

Adverse Selection, Reputation and Sudden Collapses in Secondary Loan Markets *

V.V. Chari

University of Minnesota and Federal Reserve Bank of Minneapolis
chari@econ.umn.edu

Ali Shourideh

University of Minnesota and Federal Reserve Bank of Minneapolis
shour004@umn.edu

Ariel Zetlin-Jones

University of Minnesota and Federal Reserve Bank of Minneapolis
zetli001@umn.edu

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Abstract

Loan originators often securitize some loans in secondary loan markets and hold on to others. New issuances in such secondary markets collapse abruptly on occasion, typically when collateral values used to secure the underlying loans fall and these collapses are viewed by policymakers as inefficient. We develop a dynamic adverse selection model in which small reductions in collateral values can generate abrupt inefficient collapses in new issuances in the secondary loan market by affecting reputational incentives. We find that a variety of policies intended to remedy market inefficiencies do not help resolve the adverse selection problem.

1 Introduction

Following the sharp decline in the volume of new issuances in the U.S. secondary loan market in the fall of 2007, policymakers argued that the market was not functioning normally and proposed and carried out a variety of policy interventions intended to restore the normal functioning of this market. Here we present evidence on sudden collapses and motivated by that evidence, construct a model in which new issuances in the secondary loan market abruptly collapse. This collapse, in our model, is associated with an increase in inefficiency. We also argue that reductions in the value of the collateral used to secure the underlying loans are particularly likely to trigger sudden collapses associated with increased inefficiency. Since sudden collapses are associated with increased inefficiency, our model is consistent with policymakers' views that the market was functioning poorly. We use this model to analyze proposed and actual policy interventions and argue that these interventions typically do not remedy the inefficiency associated with the market collapse.

In our model, the main economic function of the secondary loan market is to allocate originated loans to institutions that have a *comparative advantage* in holding and managing the loans. This economic function is disrupted by informational frictions. In our model, loan originators differ in their ability to originate high-quality loans. The originators are better informed about their ability to generate high-quality loans than are potential purchasers. This informational friction creates an *adverse selection* problem. The focus of our analysis is to examine the extent to which *reputational* considerations ameliorate or intensify the adverse selection problem in these markets. In order to analyze these reputational considerations, we develop a dynamic adverse selection model of the secondary loan market.

Our main finding is that our model has fragile outcomes in which sudden collapses in the volume of new issuances in secondary loan markets are associated with increased inefficiency. We say that outcomes are fragile if the model has multiple equilibria or if a large number of originators change their decisions in response to small changes in aggregate fundamentals.

In terms of fragility as multiplicity, we show that our baseline dynamic adverse selection model with reputation has multiple equilibria for a range of reputation levels. In one of these equilibria, labeled the *positive reputational equilibrium*, high-quality loan originators have incentives to sell at a current loss in order to improve their reputations and command higher prices for future loans. In the other equilibrium, labeled the *negative reputational equilibrium*, loan originators who sell their loans are perceived by future buyers to have low-quality loans. These perceptions induce high-quality loan originators to hold on to their loans. Since low-quality originators always sell their loans, the volume of new issuances is larger in the positive reputational equilibrium than in the negative reputational equilibrium. Clearly, with multiple equilibria sunspot like shocks can generate sudden collapses. We show that the positive equilibrium Pareto dominates the negative equilibrium for a range of reputation levels. In this sense, sudden collapses are associated with increased inefficiency.

Although the multiplicity of equilibria has the attractive feature that it implies that the model can be consistent with observations of sudden collapses, such multiplicity makes it difficult to conduct policy analysis. We propose a refinement adapted from the coordination games literature (see [Carlsson and van Damme \(1993\)](#) and [Morris and Shin \(2003\)](#)). Our refinement is also motivated by the idea that sudden collapses in the volume of new issuances in loan markets are associated with falls in the value of the collateral that supports the underlying loans. These considerations lead us to add aggregate shocks to collateral values and to assume that the collateral value is observed with an arbitrarily small error.

We show that shocks to collateral values make the outcomes of our model consistent with our second notion of fragility, namely, a large fraction of loan originators choose to change their decisions on whether to sell or hold their loans in response to small changes in collateral values. In this sense, reductions in collateral values can induce sudden collapses in the volume of new issuances for the market as a whole.

Both adverse selection and the dynamics induced by reputation acquisition play central roles in generating sudden collapses from small changes in collateral values. A simple way of

seeing the role of adverse selection is to note that the version of our model with symmetrically informed originators and buyers does not produce sudden collapses in new issuances. With asymmetrically informed agents, originators with high reputations receive higher prices for their loans and are therefore more willing to sell their loans. We show that a fall in collateral values makes high-quality originators who were close to being indifferent about selling versus holding to hold. Small changes in collateral values can induce a large number of originators to switch to holding from selling only if they are all close to the point of indifference. In a static model, we have no reason to expect that the distribution of originators by reputation levels will be concentrated close to the indifference point.

In a dynamic model with learning by market participants, we argue that originators' reputations are likely to be clustered. The reason is that in models like ours, the reputation levels of high-quality originators have an upward trend over time, resulting in the reputation levels of many high-quality originators tending to become similar in the long run. We show that in an infinitely repeated version of our model, the long run or invariant distribution of reputation levels displays significant clustering. This clustering in turn implies that small changes in fundamentals can lead a large number of originators to change their decisions when the fundamentals are close to the point of indifference. A related result is that small changes in collateral values, when these values are far away from the point of indifference, do not lead to large changes in the volume of new issuances.

We have argued that our model is consistent with abrupt collapses in secondary loan markets and with the widespread view among policymakers that such abrupt collapses were associated with sharp increases in the inefficiency of the operation of such markets. In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to remedy inefficiencies in the market for securitized assets. Some of these programs, such as the proposed Public-Private Partnership and TALF, were implemented at least in part. The TALF program allows participants to purchase securitized assets by borrowing from the Federal Reserve and using the assets as collateral. We use our model

to evaluate the effects of various policies. In terms of purchase policies, we show that if the purchase price is set at or below the level that prevails in the positive reputational equilibrium, the equilibrium outcomes do not change and in this sense the policy is ineffective. If the purchase price is set at a sufficiently high level, the policy implies that the government makes negative profits.

We also analyze policies that change the time path of interest rates. We show that temporary decreases in interest rates worsen the adverse selection problem. Interestingly, anticipated decreases in interest rates in the future can have beneficial current effects by reducing the range of reputations over which the economy has multiple equilibria.

1.1 Related Literature

Our work here is related to an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984), Glosten and Milgrom (1985), Kyle (1985), and Garleanu and Pedersen (2004) as well as to the related securitization literature, specifically, the work of DeMarzo and Duffie (1999) and DeMarzo (2005). See also Eisfeldt (2004), Kurlat (2010), Guerrieri et al. (2010) and Guerrieri and Shimer (2011) for analyses of adverse selection in dynamic environments. We add to this literature by analyzing how reputational incentives affect adverse selection problems.

Our assumption that buyers have less information concerning the loan quality of a bank is in line with a descriptive literature that argues that secondary loan markets feature adverse selection (see, for example, the work of Dewatripont and Tirole (1994), Ashcraft and Schuermann (2006), and Arora et al. (2009)). Also, a growing literature provides data on the presence of adverse selection in asset markets. For example, Ivashina (2009) finds evidence of adverse selection in the market for syndicated loans. Downing et al. (2009) find that loans that banks held on their balance sheets yielded more on average relative to similar loans which they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers and present evidence that these underwrit-

ers exploit their superior information when trading in the secondary market. Specifically, the tranches that such underwriters avoid bidding on exhibit much worse than average ex post performance than the tranches that they do bid on.

A recent paper by [Elul \(2009\)](#) presents evidence that is consistent with our model. [Elul \(2009\)](#) shows that returns on securitized loans and loans held by originators were similar before 2006 and that returns on securitized loans were lower than returns on comparable loans after 2006. This evidence is consistent with our model in the following sense. Our model implies that when collateral values underlying loans are relatively high, most high-quality banks with high costs of managing the loans choose to sell their loans; but when collateral values are relatively low, such banks choose to hold their loans. Before 2006, land values were rising, so it seems reasonable to suppose that collateral values were relatively high. After 2006, land values stopped rising and in some cases fell, so it seems reasonable to suppose that collateral values were lower than they had been.

Finally, [Mian and Sufi \(2009\)](#) present evidence that securitized loans were more likely to default than nonsecuritized loans. This evidence is consistent with our model in the sense that for all realizations of the aggregate shock, the default rate of securitized loans is at least as high as that of held loans, and for some realizations the default rate of securitized loans is higher than that of held loans.

Our work is also related to an extensive literature on reputation. [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#) argue that equilibrium outcomes are better in models with reputational incentives than in models without them. In the banking literature, [Diamond \(1989\)](#) develops this argument. More recently, [Mailath and Samuelson \(2001\)](#) analyze the role of reputational incentives in infinite horizon economies and provide conditions under which they can improve outcomes. In contrast, [Ely and Välimäki \(2003\)](#) and [Ely et al. \(2008\)](#) describe models in which reputational incentives can worsen outcomes. Our work here combines the results in this literature by showing that reputational models can have multiple equilibria. In some of these equilibria, reputational incentives can generate better outcomes;

in others, they can generate worse. Furthermore, using techniques from the global games literature, we develop a refinement that produces a unique, fragile equilibrium. Perhaps the work most closely related to ours is that of [Ordoñez \(2009\)](#). An important difference between our work and his is that our model has equilibria that are worse than the static equilibrium, so that reputational incentives can lead to outcomes that are ex post less efficient than those in a model without these incentives.

