# Lecture 2

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# Introduction

- In HW2, you are asked to solve some DGE models.
- Need to start with a simple test case, where its solution is known.

# Simple Model with a Known Solution

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

s.t.

$$c_t + k_{t+1} = Ak_t^{\alpha}$$
$$k_0 = \overline{k_0} : \text{given}$$

Assume we have assigned values to parameters. The solution (policy function) has the following form: $c_t = (1 - \beta\alpha)Ak_t^{\alpha}$, $k_{t+1} = \beta\alpha Ak_t^{\alpha}$.

# Equilibrium Conditions

$$
\begin{aligned}
0 &= \beta \frac{c_t}{c_{t+1}} \alpha A k_{t+1}^{\alpha-1} - 1 \\
0 &= A k_t^{\alpha} - c_t - k_{t+1}
\end{aligned}
$$

for all $t$, $k_0 = \overline{k_0}$: given, and TVC. In the steady-state,

$$
\begin{aligned}
0 &= \beta \alpha A k_{ss}^{\alpha-1} - 1 \\
0 &= A k_{ss}^{\alpha} - c_{ss} - k_{ss}
\end{aligned}
$$

# Truncated Equilibrium Conditions

$$
\begin{aligned}
0 &= \beta \frac{c_t}{c_{t+1}} \alpha A k_{t+1}^{\alpha-1} - 1, \quad \forall t = 0, 1, ..., T-1 \\
0 &= A k_t^{\alpha} - c_t - k_{t+1}, \quad \forall t = 0, 1, ..., T
\end{aligned}
$$

and

$$
\begin{aligned}
0 &= k_0 - \overline{k_0} \\
0 &= k_{T+1} - k_{ss}
\end{aligned}
$$

We have $2T + 3$ equations and $2T + 3$ unknowns. Just solve this system of equations using Newton's method.

# Implementation

- You will need a user-defined function $g$ which takes $x = (c_0, c_1, ..., c_T, k_0, ...k_{T+1})$ and $p = (T, \beta, \alpha, A, \overline{k_0}, k_{ss})$ as inputs and is equal to zero when $(c_0, c_1, ..., c_T, k_0, ...k_{T+1})$ satisfies conditions in the previous page.
- Solve for $g(x, p) = 0$ for $x$ using Newton's method.
- You can check your program in two ways. (1) Set $\overline{k_0} = k_{ss}$. (2) Set $\overline{k_0}$ arbitrarily and compare the result with the true solution.

# Extension

- Extending the program to other cases is not a big deal.
- Calibration may require some work. Matching the steady state to data would be the simplest way.