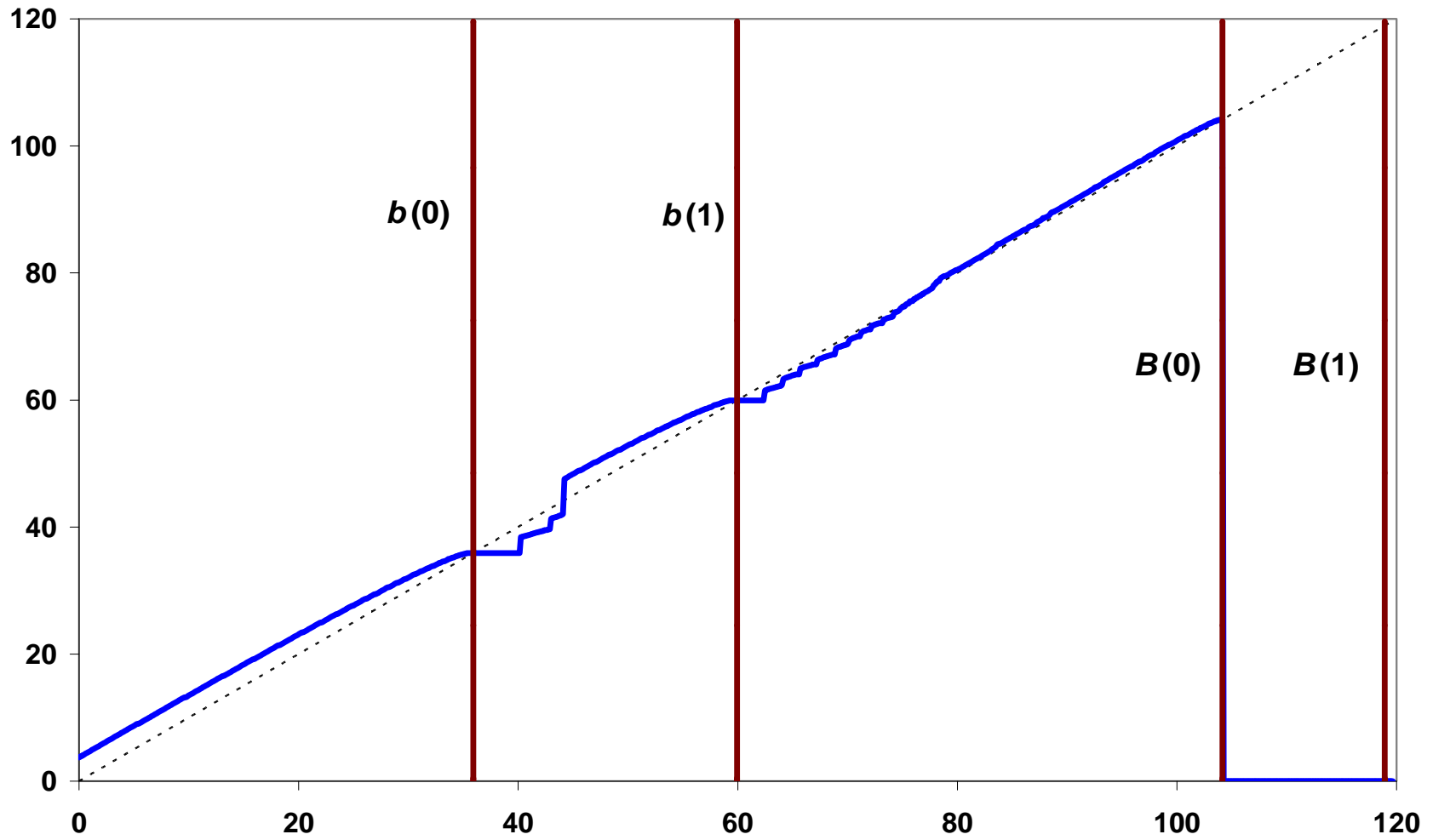
With $\delta = 0.5$



With $\delta = 0.25$



With $\delta = 0.167$



As δ becomes smaller:

The thresholds shift to the right and get closer together.

Gambling for redemption also for low (but vulnerable) levels of debt.

In the limit, $\delta = 0$ (consuls) the lower and upper thresholds coincide and huge levels of debt can be sustained (larger than 700 percent GDP).

Extensions:

Keynesian features

Panglossian borrowers *à la* Krugman (1998)

Time varying risk premia

Keynesian features

Government expenditures are close substitutes for private consumption expenditures:

$$u(c, g) = \log(c + g - \bar{c} - \bar{g}).$$

Probability of recovery $p(g)$ varies positively with government expenditures:

$$p'(g) > 0.$$

Keynesian features

Government expenditures are close substitutes for private consumption expenditures:

$$u(c, g) = \log(c + g - \bar{c} - \bar{g}).$$

Probability of recovery $p(g)$ varies positively with government expenditures:

$$p'(g) > 0.$$

Keynesian features make gambling for redemption more attractive!

Panglossian borrowers

Krugman (1998), Cohen and Villemot (2010)

The government is overly optimistic about the probability of a recovery:

$$p^g > p$$

where p is the probability that international lenders assign to a recovery.

Suppose that

$$q(B', s) = \beta(p + (1 - p)(1 - \pi))$$

or

$$q(B', s) = \beta p(1 - \pi).$$

Then holding p^g fixed and lowering p results in lower $B'(B, s)$.

Similarly, holding p fixed and increasing p^g results in lower $B'(B, s)$.

We could also analyze the case where the government is overly optimistic about the probability of a self-fulfilling crisis:

$$\pi^g < \pi$$

and obtain similar results.

Bottom line:

Optimistic governments feel the market charges too much of a premium and hence want to reduce debt.

Pessimistic governments (or governments with private information about the low probability of recovery) want to increase debt.

Time varying risk premia

Two different probabilities of a self-fulfilling crisis, $\pi_2 > \pi_1$, transitions follow a Markov process:

$$\begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}.$$

A country can be repaying its debts when faced with π_1 , then make the transition to π_2 and be forced to default.

Concluding remarks

Model provides:

- Plausible explanation for the observed behavior of PIIGS.
- Reasonable quantitative predictions for longer maturities.

Why Greece and not Belgium?

Concluding remarks

Model provides:

- Plausible explanation for the observed behavior of PIIGS.
- Reasonable quantitative predictions for longer maturities.

Why Greece and not Belgium?

Why the Eurozone and not the United States?

What about bailouts and costly reforms?

Role of bailouts: Conesa-Kehoe, “Is it too late to bail out the troubled countries in the Eurozone?”

Suppose that when a panic occurs, a third party (Troika) offers access to credit at a penalty interest rates and imposes collateral requirements (Bagehot, Clinton)

Results:

If premium is small, continue gambling

If premium is high enough to discourage gambling, then it is optimal to default instead of accepting the bailout for debt levels higher than around 80 percent GDP.