Our analysis of policy is closely related to recent work by [Philippon and Skreta \(2009\)](#) who analyze a variety of policies in a model with adverse selection. The main difference with our work is that we focus on the incentives induced by reputation, whereas they analyze a static model.

2 Evidence on Sudden Collapses

Here we present evidence on sudden collapses in the market for new issuances of asset-backed securities. Figure 1 displays the volume of new issuances of asset-backed securities for various categories from the first quarter of 2000 to the first quarter of 2009. The figure shows that the total volume of new issuances of asset-backed securities rose from roughly \$50 billion in the first quarter of 2000 to roughly \$300 billion in the fourth quarter of 2006. The volume of new issuances fell abruptly to roughly \$100 billion in the third quarter of 2007 and then fell again to near zero in roughly the fourth quarter of 2008. The figure also shows similar large fluctuations in the volume of new issuances for each category.

[Ivashina and Scharfstein \(2010\)](#) document a similar pattern for new issues of syndicated loans. Figure 1, Panel A of their paper shows that syndicated lending rose from roughly \$300 billion in the first quarter of 2000 to roughly \$700 billion in the second quarter of 2007. This lending declined sharply thereafter and fell to roughly \$100 billion by the third quarter of 2008.

The reduction in the volume of new issuances in the secondary market roughly coincided

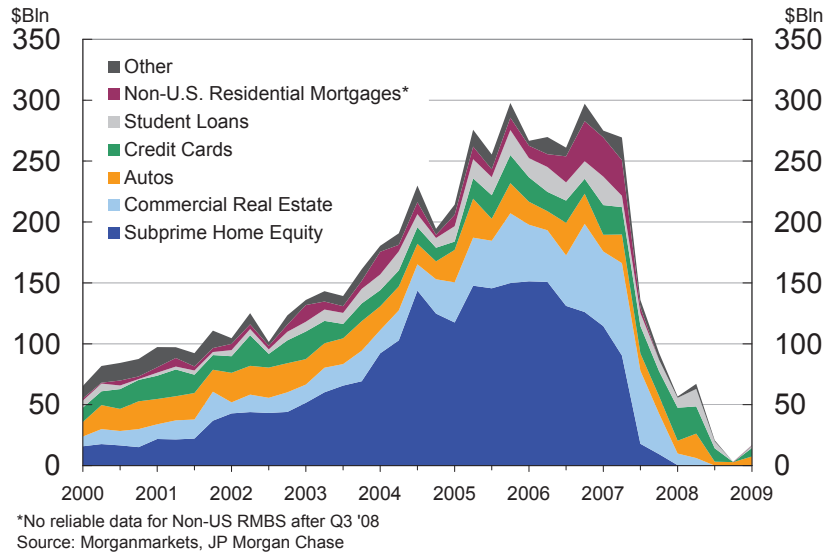


Figure 1: New Issuance of Asset-Backed Securities (Source: JP Morgan Chase)

with a reduction in collateral values. One way of seeing this coincidence is to consider the Case-Shiller home price index (available at <http://www.standardandpoors.com/indices>). This index stopped growing in late 2006 and declined through 2007. The coincidence of the reduction in the volume of new issuances and the reduction in collateral values is consistent with our model.

White (2009) has argued that in the 1920s, the United States experienced a boom-bust cycle in securitization of real estate assets that was similar to its recent experience. Figure 2 displays the change in the outstanding stock in real estate bonds in the 1920s based on data in Carter and Sutch (2006). Such bonds were issued against single large commercial mortgages or pools of commercial or real estate mortgages and were publicly traded. To make this data comparable to more recent data, we scale the data from the 1920s by nominal GDP in 2009. Specifically, we multiply the change in the nominal stock of outstanding debt in each year by the ratio of the nominal GDP in 2009 to that in the relevant year. This figure shows that the changes in the stock rose dramatically from essentially 0 in 1919 to an average of \$145

billion in the period from 1925 to 1928. The market then collapsed sharply, and changes in the stock fell to roughly \$50 billion in 1929. Such large changes in the stock are likely to have been associated with similar large changes in the volume of new issuances.

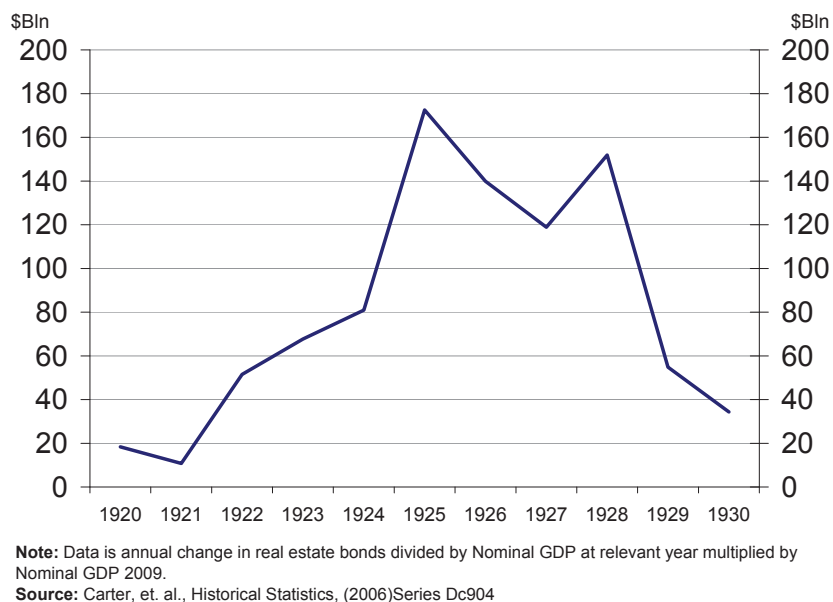


Figure 2: Change in Stock of Real Estate Bonds, 1920-1930

3 Reputation in a Secondary Loan Market Model

We develop a finite horizon model of the secondary loan market and use the model to demonstrate how adverse selection and reputation interact to yield abrupt collapses with increased inefficiency. We show that for every history, the last period of the model has a unique equilibrium which we use to construct equilibria in previous periods. We show that equilibria of the multi period model typically exhibit dynamic coordination problems in the sense that for a wide range of parameters, the game has multiple equilibria. Although reputation is always valued, loan originators choose different actions across the different equilibria based on the different inferences future buyers draw from the current actions of

originators.

3.1 Static Model: A Unique Equilibrium

We start with a static model which should be interpreted as describing the last period of a finite horizon model. We show that the static model has a unique equilibrium in which the equilibrium outcomes depend on the informed originator's reputation.

Agents. The model has three types of agents: a loan originator referred to as a bank, a continuum of buyers, and a continuum of lenders. All agents are risk neutral.

The bank is endowed with a risky loan indexed by π . The loan can also be thought of more generally as an investment opportunity such as a project, a mortgage, or an asset-backed security. Each loan requires q units of inputs, which represents the loan's size. A loan of type π yields a return of $v = \bar{v}$ if the borrower does not default and a return of $v = \underline{v}$ if the borrower does default. We refer to \underline{v} as the *collateral value* of the loan. The probability that the borrower does not default is denoted by π . For the analysis in this section, we normalize \underline{v} to 0. Later, when we allow for aggregate shocks and introduce our refinement, we will allow \underline{v} to be a random variable, possibly different from zero. We assume that $\pi \in \{\underline{\pi}, \bar{\pi}\}$ with $\underline{\pi} < \bar{\pi}$. We refer to a bank that has a loan of type $\bar{\pi}$ as a *high-quality bank* and one with a loan of type $\underline{\pi}$ as a *low-quality bank*. We assume that $\underline{\pi}\bar{v} \geq q$ so that each loan has positive net present value if sold.

The bank either can sell the loan in a secondary market or can hold the loan. Selling the loan at a price p yields a payoff to the bank of $p - q$. The purchaser of the loan is entitled to the resulting return. If the bank chooses to hold the loan, it must borrow q from lenders to finance the loan and repay $q(1 + r)$ at the end of the period, where r is the within-period interest rate paid to lenders. We allow r to be positive or negative in order to examine the effects of various policy experiments described below. If the bank holds the loan, it is entitled to the return from its projects; however, the bank then incurs a cost of holding the loan, c , in addition to the cost of repaying its debt, $q(1 + r)$.

Besides the quality of its loan, the bank is indexed by a cost type, which represents the costs, relative to the marketplace, that the bank incurs when it holds the loan to maturity. We intend the cost of the loan to represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors. We assume that $c \in \{\underline{c}, \bar{c}\}$ with $\underline{c} < -qr < 0 < \bar{c}$. We refer to a bank of type \bar{c} as a *high-cost bank* and a bank of type \underline{c} as a *low-cost bank*. We normalize the cost of holding and managing the loan for the market to be zero.

