Numerical Methods for Ph. D Students in Economics

Homework 1.2 (Optional Questions)

1. (Newton's method for vector-valued functions) Extend your subroutine that computes a gradient of a scalar-valued function so that it can deal with vector-valued functions. I.e. write a subroutine that caclulates Jacobian of functions. Using that subroutine, write a program which implements the Newton's method for nonlinear system of equations.

2. (Numerical Optimization) Consider the following model: $y_n \sim N(x_n'\beta, \gamma)$ i.i.d. for all $n = 1, 2, ...N$ (γ is the variance), where x_n's are K-dimensional vectors and y_n's are scalars. You are asked to generate data using a given set of parameters and estimate the model via the maximum likelihood estimation method. In this question, we set $\beta = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\gamma = 10$, $K = 2$ and $N = 100$.

   (a) Generate data set (y, X) using a random number generator. (Hint: use `randn` to generate standard normal random variables. You can choose the distribution of X arbitrarily.)

   (b) Create the log likelihood function as a user-defined function. (Hint: inputs for your user-defined function will be a stacked vector of parameters `[beta; gamma]` and a data matrix `[y,X]`.)

   (c) Find the solution to the maximization problem. (Hint: you are asked to find a point where the gradient of log likelihood is equal to zero. Can you use the Newton's method? You also need to check if the Hessian of log likelihood function is negative semi-definite at that point. Check its eigenvalues using `eig`.)

   (d) Check your results with those you can obtain using `ols.m` . (You can find it in the slides "Introduction to MATLAB.") Note that the variance estimates will differ unless you make some adjustment.