Numerical Methods for Ph. D Students in Economics

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Problem Set

1. Use the simplex method to solve the two-person, zero-sum games that correspond to the two versions of the poker problem in the lecture notes. Read Chapter 14 in George Dantzig, *Linear Programming and Extensions*.

2. Write a computer program to solve a "Great Depressions" dynamic general equilibrium model. Calibrate the model to data from

http://www.greatdepressionsbook.com

and solve it.

3. Consider an overlapping generations economy in which the representative consumer in generation t, $t = 1, 2, \ldots,$ has preferences over the consumption of the single good in each of the two periods of her life given by the utility function

$$u(c_t^t, c_{t+1}^t) = \log\ c_t^t + \log\ c_{t+1}^t .$$

This consumer is endowed with quantities of labor $(\ell_t^t, \ell_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$. In addition, there is a generation 0 who representative consumer lives only in period 1 and has the utility function

$$u^0(c_1^0) = \log\ c_1^0 ,$$

and the endowment of $\ell_1^0 = \bar{\ell}_2$ units of labor and $\bar{k}_1^0$ units of capital in period 1. This consumer also has an endowment of fiat money m, which can be positive, negative or zero.

The production function is

$$f(k_t, \ell_t) = \theta k_t^\alpha \ell_t^{1-\alpha} ,$$

and capital depreciates at the rate δ per period, $0 \le \delta \le 1$.

a) Define a sequential market equilibrium for this economy.

b) Define an Arrow-Debreu equilibrium for this economy. State and prove two theorems that establish the equivalence between a sequential market equilibrium and an Arrow-Debreu equilibrium.

c) Reduce the equilibrium conditions to a second-order difference equation in k_t, that is, an equation in k_{t+1}, k_t, k_{t-1} that includes no other endogenous variables.

d) Suppose that $m = 0$. Reduce the equilibrium conditions to a first-order difference equation in k_t. (Hint: in this case you know that the savings of generation t in period t in the sequential market equilibrium must equal k_{t+1}.)

4. Suppose that $\theta = 100$, $\alpha = 0.4$, $\delta = 0.8$, $\bar{\ell}_1 = 1$, and $\bar{\ell}_2 = 0$ in question 2.

a) Define a steady state for this economy. Calculate the two steady states.

b) Suppose that $\bar{k}_1^0 = 10$ and $m = 0$. Use your answer to question 2d to calculate the equilibrium in the first 10 periods both by hand and on the computer.

c) Suppose now that $\bar{\ell}_2 = 0.5$. Repeat parts a and b, doing your calculations on the computer.

d) Suppose now that $\bar{\ell}_2 = 2$. Repeat parts a and b, doing your calculations on the computer.

5. Consider now an economy like that in questions 3 and 4 except that consumers live for 4 periods rather 2. The utility function of the consumer born in period t, $t = 1, 2, \ldots$, is

$$u(c_t^t, c_{t+1}^t, c_{t+2}^t, c_{t+3}^t) = \log c_t^t + \beta \log c_{t+1}^t + \beta^2 \log c_{t+2}^t + \beta^3 \log c_{t+3}^t .$$

This consumer is endowed with quantities of labor $(\ell_t^t, \ell_{t+1}^t, \ell_{t+2}^t, \ell_{t+3}^t) = (\bar{\ell}_1, \bar{\ell}_2, \bar{\ell}_3, 0)$.

In addition, there are three other consumers, -2, -1, and 0. Consumer -2 has the utility function

$$u^{-2}(c_1^{-2}) = \log c_1^{-2}$$

and the endowments $\ell_1^{-2} = 0$, $\bar{k}_1^{-2}$. Consumer -1 has the utility function

$$u^{-1}(c_1^{-1}, c_2^{-1}) = \log c_1^{-1} + \beta \log c_2^{-1}$$

and the endowments $(\ell_1^{-1}, \ell_2^{-1}) = (\bar{\ell}_3, 0)$, $\bar{k}_1^{-1}$. Consumer 0 has the utility function

$$u^0(c_1^0, c_2^0, c_3^0) = \log\ c_1^0 + \beta \log\ c_2^0 + \beta^2 \log\ c_3^0$$

and endowments $(\ell_1^0, \ell_2^0, \ell_3^0) = (\bar{\ell}_2, \bar{\ell}_3, 0_4)$, $\bar{k}_1^0$. Consumers -2, -1, and 0 are also endowed with m^{-2}, m^{-1}, and m^0 units of fiat money respectively.

The production function remains

$$f(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha},$$

and capital depreciates at the rate δ per period, $0 \le \delta \le 1$.

a) Define a sequential market equilibrium for this economy.

b) Define a steady state for this economy.

c) Reduce the equilibrium conditions in part a to a difference equation in k_t. Show that, if $m^{-2} = m^{-1} = m^0 = 0$, you can reduce the equilibrium conditions to a difference equation of lower order.

d) Suppose that, after $T = 9$ periods, the capital stock becomes constant at $\hat{k}$. That is,

$$k_{10} = k_{11} = ... = \hat{k}.$$

Considering the general case — not the case where $m^{-2} = m^{-1} = m^0 = 0$ — write out the equilibrium conditions in part c as a system of 8 equations in the 8 unknowns k_2, k_3, k_3, k_4, k_5, k_6, k_7, k_8, and k_9 by considering the market clearing conditions in periods 1, 2, 3, 4, 5, 6, 7, and 8.

e) Suppose that T — the date immediately after which the capital stock becomes constant — increases. What happens to the system of equations and unknowns in part d?

f) Redo parts d and e for the case where $m^{-2} = m^{-1} = m^0 = 0$. (Hint: you can reduce the number of equations and unknowns.)

g) Write a computer program to solve parts d, e, and f and solve the model for $T = 30$. Assign values to the parameters. Solve the same model for $T = 35$ and compare the solution with that of the model where $T = 30$.

6. Repeat question 5 for a model in which the utility function is

$$u(c_t^t, \ell_t^t, c_{t+1}^t, \ell_{t+1}^t, c_{t+2}^t, \ell_{t+2}^t, c_{t+3}^t) = \sum_{j=1}^{3} \beta^{j-1}\left(\gamma \log\ c_{t+j-1}^t + (1-\gamma)\log\ (\bar{\ell}_j - \ell_{t+j-1}^t)\right) + \beta^3 \gamma \log\ c_{t+3}^t .$$