Hence, the model has four types of banks: $(\pi, c) \in \{\underline{\pi}, \bar{\pi}\} \times \{\underline{c}, \bar{c}\}$. We refer to the different types of banks, $(\bar{\pi}, \bar{c})$, $(\bar{\pi}, \underline{c})$, $(\underline{\pi}, \bar{c})$, $(\underline{\pi}, \underline{c})$, as HH, HL, LH, LL banks, respectively.

Timing of the Static Game. We formalize the interactions in this economy as an extensive form game with the following timing. First, nature draws the quality and cost types of the bank. Then, buyers simultaneously offer a price, p , to purchase the loan. Finally, the bank sells the loan to one of the buyers or holds the loan to maturity.

We assume that, as perceived by buyers and lenders, the bank has quality type $\bar{\pi}$ with probability μ_2 and quality type $\underline{\pi}$ with probability $1 - \mu_2$. (The subscript 2 on the probability is meant to indicate that these are the beliefs of lenders associated with the second period of our two-period model described below.) Following the work of [Kreps and Wilson \(1982\)](#) and [Milgrom and Roberts \(1982\)](#), we refer to μ_2 as the bank's reputation. Also, buyers believe that the bank has cost type \underline{c} with probability α and cost type \bar{c} with probability $1 - \alpha$. The cost and quality types are independently drawn.

Strategy and Equilibrium. A strategy for the bank consists of a decision of whether to sell or hold its loan as a function of prices offered by buyers, and which buyer to sell to if the bank chooses to sell. Clearly, the bank will choose the buyer offering the highest price if the bank decides to sell, so we suppress this aspect of the bank's strategy. Let $a = 1$ denote the decision of the bank to sell the loan, and let $a = 0$ denote the decision to hold the loan. A strategy for the bank is a function $a(\cdot)$ that maps the highest offered price, p , into

a decision of whether to sell or hold the loan. The payoffs to a type (π, c) bank are given by

$$w_2(a|p, \pi, c) = a(p - q) + (1 - a) [\pi \bar{v} - q(1 + r) - c].$$

A strategy for a buyer consists of the choice of a price to offer a bank for its loan. The payoffs to a buyer with an accepted price p and a strategy $a_2(\cdot|\pi, c)$ for each type of bank is

$$u_2(p|a_2) = E_{\pi, c}[v|a_2(p|\pi, c) = 1] - p.$$

Since buyers move simultaneously, they engage in a form of Bertrand competition, so that the price is equal to the expected return on the loan.

A (pure strategy) Perfect Bayesian Equilibrium is a price p_2 and a strategy for each bank type, $a_2(\cdot|\pi, c)$, such that for all p , each bank type chooses the optimal loan decision and buyers offer the highest price that yields a payoff of 0; i.e., $p_2 = \max\{p|u_2(p|r, a_2) = 0\}$.

With full information, when the bank's type is known by buyers, under the assumption that $\underline{c} < -qr$, it is easy to show that the high cost bank sells its loan and a low cost bank holds its loan. In particular, the decision of whether to sell or hold the loan does not depend on the quality type of the bank. The reason is that the return on the loan, ignoring the holding cost, is the same for both the bank and the buyers. Notice that the equilibrium allocation under full information is ex post efficient. Low-cost banks have a comparative advantage (over buyers) in holding loans to maturity, while buyers have a comparative advantage over high-cost banks. The full information equilibrium allocates loans to agents with a comparative advantage in holding and managing the loan.

Next, we show that the private information model has a unique equilibrium. For expositional simplicity, we focus on the decisions of the high-quality, high-cost bank (HH) and restrict the strategy sets of the low-cost bank as well as the low-quality, high-cost bank (LH). Specifically, we assume that HL and LL banks hold their loans and the LH bank sells its loan. In [Appendix B](#), we show that if \underline{c} is sufficiently negative, the assumed strategies for these three types of banks are indeed optimal.

To construct our (unique) equilibrium, we show that the HH bank sells its loan for

reputation levels higher than a critical threshold, μ_2^* , and holds its loan otherwise. To see this result, note that facing price p , the HH bank sells its loan if and only if

$$p - q \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}. \quad (1)$$

Bertrand competition among buyers implies that buyers must make zero profits so that any candidate equilibrium price at which the HH bank sells must satisfy the following equality:

$$\hat{p}(\mu_2) := [\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\bar{v}. \quad (2)$$

To determine the threshold, μ_2^* , above which the equilibrium involves the HH selling its loan, substitute from (2) into (1) and find the thresholds for μ_2 at which (1) holds with equality.

We obtain

$$\mu_2^* = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})\bar{v}}. \quad (3)$$

Above this threshold, the bank sells and below it the bank holds the loan. To establish uniqueness of equilibrium, note that below μ_2^* , the bank clearly holds the loan. Below μ_2^* , the equilibrium price must satisfy

$$p = \underline{\pi}\bar{v}. \quad (4)$$

To see that when $\mu_2 \geq \mu_2^*$ the equilibrium must have the HH bank selling, note that if $\mu_2 \geq \mu_2^*$ and the offered price is below $\hat{p}(\mu_2)$, one of the buyers can deviate and offer a price just below $\hat{p}(\mu_2)$ and induce the HH bank to sell. This deviation yields strictly positive profits. For reputation levels below μ_2^* , the HH bank holds even if offered $\hat{p}(\mu_2)$.

We use this characterization of the static equilibrium to calculate the payoffs associated with a given level of reputation μ_2 at the beginning of the period before a bank's cost type is realized. These payoff calculations play a crucial role in our dynamic game. They are given by

$$V_2(\mu_2) = \begin{cases} \bar{\pi}\bar{v} - q(1 + r) - Ec, & \mu_2 < \mu_2^* \\ (1 - \alpha) \{[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\bar{v} - q\} + \alpha[\bar{\pi}\bar{v} - q(1 + r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases} \quad (5)$$

Similarly, we can define the value of the equilibrium for a low-quality bank:

$$W_2(\mu_2) = \begin{cases} (1 - \alpha) [\underline{\pi}\bar{v} - q] + \alpha[\underline{\pi}\bar{v} - q(1 + r) - \underline{c}], & \mu_2 < \mu_2^* \\ (1 - \alpha) \{[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\bar{v} - q\} + \alpha[\underline{\pi}\bar{v} - q(1 + r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases}$$

It is clear that V_2 is weakly increasing and convex in μ_2 . We have proved the following proposition.

Proposition 1 *If $\underline{\pi}\bar{v} > q$ and $qr + \bar{c} > 0$, then for any $\mu \in [0, 1]$, the static model has a unique equilibrium. Let μ_2^* be defined by (3). For $\mu_2 < \mu_2^*$, the HH bank holds its loan and for $\mu_2 \geq \mu_2^*$, the HH bank sells its loan.*

Note that we have modeled buyers as behaving strategically. This modeling choice plays an important role in ensuring that the static game has a unique equilibrium. Suppose that rather than modeling buyers as behaving strategically, we had instead simply required that market prices satisfy a zero profit condition. One rationale for this requirement is that buyers take prices as given and choose how many loans to buy as in a competitive equilibrium. It is easy to show that with this requirement the economy has multiple equilibria in the static game if $\mu_2 \geq \mu_2^*$. One of these equilibria corresponds to the unique equilibrium of our game. In the other equilibrium, the buyers offer a price of $\underline{\pi}\bar{v}$. At this offered price, the HH bank holds its loan and only the low-quality, high-cost bank sells its loan. We find multiplicity of this kind unattractive in our model because obvious bilateral gains to trade are not being exploited. Each of the buyers has a strong incentive to offer a price slightly below $[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\bar{v}$. At this offered price, the HH bank strictly prefers to sell, and the buyer making such an offer makes strictly positive profits. In our formulation, with strategic behavior by the buyers, this low price outcome cannot be an equilibrium.

Although we prefer our strategic formulation, we emphasize that our results that reputational incentives induce multiplicity do not rely on the static game having a unique equilibrium. We chose a formulation in which the static game has a unique equilibrium in order to argue that reputational incentives by themselves can induce multiplicity.

3.2 Two-Period Benchmark Model

Consider now a two-period repetition of our static game in which the bank's quality type is the same in both periods. We assume that the bank's second-period payoffs are discounted at rate β . In period 1, a continuum of buyers who are present in the market for only one period choose to offer prices for loans sold in that period. In period 2, a new set of buyers each offer prices for loans sold in that period. This new set of buyers observes whether the bank sold or held its loan in the previous period, and, if the bank sold its loan, buyers observe the realized value of the loan. If the loan is held, we assume that period 2 buyers do not observe the realized value of the loan.

The assumption that period 2 buyers receive no information about the realized value of the loan is convenient but not essential in generating multiplicity of equilibria. Our multiplicity results go through if period 2 buyers receive a sufficiently noisy signal of the realized value of the loan. The critical assumption in generating multiplicity is that the market receives more precise information about the value of the loan if it is sold than if it is held. We think this assumption is natural in that market participants typically receive information only about aggregate returns to bank portfolios and do not receive information on the returns to specific assets. Banks typically hold a variety of assets in their portfolios, some of which can be securitized and others which cannot. In such a setting, the information investors receive about returns on specific assets is typically not as precise if a bank holds an asset as it would be if the bank sold the asset.

