MONOPOLISTIC COMPETITION MODEL

Key ingredients

- Consumer utility: $\log\ c_0 + (1/\rho)\log\left(\sum_{j=1}^{n} c_j^{\rho}\right)$ (taste for variety of differentiated goods)
- Production of differentiated products: $y_j = (1/b)\max[\ell_j - f,\ 0]$ (increasing returns/fixed costs)

Assume that good 0, the agricultural good, is produced with perfect competition and the constant returns to scale production function

$$y_0 = \ell_0 ,$$

but that there are n manufactured goods that are produced with monopolistic competition and the increasing returns to scale production function specified above.

The representative consumer solves

$$\begin{aligned} &\max\ \ \log\ c_0 + (1/\rho)\log\left(\textstyle\sum_{j=1}^{n} c_j^{\rho}\right) \\ &\text{s. t.}\ \ p_0 c_0 + \textstyle\sum_{j=1}^{n} p_j c_j \le w\bar{\ell} \\ &\qquad\qquad\qquad\qquad c_j \ge 0 \end{aligned}$$

$$\frac{\partial u / \partial c_i}{\partial u / \partial c_0} = \frac{p_i}{p_0} \quad \text{(MRS equals price ratio)}$$

$$\frac{c_i^{\rho-1} / \sum_{j=1}^{n} c_j^{\rho}}{1/c_0} = \frac{p_i}{p_0}$$

Producer of good i solves the consumer's problem to find the indirect demand function:

$$\frac{c_i^{\rho}}{\sum_{j=1}^{n} c_j^{\rho}} = \frac{p_i c_i}{p_0 c_0} \quad \text{(multiplying by } \frac{c_i}{c_0}\text{)}$$

$$\sum_{i=1}^{n} \frac{c_i^{\rho}}{\sum_{j=1}^{n} c_j^{\rho}} = \sum_{i=1}^{n} \frac{p_i c_i}{p_0 c_0} \quad \text{(summing over } i\text{)}$$

$$1 = \frac{\sum_{i=1}^{n} p_i c_i}{p_0 c_0} \quad \text{(simplifying)}$$

$$2 p_0 c_0 = w\bar{\ell} \quad \text{(from previous equation and budget constraint)}$$

$$c_0 = \frac{w\bar{\ell}}{2 p_0}$$

Indirect demand function

$$p_i = \left(\frac{p_0 c_0}{c_i}\right) \left(\frac{c_i^{\rho}}{\sum_{j=1}^{n} c_j^{\rho}}\right) = \frac{w\bar{\ell}}{2} \left(\frac{c_i^{\rho-1}}{\sum_{j=1}^{n} c_j^{\rho}}\right)$$

Profits of firm *i*:

$$p_i y_i - wby_i - wf$$

(revenue-variable costs -fixed costs)

We suppose that the firm chooses its output y_i to maximize its profits, assuming that the outputs of all other firms are constant and that prices will adjust to clear the markets of each good. (This is the Cournot competition assumption.)

To maximize profits, the firm sets $MR = MC$: We set $c_i = y_i$ in the indirect demand function (this is the assumption that the price of good i adjusts to clear the market for good i) and plug this function into expression for profits:

$$\frac{w\bar{\ell}}{2} \left(\frac{y_i^{\rho-1}}{\sum_{j=1}^{n} y_j^{\rho}}\right) y_i - wby_i - wf \ .$$

To maximize profits, the firms sets the first derivative of this expression equal to 0, that is, $MR - MC = 0$:

$$\frac{w\bar{\ell}}{2} \left(\frac{(\sum_{j=1}^{n} y_j^{\rho}) \rho y_i^{\rho-1} - y_i^{\rho} \rho y_i^{\rho-1}}{(\sum_{j=1}^{n} y_j^{\rho})^2}\right) - wb = 0 \ .$$

Set $w = 1$ as numeraire.

Since firms are symmetric, we know that there is an equilibrium in which $y_i = \bar{y}$ if $y_i > 0$:

$$\frac{\bar{\ell}}{2}\left(\frac{(n\bar{y}^{\rho})\rho\bar{y}^{\rho-1} - \bar{y}^{\rho}\rho\bar{y}^{\rho-1}}{(n\bar{y}^{\rho})^2}\right) = b$$

$$\frac{\bar{\ell}}{2}\left(\frac{\rho(n-1)}{n^2\bar{y}}\right) = b$$

$$\bar{y} = \frac{\rho(n-1)\bar{\ell}}{2n^2 b}$$

$$p_i = \bar{p} = \frac{\bar{\ell}\bar{y}^{\rho-1}}{2n\bar{y}^{\rho}} = \frac{\bar{\ell}}{2n\bar{y}} = \frac{bn}{\rho(n-1)}.$$

The profits of typical firm are

$$\overline{py} - b\bar{y} - f = \frac{\bar{\ell}}{2n} - \frac{\rho(n-1)\bar{\ell}}{2n^2} - f \ .$$