The timing of the game is an extension of that described in the static game. As in that game, at the beginning of period 1, nature draws the bank's quality and cost type. We assume that the bank's quality type is fixed for both periods. At the beginning of period 2, nature draws a new cost type for the bank. In any period, the bank's quality and cost types are unknown to buyers. The timing within each period is the same as in the static game. We also assume that the returns to successful loans, $v = \bar{v}$, and to unsuccessful loans, $v = 0$, are the same in both periods.

In order to define an equilibrium in this repeated game, we describe how second-period buyers update their beliefs about the bank's type based on observations from period 1. To do so, we let the public history at the beginning of period 2 be denoted by θ_1 , where $\theta_1 \in \{h, s0, s\bar{v}\}$ where $\theta_1 = h$ denotes that the bank held its loan in period 1, $\theta_1 = s0$ denotes that the bank sold its loan and the loan paid off $v = 0$, and $\theta_1 = s\bar{v}$ denotes that the bank sold its loan and the loan paid off $v = \bar{v}$.

As in the static game, we focus on the strategic incentives of the HH bank and restrict the strategy sets of the low-cost bank as well as the low-quality, high-cost bank. Specifically, we assume that the low-cost bank must hold its loan and the LH bank must sell its loan. A strategy for the high-quality, high-cost bank is now given by a pair of functions, $a_1(p_1)$ representing the decision in period 1 and $a_2(p_2, \theta_1)$ representing the loan decision in period 2 if the bank realizes a high cost in period 2, as a function of offered prices.

Consider next how the buyers in the last period update their beliefs about the bank's type. This updating depends through Bayes' rule on the prior belief of the buyers, the loan decision of the bank and the loan return realization if the bank sold, as well as on the first-period strategies chosen by the HH bank and period 1 buyers. From Bayes' rule, these posterior probabilities are given by

$$\mu_2(\mu_1, \theta_1 = h, a_1(\cdot), p_1) = \frac{\mu_1(\alpha + (1 - \alpha)(1 - a_1(p_1)))}{\mu_1(\alpha + (1 - \alpha)(1 - a_1(p_1))) + (1 - \mu_1)\alpha} \quad (6)$$

$$\mu_2(\mu_1, \theta_1 = s\bar{v}, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1 - \alpha)\bar{\pi}}{\mu_1 a_1(p_1)(1 - \alpha)\bar{\pi} + (1 - \mu_1)(1 - \alpha)\underline{\pi}} \quad (7)$$

$$\mu_2(\mu_1, \theta_1 = s0, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1 - \alpha)(1 - \bar{\pi})}{\mu_1 a_1(p_1)(1 - \alpha)(1 - \bar{\pi}) + (1 - \mu_1)(1 - \alpha)(1 - \underline{\pi})}. \quad (8)$$

For notational convenience, we suppress the dependence on strategies and priors and let μ_h denote the posterior associated with the bank holding its loan, and $\mu_{s\bar{v}}$ and μ_{s0} denote the posteriors associated with selling and yielding a high or low return.

Given the updating rules, the period 1 payoffs for the HH bank are given by

$$w_1(a|p) = a [p - q + \beta (\bar{\pi} V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi}) V_2(\mu_{s0}))] \\ + (1 - a) [(\bar{\pi} \bar{v} - q(1 + r) - \bar{c}) + \beta V_2(\mu_h)]$$

where μ_h , $\mu_{s\bar{v}}$, and μ_{s0} are given by equations (6), (7), and (8). Buyers' payoffs associated with an accepted price, p , in period t are given by

$$u_t(p|r, a_t, \mu_t) = \frac{\mu_t(1 - \alpha)a_t(p)\bar{\pi} + (1 - \mu_t)(1 - \alpha)\underline{\pi}\bar{v}}{\mu_t(1 - \alpha)a_t(p) + (1 - \mu_t)(1 - \alpha)} \bar{v} - p.$$

A Perfect Bayesian Equilibrium is a first-period price, p_1 , a first-period loan decision for the high-quality, high-cost bank $a_1(\cdot)$ that maps accepted prices into loan decisions, updating rules $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ that map observations on loan decisions into posterior beliefs, a second-period price, p_2 , that maps second-period beliefs into prices, and a second-period loan decision $a_2(\cdot)$ that maps accepted prices and histories into loan decisions such that (i) for all p , the HH bank chooses the optimal action in period 1 so that $w_1(a_1(p)|p) \geq \max_{a'} w_1(a|p)$, (ii) for all p , the HH bank chooses the optimal action in period 2 so that $w_2(a_1(p)|p) \geq \max_{a'} w_2(a|p)$, (iii) the first-period price, p_1 , satisfies $p_1 \in \max\{p|u_1(p|a_1) = 0\}$, (iv) the second-period price, p_2 , satisfies $p_2 \in \max\{p|u_2(p|a_2) = 0\}$, (v) the updating rules, $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$, satisfy Bayes' rule, namely, (6), (7), and (8).

To show that our model has mutliplicity of equilibria, we begin by showing that the game has two (pure strategy) equilibria when prior beliefs in period 1, μ_1 , are equal to the static threshold, μ_2^* . Continuity of payoffs then implies that the game has two equilibria in an interval around the static threshold.

In one equilibrium, labeled the positive reputational equilibrium, the HH bank chooses to sell its loan in period 1. To see that such a choice is part of an equilibrium, note that in this case, the period 1 price is given by

$$\hat{p}(\mu_2) := [\mu_2 \bar{\pi} + (1 - \mu_2) \underline{\pi}] \bar{v}. \quad (9)$$

Given this price, selling is optimal if the following incentive constraint is satisfied:

$$(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})\bar{v} - q + \beta(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi})V_2(\mu_{s0})) \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c} + \beta V_2(\mu_h) \quad (10)$$

where the posterior beliefs are obtained from (6) through (8) substituting $a_1(p_1) = 1$ so that

$$\mu_h = \mu_1, \quad \mu_{s\bar{v}} = \frac{\mu_1\bar{\pi}}{\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}}, \quad \text{and} \quad \mu_{s0} = \frac{\mu_1(1 - \bar{\pi})}{\mu_1(1 - \bar{\pi}) + (1 - \mu_1)(1 - \underline{\pi})}. \quad (11)$$

Notice from (11) that if a bank holds the loan, the posterior beliefs are unchanged. The reason is that the beliefs of period 2 buyers is that low cost banks of both qualities hold their loans and period 2 buyers receive no information about the return to the loan if it is held.

We show that at μ_2^* , the incentive constraint (10) holds as a strict inequality. Note that using (9), (10) evaluated at μ_2^* can be written as

$$\beta(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi})V_2(\mu_{s0})) \geq \beta V_2(\mu_h)$$

Further, from the updating rules for posterior beliefs, (11), $\mu_{s\bar{v}} > \mu_h = \mu_2^* > \mu_{s0}$. Hence, using the second period payoffs from (5), it follows that $V_2(\mu_{s\bar{v}}) > V_2(\mu_{s0}) = V_2(\mu_h)$ so that (10) holds as a strict inequality at μ_2^* . Thus, the HH bank has a strict incentive to sell. The reason for this strict incentive is that the worst outcome associated with selling is that the loan is unsuccessful and this payoff is the same as that associated with holding the loan. If the loan is succesful, the HH bank's payoff is strictly higher than the payoff to holding the loan. Not surprisingly, this result suggests that for reputation levels in an interval around μ_2^* , given beliefs that the HH bank sells in period 1, the bank finds it optimal to do so, and hence the model has a positive equilibrium for an interval around μ_2^* .

In the second type of equilibrium, labeled the negative reputational equilibrium, the HH bank chooses to hold its loan. In this case the equilibrium price is given by $\underline{\pi}\bar{v}$ using (4). A bank holds its loan if and only if

$$(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})\bar{v} - q + \beta(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi})V_2(\mu_{s0})) \leq \bar{\pi}\bar{v} - q(1 + r) - \bar{c} + \beta V_2(\mu_h), \quad (12)$$

where

$$\mu_h = \frac{\mu_1}{\mu_1 + (1 - \mu_1)\alpha}, \text{ and } \mu_{s\bar{v}} = \mu_{s0} = 0. \quad (13)$$

Note that in the negative equilibrium, only low quality banks sell, and uninformed agents assign a posterior reputation of zero if the bank sells and rationally disregard the information from the realized value of the loans. Note also that if a bank chooses to hold its loan, buyers perceive that it is more likely to be a high quality bank and the posterior belief rises.

The argument that at μ_2^* , the incentive constraint (12) holds as a strict inequality parallels that of the positive equilibrium. Using the updating rules in (13), it follows that $\mu_{s\bar{v}} = \mu_{s0} < \mu_2^* < \mu_h$. Hence, using the second period payoffs given in (5), it follows that $V_2(\mu_{s\bar{v}}) = V_2(\mu_{s0}) = V_2(\mu_2^*) < V_2(\mu_h)$ so that the incentive constraint (12) holds as a strict inequality at μ_2^* . This result suggests that for reputation levels in an interval around μ_2^* , given beliefs that the HH bank holds in period 1, the bank finds it optimal to do so, and hence the model has a negative equilibrium.

Continuity of payoffs implies that (10) and (12) hold as strict inequalities in some interval of prior beliefs around μ_2^* so that our model has multiple equilibria in this interval. In [Appendix B](#), we show that our model has unique equilibria outside this interval under the assumption that $\beta(1 - \alpha) \leq 1$.

Proposition 2 (*Multiplicity of Equilibria*) *Suppose $0 < \mu_2^* < 1$. Then, there exist $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that if $\mu_1 \in [\underline{\mu}, \bar{\mu}]$, the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan in the first period.*

In the proposition, we have shown that introducing reputation as a device for mitigating lemons problems results in equilibrium multiplicity, that is, reputation can be both a blessing and a curse. The game has a positive reputational equilibrium in which, encouraged by reputational incentives, banks with a high-quality asset sell their asset. In this equilibrium, reputation helps sustain market activity in a market that would be illiquid without reputational incentives. The game also has a negative reputational equilibrium in which

reputational incentives discourage selling and banks with a high-quality asset hold on to their asset. In this equilibrium, reputation helps depress market activity in a market that would be liquid without reputational incentives.

In terms of the relationship to the literature on reputation, our model nests features of the model in [Mailath and Samuelson \(2001\)](#) and [Ordoñez \(2009\)](#) as well as that of [Ely and Välimäki \(2003\)](#). In [Mailath and Samuelson \(2001\)](#) and [Ordoñez \(2009\)](#), strategic types are good and want to separate from nonstrategic types, although in [Mailath and Samuelson \(2001\)](#) reputation generally fails to deliver this type of equilibria. Nevertheless, in their environments, there is no long-run reputational loss from good behavior. [Ely and Välimäki \(2003\)](#) share the property that strategic types are good and want to separate; however, the structure of learning is such that good behavior never implies long-run positive reputational gains, and therefore reputational incentives exacerbate bad behavior in equilibrium.

3.3 Sudden Collapses and Increased Inefficiency

In this section, we study the efficiency properties of the positive and negative reputational equilibria. We provide sufficient conditions under which the positive reputational equilibrium Pareto dominates the negative reputational equilibrium in the sense of interim utility (see [Holmstrom and Myerson \(1983\)](#)), and sufficient conditions under which the positive equilibrium dominates the negative equilibrium in the sense of ex ante utility. In this sense, sudden collapses of trade volume in our model due to switches between equilibria are associated with increased inefficiency.

In order to develop these sufficient conditions, suppose that $\mu_1 \in [\underline{\mu}, \mu_2^*]$. Consider the welfare of the HH bank. Let μ_h^n denote the posterior beliefs in the negative equilibrium, conditional on future buyers observing a hold decision by a bank in the first period. Suppose μ_h^n is less than the static cutoff, μ_2^* . Using the form of second period payoffs (5), it follows that the present value of payoffs in the negative equilibrium is given by the right side of the incentive constraint in the positive equilibrium, (10). The left side of (10) is the equilibrium

payoff in the positive equilibrium. Clearly, the payoff for the HH bank is higher in the positive equilibrium than it is in the negative equilibrium.

Consider next the low quality, high cost, or LH bank. This bank sells in both equilibria in the first period, but receives a higher price in the positive equilibrium than in the negative equilibrium. In terms of continuation values, note that the reputation level in the negative equilibrium falls to zero and is positive in the positive equilibrium. It follows that this bank is strictly better off in the positive equilibrium than in the negative equilibrium. Since $\mu_h^n \leq \mu_2^*$, the continuation values for low-cost types are the same in the two equilibria, and since they are holding in the first period, their utility levels are the same. In [Appendix B](#), we show that $\mu_2^* < (\beta\bar{\pi} - \underline{\pi}/(\bar{\pi}\alpha - \underline{\pi})) / (1 + \beta\bar{\pi}(1 - \alpha))$ and μ_1 close to $\underline{\mu}$ is a sufficient condition for μ_h^n to be less than or equal to μ_2^* . Since buyers make zero profits in both equilibria, we have established the following proposition.

Proposition 3 *Suppose that $0 < \mu_2^* < (\beta\bar{\pi} - \underline{\pi}/(\bar{\pi}\alpha - \underline{\pi})) / (1 + \beta\bar{\pi}(1 - \alpha))$ and that $\mu_2^* < 1$. Then for all μ_1 in some neighborhood of $\underline{\mu}$, the utility level for each type of bank and the buyers in the positive equilibrium is at least as large as the utility level for the corresponding type of bank and the buyers in the negative equilibrium.*

If $\mu_h^n > \mu_2^*$, one can show that the utility level of the low-cost types is lower in the positive reputational equilibrium than in the negative reputational equilibria. Hence, the two equilibria are not comparable in interim utility terms. However, under appropriate sufficient conditions, the positive equilibrium yields a higher ex ante utility than the negative equilibrium. Consider the allocations in the two equilibria in the first period. The only difference in allocations is that in the positive equilibrium the high-quality, high-cost type sells, whereas in the negative equilibrium this type holds. Thus, the difference in ex ante utility (or social surplus) in the first period between the two equilibria is given by $(1 - \alpha)\mu(qr + \bar{c})$. Clearly, first-period utility is higher in the positive equilibrium than in the negative equilibrium. However, in the second period social surplus is higher in the negative

equilibrium than in the positive equilibrium because the high-cost types always sell in the negative equilibrium, whereas in the positive equilibrium they hold the asset some fraction of the time – when the signal quality is bad in the first period or after a hold decision in the first period. Therefore, the change in social surplus in the second period is given by $-\mu(1-\alpha)((1-\alpha)(1-\bar{\pi})+\alpha)(qr+\bar{c})$. Thus, the overall change in the social surplus is given by

$$\mu(1-\alpha)(1-\beta(1-\bar{\pi}(1-\alpha)))(qr+\bar{c}).$$

Clearly, this overall change is positive if and only if $\beta(1-\bar{\pi}(1-\alpha)) < 1$. We have established the following proposition.

Proposition 4 *Suppose that $\beta(1-\bar{\pi}(1-\alpha)) < 1$. Then the ex-ante utility of the bank is higher in the positive reputational equilibrium than in the negative reputational equilibrium and the ex-ante utility of the buyers is the same in the two equilibria.*

4 Aggregate Shocks and Uniqueness

In this section, we show that with aggregate shocks to collateral values which are imperfectly observed, our model has a unique equilibrium. In the model, small fluctuations in collateral values in a critical region lead to sudden collapses in the volume of trade and fluctuations outside the critical region lead to insignificant changes in the volume of trade.

Adding aggregate shocks with imperfect observability ensures that our model has a unique equilibrium and is, in this sense, a type of refinement. This refinement is in the spirit of the literature on equilibrium selection in static coordination games (see, for example, Carlsson and van Damme (1993), Morris and Shin (2003)). One reason for using such a refinement is to compare outcomes under various policies. Uniqueness is desirable because such comparison is difficult in models with multiple equilibria. Furthermore, we want to develop a well-defined notion of fragility. In many macroeconomic environments with multiple equilibria, small shocks to the environment can cause sudden changes in behavior. Without a

selection device, multiplicity leads to a lack of discipline on how equilibrium behavior changes in response to shocks. We impose discipline by adapting techniques from the literature on coordination games.

We assume that the collateral value, \underline{v} , is affected by an aggregate shock common for all banks. One example of the situation in which collateral values are subject to aggregate shocks is a mortgage on a residential or a commercial property. The value of real estate is often subject to aggregate shocks.

Consider the following model with aggregate shocks and imperfect observability. In each period $t = 1, 2$, an aggregate shock $\underline{v}_t \sim F_t(\underline{v}_t)$ is drawn. These shocks are drawn independently across periods. Banks and buyers at the beginning of each period observe a noisy signal of \underline{v}_t given by $v_t = \underline{v}_t + \sigma \varepsilon_t$, where $\varepsilon_t \sim G(\varepsilon_t)$ with $E[\varepsilon_t] = 0$ is i.i.d. across periods. When $\sigma > 0$ the aggregate shock is imperfectly observed. We assume that F_t and G have full support over \mathbb{R} .

We assume that the distributions F_1 and G satisfy a monotone likelihood property. To develop this property note that, when $\sigma > 0$, the updating rules for the signal of the aggregate shock are given by

$$\Pr(v_1 \leq \hat{v}_1 | \underline{v}_1) = \Pr(\underline{v}_1 + \sigma \varepsilon_1 \leq \hat{v}_1) = G\left(\frac{\hat{v}_1 - \underline{v}_1}{\sigma}\right)$$

$$\Pr(\underline{v}_1 \leq \hat{v}_1 | v_1) = \frac{\int_{-\infty}^{\hat{v}_1} f_1(v) g\left(\frac{v_1 - v}{\sigma}\right) dv}{\int_{-\infty}^{\infty} f_1(v) g\left(\frac{v_1 - v}{\sigma}\right) dv} = H(\hat{v}_1 | v_1)$$

Assumption 1 (*Monotone Likelihood Ratio*) *The posterior belief function $H(\underline{v}_1 | v_1)$ is a decreasing function of v_1 .*

This assumption implies that when the signal, v_1 , about the shock is high, the value of the shock, \underline{v}_1 , is likely to be high. Straightforward algebra can be used to show that this assumption is satisfied if a monotone likelihood ratio property on g holds, namely, that for any $v_1 > v'_1$, $g(v_1 - \underline{v}_1)/g(v'_1 - \underline{v}_1)$ is increasing in \underline{v}_1 .