We assume that there is free entry/exit until profits equal zero:

$$2fn^2 - (1-\rho)\bar{\ell}n - \rho\bar{\ell} = 0$$

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$$n = \frac{(1-\rho)\bar{\ell} + \sqrt{(1-\rho)^2\bar{\ell}^2 + 4(\rho\bar{\ell})(2f)}}{4f}.$$

Equilibrium

An **equilibrium** of the monopolistic competition model is
the number of manufacturing firm $\hat{n}$,
a price $\hat{p}_0$ for the agricultural good,
a price $\hat{p}_j$ for each manufacturing firm that operates at a positive level,
a wage rate $\hat{w}$,
a consumption plan $\hat{c}_0, \hat{c}_1, \hat{c}_2, ..., \hat{c}_{\hat{n}}$,
production plans, $\hat{y}_0$, $\hat{\ell}_0$ for the agricultural good and $\hat{y}_j$, $\hat{\ell}_j$ for each manufacturing firm that operates at a positive level
such that

- Given $\hat{p}_0, \hat{p}_1, \hat{p}_2, ..., \hat{p}_{\hat{n}}$, and $\hat{w}$, the consumer chooses $\hat{c}_0, \hat{c}_1, \hat{c}_2, ..., \hat{c}_{\hat{n}}$ to solve

$$\begin{aligned} &\max \ \log \ c_0 + (1/\rho)\log\left(\sum_{j=1}^{n} c_j^{\rho}\right) \\ &\text{s. t. } \ \hat{p}_0 c_0 + \sum_{j=1}^{\hat{n}} \hat{p}_j c_j \leq \hat{w}\bar{\ell} \\ &\qquad\qquad\qquad c_j \geq 0. \end{aligned}$$

- $\hat{p}_0 - \hat{w} \leq 0, \ = 0 \text{ if } \hat{y}_0 > 0$.

- Given the indirect demand function $p_j(c_1, ..., c_j, ..., c_n)$ that comes from solving the representative consumer's utility maximization problem, firm j chooses $\hat{y}_j$ to solve

$$\max \ p_j(\hat{y}_1, ..., y_j, ..., \hat{y}_{\hat{n}}) y_j - \hat{w} b y_j - \hat{w} f$$

- $\hat{p}_j = \hat{p}_j(\hat{y}_1, ..., \hat{y}_j, ..., \hat{y}_n)$.

- $\hat{p}_j \hat{y}_j - \hat{w} b \hat{y}_j - \hat{w} f \leq 0, \ = 0 \text{ if } \hat{y}_j > 0$ where $\hat{p}_j = \hat{p}_j(\hat{y}_1, ..., \hat{y}_j, ..., \hat{y}_n)$.

- $\hat{y}_0 = \hat{\ell}_0$.

- $\hat{y}_j = (1/b)\max[\hat{\ell}_j - f, \ 0], \ j = 1, 2, ..., \hat{n}.$

- $\hat{c}_j = \hat{y}_j, \ j = 0, 1, 2, ..., \hat{n}$.

- $\hat{\ell}_0 + \sum_{j=1}^{\hat{n}} \hat{\ell}_j = \bar{\ell}$.

Numerical example

$$b=1,\ f=2,\ \rho=1/2,\ \bar{\ell}=49$$

$$n=\frac{24.5+\sqrt{(24.5)^2+4(24.5)(4)}}{8}$$

$$n=7$$

$$\bar{y}=1.5,\ \bar{p}=2.3333$$

$$p_0=w=1,\ y_0=24.5$$

Utility:

$$\log 24.5+2\log(7(1.5)^{1/2})=7.4960$$

Homogenous of degree one representation of utility (a real income index):

$$\exp[(1/2)(\log 24.5+2\log(7(1.5)^{1/2}))]=\exp[(1/2)(7.4960)]=42.44\,.$$

An integral number of firms?

There is a problem with our concept of equilibrium if the number of firms, $\hat{n}$, does not turn out to be an integer. Suppose, for example, that

$$b = 1,\ f = 2,\ \rho = 1/2,\ \bar{\ell} = 490\ .$$

Then, when we solve

$$n = \frac{245 + \sqrt{(245)^2 + 4(245)(4)}}{8}\ ,$$

we obtain $\hat{n} = 62.2342$. How do we interpret this solution? There are two approaches that we could take:

1. We could restrict $\hat{n}$ to be an integer, and let it be the largest number of firms for which profits are nonnegative. In this case, however, there can be positive profits in equilibrium. These profits need to be earned by someone. If we give them to the representative consumer, then the consumer's budget constraint becomes

$$\hat{p}_0 c_0 + \sum_{j=1}^{\hat{n}} \hat{p}_j c_j \leq \hat{w}\bar{\ell} + \hat{\Pi}$$

where $\hat{\Pi}$ are profits. Everything becomes a more complicated even in this simple model with only one market with monopolistic competition. Things become much more complicated in applied models with many such markets.