The timing of the game is as follows: (i) At the beginning of each period t , agents observe the aggregate shock in the previous period \underline{v}_{t-1} . Buyers do not observe previous

period signals v_{t-1} or the market price p_{t-1} . (We believe that our uniqueness result goes through if future buyers receive a noisy signal about previous prices.), (ii) The new aggregate state \underline{v}_t is drawn, the bank and current period buyers do not observe the current state, \underline{v}_t , but they do observe the noisy signal, v_t , (iii) Buyers offer prices, (iv) The bank decides whether to sell or hold.

With aggregate shocks and perfect observability, $\sigma = 0$, it is immediate that a version of Proposition 2 applies and the two period model has multiple equilibria.

To establish uniqueness in our two period model with imperfect observability we begin from the last period. We will show that in the last period, the unique equilibrium is characterized by a cutoff threshold $\mu_2^*(v_2)$ such that banks with reputation levels above $\mu_2^*(v_2)$ sell their loans and banks below this threshold hold their loans and a fall in v_2 raises $\mu_2^*(v_2)$. In this sense, a fall in collateral values worsens the adverse selection problem. To see this result, note that an HH bank sells its loan if and only if

$$\hat{p}(\mu_2; v_2) - q \geq \bar{\pi}\bar{v} + (1 - \bar{\pi})E[\underline{v}_2|v_2] - q(1 + r) - \bar{c}, \quad (14)$$

where

$$\hat{p}(\mu_2; v_2) := [\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\bar{v} + [\mu_2(1 - \bar{\pi}) + (1 - \mu_2)(1 - \underline{\pi})]E[\underline{v}_2|v_2]. \quad (15)$$

Substituting for $\hat{p}(\mu_2; v_2)$ from (15) into (14) and noting that $E[\underline{v}_2|v_2] = v_2$, we obtain that the threshold reputation at which the HH bank is just indifferent between holding and selling is given by

$$\mu_2^*(v_2) = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - v_2)} \quad (16)$$

whenever the right hand side of (16) is between zero and one and at the appropriate extreme points otherwise. Clearly $\mu_2^*(v_2)$ is decreasing in v_2 . We summarize this discussion in the following proposition.

Proposition 5 *In the second period, given a reputation level μ_2 and a default value signal v_2 , there is a unique equilibrium outcome in which the HH bank's decision is to sell if $\mu_2 \geq \mu_2^*(v_2)$*

and to hold otherwise, where

$$\mu_2^*(v_2) = \max \left\{ \min \left\{ 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - v_2)}, 1 \right\}, 0 \right\}.$$

Given this characterization of the second period equilibrium, we can calculate the payoff to the HH bank before the aggregate shock (as well as the second period signal) or the cost type is realized for every value of reputation at the beginning of the second period. These payoffs are given by

$$V_2(\mu_2) = \int \int \hat{V}_2(\mu_2, v_2) dG \left(\frac{v_2 - \underline{v}_2}{\sigma} \right) dF_2(\underline{v}_2) \quad (17)$$

where

$$\hat{V}_2(\mu_2, v_2) = \alpha [\bar{\pi}\bar{v} - q(1+r) - \underline{c}] + (1-\alpha) \max\{\hat{p}(\mu_2; v_2) - q, \bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}\}.$$

Next, we use the characterization of the payoffs given in (17) to prove that the two period model has a unique equilibrium. Proving that the perturbed game has a unique equilibrium is easiest when F_1 is an improper uniform distribution, $U[-\infty, \infty]$. In Section 4.2, we prove uniqueness as $\sigma \rightarrow 0$, when F_1 is a proper distribution.

We have the following proposition:

Proposition 6 *For each $\sigma > 0$ and $V_2(\mu_2)$ given by (17), the game with uniform improper priors has a unique equilibrium in which in period 1, HH bank's action is characterized by a cutoff $v_1^*(\sigma) \in \mathbb{R}$ above which the HH bank sells and below which the HH bank holds.*

We prove this proposition using a method similar to that in Carlsson and van Damme (1993). We begin by restricting attention to switching strategies in which the bank sells for all default values above a threshold and holds for all default values below that threshold. We show that the game has a unique equilibrium in switching strategies. We then prove that the equilibrium switching strategy is the only strategy that survives iterated elimination of strictly dominant strategies so that we have a unique equilibrium.

The intuition for the iterated elimination argument is as follows. Note that we can define equilibrium as a strategy for the bank in period 1, and a belief – about the bank's action

in period 1 – by period 2 buyers used for Bayesian updating. In equilibrium beliefs have to coincide with strategies. Obviously reputational incentives depend on future buyers’ beliefs. When v_1 is very large, independent of future buyers’ beliefs, an HH bank sells the asset. Similarly, when v_1 is very low, an HH bank holds on to the asset, independent of future beliefs. This argument establishes two bounds $\hat{v}^1 > \tilde{v}^1$, such that any equilibrium strategy must prescribe a sale for v_1 higher than \hat{v}^1 and holding for v_1 lower than \tilde{v}^1 . This result means that the set of beliefs by future buyers have to satisfy the same property. Limiting the set of beliefs puts tighter upper and lower bounds on reputational incentives, which in turn implies new bounds $\hat{v}^2 > \tilde{v}^2$. We show that iterating in this manner implies that the bounds \hat{v}^n and \tilde{v}^n converge to a common limit.

Here we sketch the key steps of the proof and leave the details to [Appendix A](#).

4.1 Outline of Proof with Improper Priors

1. Unique Equilibrium in Switching Strategies: We begin by restricting attention to switching strategies of the form:

$$d_k(v_1) = \begin{cases} 1 & v_1 \geq k \\ 0 & v_1 < k, \end{cases}$$

where k represents the switching point. We characterize the best response of the HH bank when future buyers use d_k to form their posteriors over the bank’s type. To do so, we use Bayes rule. Consider an arbitrary belief $\hat{a}_1(\cdot)$ by period 2 buyers about the HH bank’s period 1 action. Based on the observed history and signal \underline{v}_1 , Bayes rule implies the following updating formulas:

$$\begin{aligned} \mu_{sg}(\underline{v}_1; \hat{a}_1) &= \frac{\mu_1 \bar{\pi} \int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right)}{\mu_1 \bar{\pi} \int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) + (1 - \mu_1) \bar{\pi}} \\ \mu_{sd}(\underline{v}_1; \hat{a}_1) &= \frac{\mu_1 (1 - \bar{\pi}) \int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right)}{\mu_1 (1 - \bar{\pi}) \int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) + (1 - \mu_1) (1 - \bar{\pi})} \\ \mu_h(\underline{v}_1; \hat{a}_1) &= \frac{\mu_1 [(1 - \alpha) \int [1 - \hat{a}_1(v_1)] dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) + \alpha]}{\mu_1 [(1 - \alpha) \int [1 - \hat{a}_1(v_1)] dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) + \alpha] + (1 - \mu_1) \alpha}. \end{aligned}$$

For switching strategies, these formulas simplify to

$$\begin{aligned}
\mu_{sg}(\underline{v}_1; d_k) &= \frac{\mu_1 \bar{\pi} \left[1 - G\left(\frac{k - \underline{v}_1}{\sigma}\right) \right]}{\mu_1 \bar{\pi} \left[1 - G\left(\frac{k - \underline{v}_1}{\sigma}\right) \right] + (1 - \mu_1) \underline{\pi}} \\
\mu_{sd}(\underline{v}_1; d_k) &= \frac{\mu_1 (1 - \bar{\pi}) \left[1 - G\left(\frac{k - \underline{v}_1}{\sigma}\right) \right]}{\mu_1 (1 - \bar{\pi}) \left[1 - G\left(\frac{k - \underline{v}_1}{\sigma}\right) \right] + (1 - \mu_1) (1 - \underline{\pi})} \\
\mu_h(\underline{v}_1; d_k) &= \frac{\mu_1 \left[(1 - \alpha) G\left(\frac{k - \underline{v}_1}{\sigma}\right) + \alpha \right]}{\mu_1 \left[(1 - \alpha) G\left(\frac{k - \underline{v}_1}{\sigma}\right) + \alpha \right] + (1 - \mu_1) \alpha}.
\end{aligned} \tag{18}$$

Next, given any belief \hat{a}_1 and noting that with improper priors $H(\underline{v}_1 | v_1) = G\left(\frac{v_1 - \underline{v}_1}{\sigma}\right)$, we define the gain from reputation as

$$\Delta(v_1; \hat{a}_1) = \beta \int \left[\bar{\pi} V_2(\mu_{sg}(\underline{v}_1; \hat{a}_1)) + (1 - \bar{\pi}) V_2(\mu_{sd}(\underline{v}_1; \hat{a}_1)) - V_2(\mu_h(\underline{v}_1; \hat{a}_1)) \right] dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right). \tag{19}$$

In [Appendix A](#) we prove the following Lemma, which characterizes the gain from reputation for general strategies and switching strategies.

Lemma 1 *The gain from reputation $\Delta(v_1; \hat{a}_1)$ is uniformly bounded and strictly increasing in \hat{a}_1 according to a point-wise ordering on beliefs. In particular, if \hat{a}_1 is a switching strategy, d_k , then $\Delta(v_1; d_k)$ is strictly decreasing in k . Moreover, when \hat{a}_1 is a switching strategy, $\Delta(v_1; \hat{a}_1)$ is strictly increasing in v_1 .*

Facing a switching strategy belief of future buyers, d_k , clearly, the HH bank sells if and only if

$$\hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_k) \geq \bar{\pi} \bar{v} + (1 - \bar{\pi}) v_1 - q(1 + r) - \bar{c}. \tag{20}$$

Note that the value of selling, given by the left side of (20), is increasing in v_1 and its partial derivative with respect to v_1 is at least the derivative of $\hat{p}(\mu_1; v_1)$, given by $\mu_1(1 - \bar{\pi}) + (1 - \mu_1)\underline{\pi}$. The value of holding, given by the right side of (20), is increasing in v_1 and its derivative is $1 - \bar{\pi}$. Since the derivative for the value of selling is greater than the value of holding, there

exists a unique solution, $b(k)$, that solves the equation

$$\hat{p}(\mu_1; b(k)) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} + (1 - \bar{\pi})b(k) - q(1 + r) - \bar{c}. \quad (21)$$

Hence, the best response of the HH bank to a switching strategy belief of future buyers, d_k , is a switching strategy, $d_{b(k)}$, in which the bank sells for all returns above $b(k)$ and holds for all return values below $b(k)$. With some abuse of terminology, we refer to $b(k)$ as the *best response function*. An equilibrium in switching strategies must be a fixed point of the above equation, so an equilibrium switching point, k^* , satisfies

$$\hat{p}(\mu_1; k^*) - q + \Delta(k^*; d_{k^*}) = \bar{\pi}\bar{v} + (1 - \bar{\pi})k^* - q(1 + r) - \bar{c}.$$

In [Appendix A](#), we prove the following lemma.

Lemma 2 *The best response function $b(k)$ has a unique fixed point k^* which is globally stable.*

Hence, the game with switching strategies has a unique equilibrium.

2. Restriction to Switching Strategies Is without Loss of Generality: We follow [Morris and Shin \(2003\)](#) in showing that the restriction to switching strategies is without loss of generality. We do so by showing that regardless of future buyers' belief functions, the bank has a dominant strategy for extreme values of default values. Consider two numbers $\hat{v} < \tilde{v}$. We define an *extreme monotone strategy* to be a strategy that calls for selling when $v_1 \geq \tilde{v}$ and holding for $v_1 \leq \hat{v}$. We define $A_{\hat{v}, \tilde{v}}$ to be the set of such strategies. Notice that $A_{-\infty, \infty}$ is the set of all strategies. Define the best response set operator on a subset of beliefs, A , as

$$BR(A) = \{a_1 | \exists \hat{a}_1 \ni a_1(v_1) = 1 \Leftrightarrow \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1 - \bar{\pi})v_1 - q(1 + r) - \bar{c}\}.$$

We show that there exist bounds $\hat{v}^0 < \tilde{v}^0$ such that the HH bank holds for $v_1 \leq \hat{v}^0$ and it

sells the asset for $v_1 \geq \tilde{v}^0$, independent of future buyers' belief function \hat{a}_1 . That is,

$$\forall \hat{a}_1, v_1 \geq \tilde{v}^0; \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1 - \bar{\pi})v_1 - q(1 + r) - \bar{c} \quad (22)$$

$$\forall \hat{a}_1, v_1 \leq \hat{v}^0; \hat{p}(\mu_1; v_1) - q - q + \Delta(v_1; \hat{a}_1) \leq \bar{\pi}\bar{v} + (1 - \bar{\pi})v_1 - q(1 + r) - \bar{c}.$$

Using the result from Lemma (1) that $\Delta(v_1; \hat{a}_1)$ is uniformly bounded in (22), it follows that these bounds exist. We have established that any equilibrium strategy must be an extreme monotone strategy with cutoffs $\hat{v}^0 < \tilde{v}^0$. That is,

$$BR(A_{-\infty, \infty}) \subseteq A_{\hat{v}^0, \tilde{v}^0}.$$

Thus, we can restrict attention to extreme monotone strategies without loss of generality.

Next, we show that the best response set operator is decreasing in the sense that it induces a best response set, which is a strict subset of any arbitrary set of extreme monotone beliefs. Repeatedly applying this operator induces a decreasing sequence of sets, which converges to a unique equilibrium.

To show that the best response set operator is decreasing, we show that for any $\hat{v} < \tilde{v}$, $BR(A_{\hat{v}, \tilde{v}}) \subseteq A_{b(\hat{v}), b(\tilde{v})}$. To show the set inclusion, we need to show that the best response a_1 to an arbitrary $\hat{a}_1 \in A_{\hat{v}, \tilde{v}}$ has the property that

$$\forall v_1 < b(\hat{v}), \quad a_1(v_1) = 0, \quad \forall v_1 > b(\tilde{v}), \quad a_1(v_1) = 1.$$

To show that $\forall v_1 < b(\hat{v}), \quad a_1(v_1) = 0$, we have to show that

$$\bar{\pi}\bar{v} + (1 - \bar{\pi})v_1 - q(1 + r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \quad (23)$$

for all $v_1 < b(\hat{v})$. Note that from the definition of the best response function $b(\cdot)$, see equation (21),

$$\bar{\pi}\bar{v} + (1 - \bar{\pi})v_1 - q(1 + r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\hat{v}}). \quad (24)$$

if and only if $v_1 < b(\hat{v})$. Since, by Lemma 1, $\Delta(v_1; \hat{a}_1)$ is increasing in \hat{a}_1 and $d_{\hat{v}}$ is pointwise higher than $\hat{a}_1 - \hat{a}_1 \in A_{\hat{v}, \tilde{v}}$, the right side of (24) is greater than the right side of (23). Thus $\forall v_1 < b(\hat{v}), \quad a_1(v_1) = 0$. A similar argument establishes that $\forall v_1 > b(\tilde{v}), \quad a_1(v_1) = 1$.

We have proved that $BR(A_{\hat{v},\bar{v}}) \subseteq A_{b(\hat{v}),b(\bar{v})}$. Iterated application of the best response operator and applying the set inclusion repeatedly, implies that

$$BR^n(A_{\hat{v},\bar{v}}) \subseteq A_{b^n(\hat{v}),b^n(\bar{v})}$$

Finally, because $b(k)$ has a unique fixed point and is globally stable, $A_{b^n(\hat{v}),b^n(\bar{v})}$ converges to $A_{k^*,k^*} = \{d_{k^*}\}$ so that $BR^n(A_{-\infty,\infty})$ also converges to $\{d_{k^*}\}$.

4.2 Uniqueness Result with Proper Priors

In this section, we provide a characterization of equilibria in the limiting perturbed game with general proper priors. In particular, we prove that in the perturbed game as $\sigma \rightarrow 0$, the set of period 1 equilibrium strategies converges to a unique strategy. We use the method of Laplacian beliefs introduced by [Frankel et al. \(2003\)](#) and reviewed by [Morris and Shin \(2003\)](#) to prove our uniqueness result. In fact, we show that the game described above is equivalent to a game discussed by [Morris and Shin \(2003\)](#). We then use their result to prove the following theorem. The proof is in [Appendix A](#).

Theorem 1 *Given the value function $V_2(\mu_2)$ given by (17), as $\sigma \rightarrow 0$ the set of first period equilibrium strategies in the game with proper priors converges to a unique strategy by the HH bank in which the bank sells if $v_1 \geq v_1^*$ and holds if $v_1 < v_1^*$ where v_1^* satisfies*

$$\hat{p}(\mu_1; v_1^*) - q + \beta \int_0^1 [\bar{\pi} V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi}) V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))] dl = \bar{\pi}\bar{v} + (1 - \bar{\pi})v_1^* - q(1+r) - \bar{c}$$

and

$$\begin{aligned} \hat{\mu}_{sg}(l) &= \frac{\mu_1 \bar{\pi} l}{\mu_1 \bar{\pi} l + (1 - \mu_1) \bar{\pi}} \\ \hat{\mu}_{sd}(l) &= \frac{\mu_1 (1 - \bar{\pi}) l}{\mu_1 (1 - \bar{\pi}) l + (1 - \mu_1) (1 - \bar{\pi})} \\ \hat{\mu}_h(l) &= \frac{\mu_1 [(1 - \alpha)(1 - l) + \alpha]}{\mu_1 [(1 - \alpha)(1 - l) + \alpha] + (1 - \mu_1) \alpha}. \end{aligned}$$

5 The Multi-Period Model

In this section, we extend the model to many periods. The qualitative properties of the model are very similar to the model with two periods. In particular, we show that the game with noisy signals has a unique equilibrium in the limit as the observation error converges to zero.

The extension of the model to multi periods is as follows: time is discrete and $t = 1, \dots, T$, $T < \infty$. The bank's quality type is drawn at the beginning of period 1. The bank's cost type is drawn independently over time and is independent of the quality type. The collateral value \underline{v}_t is drawn from a distribution function $F(\underline{v}_t)$ and is independent across periods. A new set of buyers arrives each period and lives only for that period. The information structure of the game is as in the two-period model in Section 4. In each period before trading occurs, all agents in the economy observe $v_t = \underline{v}_t + \sigma_t \varepsilon_t$ where ε_t is i.i.d. and distributed according to $G(\varepsilon)$. They do not, however, observe \underline{v}_t . Given this information, the agents trade in the market. After the trade, the collateral value \underline{v}_t becomes public information. Previous prices are not observed by current buyers. Based on observables, agents update their beliefs at the end of period t .