2. We could think of $\hat{n}$ as being an integer up until we compute the number of firms, at which we point we simply calculate a real number. This is the approach that economists typically use in applying this sort of model.

Reinterpreting the model as a model of international trade

We can reinterpret this model as a model of international trade among countries that are identical except for their sizes as measured by their labor forces, $\overline{\ell}^i$.

Consider the numerical example in which $b=1,\ f=2,\ \rho=1/2$ and there are two countries, one in which $\overline{\ell}^1 = 441$ and the other in which $\overline{\ell}^2 = 49$. (We can think of these countries as being the United States and Canada respectively.)

In the integrated equilibrium of the world economy

$$p_0 = w = 1$$
$$n = \frac{245 + \sqrt{(245)^2 + 4(245)(4)}}{8} = 62.2342.$$
$$\overline{y} = \frac{\rho(n-1)\overline{\ell}}{2n^2 b} = \frac{61.2342 \cdot 490}{2 \cdot 2 \cdot 62.2342^2} = 1.9367$$
$$\overline{p} = \frac{\overline{\ell}\overline{y}^{\rho-1}}{2n\overline{y}^{\rho}} = \frac{\overline{\ell}}{2n\overline{y}} = \frac{bn}{\rho(n-1)} = \frac{2 \cdot 62.2342}{61.2342} = 2.0327$$
$$y_0 = \frac{\overline{\ell}}{2p_0} = \frac{490}{2 \cdot 1} = 245.$$

To calculate consumption of each variety in each country, we just divide the world production of the variety $\overline{y}$ proportionally. In country 1, for example,

$$\overline{c}^1 = \frac{\overline{\ell}^1}{\overline{\ell}^1 + \overline{\ell}^2}\overline{y} = \frac{441}{490}1.9367 = 1.7430.$$

We also divide the production and the consumption of the agricultural good proportionally:

$$\hat{n}^1 = (441/490)\hat{n} = (441/490)62.2342 = 56.0108$$
$$\hat{n}^2 = (49/490)\hat{n} = (49/490)62.2342 = 6.2234,$$

and

$$\hat{y}_0^1 = (441/490)\hat{y}_0 = (441/490)245 = 220.5 \quad \hat{y}_0^2 = (49/490)\hat{y}_0 = (49/490)245 = 24.5$$

(Strictly speaking, there is nothing in this model that pins down the location of production of the agricultural good. We are calculating a symmetric equilibrium.)

Trade Equilibrium

	$\hat{n}^i$	$\hat{p}_0^i$	$\overline{p}$	$\hat{w}^i$	$\hat{c}_0^i$	$\overline{c}^i$	$\hat{y}_0^i$	$\hat{\ell}_0^i$	$\overline{y}^i$	$\overline{\ell}^i$
country 1	56.0108	1.0	2.0327	1.0	220.5	1.7430	220.5	220.5	1.9367	3.9367
country 2	6.2234	1.0	2.0327	1.0	24.5	0.1937	24.5	24.5	1.9367	3.9367

Utility:

$$\hat{u}^1 = \log 220.5 + 2\log 62.2342(1.7430)^{1/2} = 14.2133$$
$$\hat{u}^2 = \log 24.5 + 2\log 62.2342(0.1937)^{1/2} = 9.8190\,.$$

Real income index:

$$e^{\hat{u}^1/2} = 1220.05$$
$$e^{\hat{u}^2/2} = 135.57$$

(Notice that, not surprisingly, the real income in country 1 is 9 times greater than that in country 2.)

Gains from Trade

To calculate the gains from trade, we can compute the autarky equilibria for both countries. (We have already calculated this equilibrium for country 2.

Autarky Equilibrium

	$\hat{n}^i$	$\hat{p}_0^i$	$\overline{p}$	$\hat{w}^i$	$\hat{c}_0^i$	$\overline{c}^i$	$\hat{y}_0^i$	$\hat{\ell}_0^i$	$\overline{y}^i$	$\overline{\ell}^i$
country 1	56.1075	1.0	2.0363	1.0	220.5	1.9300	220.5	220.5	1.9300	3.9300
country 2	7.0000	1.0	2.3333	1.0	24.5	1.5000	24.5	24.5	1.5000	3.5000

Utility:

$$\hat{u}^1 = \log 220.5 + 2\log 56.1075(1.9300)^{1/2} = 14.1080$$
$$\hat{u}^2 = \log 24.5 + 2\log 7(1.5)^{1/2} = 7.4960\,.$$

Real income index:

$$e^{\hat{u}^1/2} = 1157.48$$
$$e^{\hat{u}^2/2} = 42.44\,.$$

The smaller country, country 2, has the most to gain from trade:

In country 1, real income goes up by 5.4 percent ($1220.05/1157.48 = 1.0541$).

In country 2, real income goes up by 219.4 percent ($135.57/42.44 = 3.1944$).