In [Appendix A](#), we recursively construct the payoff of the HH bank and its equilibrium strategy and prove the following proposition.

Proposition 7 *Suppose that for some period $t + 1$ and for any μ_{t+1} , the multiperiod model has a unique equilibrium with payoff for the HH bank given by $V_{t+1}(\mu_{t+1})$. If $V_{t+1}(\mu_{t+1})$ is increasing in μ_{t+1} , there is a unique equilibrium strategy in period t as $\sigma_t \rightarrow 0$ for all μ_t . The equilibrium strategy for the HH bank in period t is given by a cutoff strategy in which the HH bank sells if $v_t \geq v_t^*(\mu_t)$ and holds if $v_t < v_t^*(\mu_t)$ where $v_t^*(\mu_t)$ satisfies the following equation*

$$\hat{p}(\mu_t; v_t^*) - q + \beta \int_0^1 [\bar{\pi} V_{t+1}(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi}) V_{t+1}(\hat{\mu}_{sb}(l)) - V_{t+1}(\hat{\mu}_h(l))] dl = \bar{\pi} \bar{v} + (1 - \bar{\pi}) v_t^* - q(1 + r) - \bar{c}.$$

Furthermore, the model has a unique equilibrium in the last period.

This proposition shows that the finite horizon version of the model has a unique equilibrium under the assumption that the value function is increasing in the reputation of the bank. This assumption can be replaced by assumptions on parameter values. One such assumption is that α , the probability that the bank's cost type is low, is sufficiently small. In the numerical examples described below, we found that the value function is increasing in the reputation of the bank for all of the parameter values we studied.

6 Fragility

We think of equilibrium outcomes as *fragile* in two ways. One notion of fragility is simply that the economy has multiple equilibria so that sunspot-like fluctuations can induce changes in outcomes. A second notion of fragility is that small changes in fundamentals induce large changes in aggregate outcomes.

Equilibrium outcomes in our unperturbed game are clearly fragile under the first notion because that game has multiple equilibria. They are also fragile under the second notion if agents in the model coordinate on different equilibria depending on the realization of the fundamentals and if a large mass of agents have reputation levels in the multiplicity region.

Since our perturbed game has a unique equilibrium, it is not fragile under the first notion. We argue that it is fragile under our second notion. In our multi-period model, the history of past outcomes induces dispersion in the reputation levels of different banks. In order for our equilibrium to display fragility under the second notion, we must have that either banks with a wide variety of reputation levels change their actions in the same way in response to aggregate shocks or that the reputation levels of banks cluster close to each other. We conducted a wide variety of numerical exercises and found that the clustering effect is very strong in our model. This clustering effect clearly depends on the details of the history of exogenous shocks. To abstract from these details, we consider the invariant distribution associated with our model and show that this invariant distribution displays clustering. The

invariant distribution is that associated with the infinite horizon limit of our multi-period model. We allow for a small probability of replacement in order to ensure that the invariant distribution is not concentrated at a single point.

Figure 3 displays the cutoff values for each reputation type for the ergodic set associated with the invariant distribution.¹ This ergodic set contains reputation levels between roughly 0.25 and 0.85. For collateral values above the cutoffs shown in Figure 3, banks sell their loans and below the cutoffs banks hold their loans. This figure illustrates that as the collateral value falls, the adverse selection problem worsens in the sense that banks with a wider range of reputations hold their loans. For example, at a collateral value of 5, banks with reputation levels below roughly 0.4 hold their loans and the banks with higher reputation levels sell their loans. At a collateral value of 4, banks with reputation levels below roughly 0.65 hold their loans and banks with higher reputation levels sell their loans. Thus, a fall in collateral values from 5 to 4 induces banks with reputation levels roughly between 0.4 and 0.65 to switch from selling to holding their loans.

Figure 4 displays the invariant distribution of reputation levels for high-quality banks. This figure shows that the invariant distribution displays significant clustering. Roughly 70 percent of high-quality banks have reputation levels between 0.8 and 0.85. Small fluctuations in the default value of loans around the cutoff values for such banks can induce a large mass of banks to alter their behavior.

Figure 5 plots the volume of trade, measured as the fraction of all banks that sell their loans. A decrease in the default value from 1.3 to 1.1 induces a 50 percent decrease in the volume of trade. In this sense, Figure 5 suggests that equilibrium outcomes in our model are fragile under the second notion.

Next we analyze the forces that induce clustering in our model. Bayes' rule implies that $\frac{1}{\mu_t}$ is a martingale. Since $\frac{1}{\mu_t}$ is a convex function, Jensen's inequality implies that the reputation

¹The parameters used in this simulation are the following: $\bar{\pi} = 0.8, \underline{\pi} = 0.3, \bar{v} = 7, \bar{c} = 0.5, \underline{c} = -3, \alpha = 0.15, q = .1, r = 0.5, \beta(1 - \lambda) = .99, \lambda = .4, \mu_0 = .6$, where λ represents the exogenous probability of replacement and μ_0 is the reputation of a newly replaced bank. The distribution of \underline{v} is $N(0, 2)$.

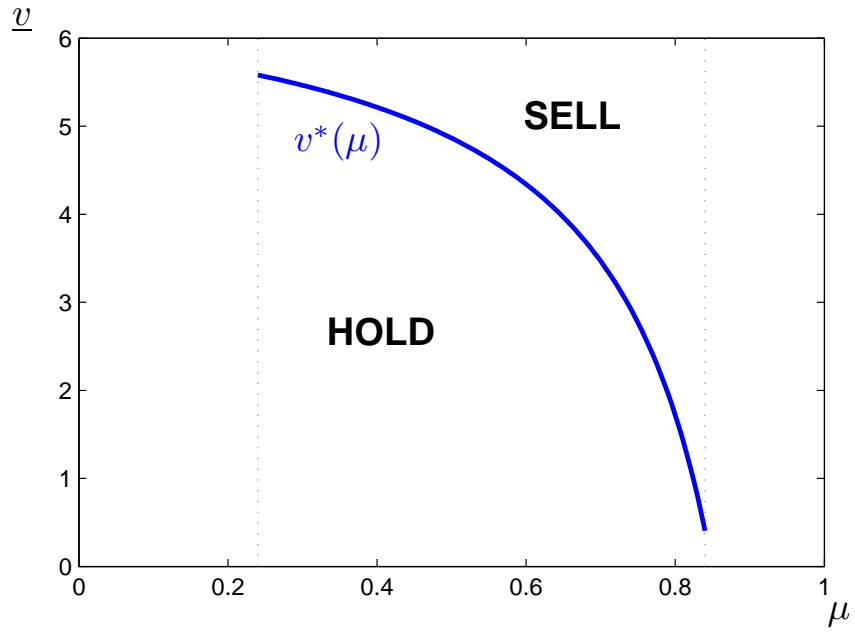


Figure 3: Cutoff Thresholds for High-Quality Banks.

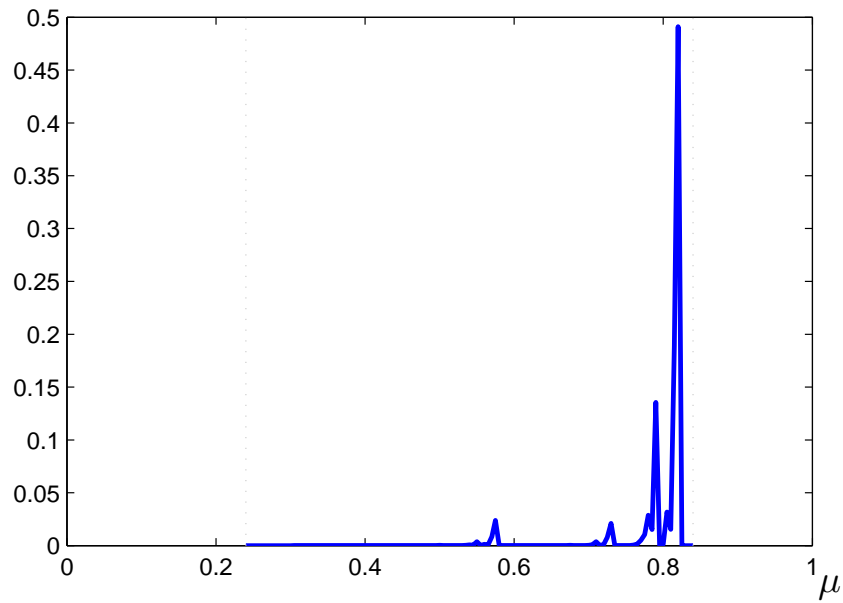


Figure 4: Invariant Distribution of Reputations of High-Quality Banks

of a bank, μ_t , is a submartingale so that μ_t tends to rise. Conditional on a high-quality, high-cost bank holding, the analysis of our equilibrium implies that the reputation of such a bank also rises. These forces imply that the reputation of a high-quality bank displays an upward trend. This upward trend is dampened by replacement. Since all high-quality banks tend to have an upward trend in their reputations, these reputations tend to cluster toward each other.

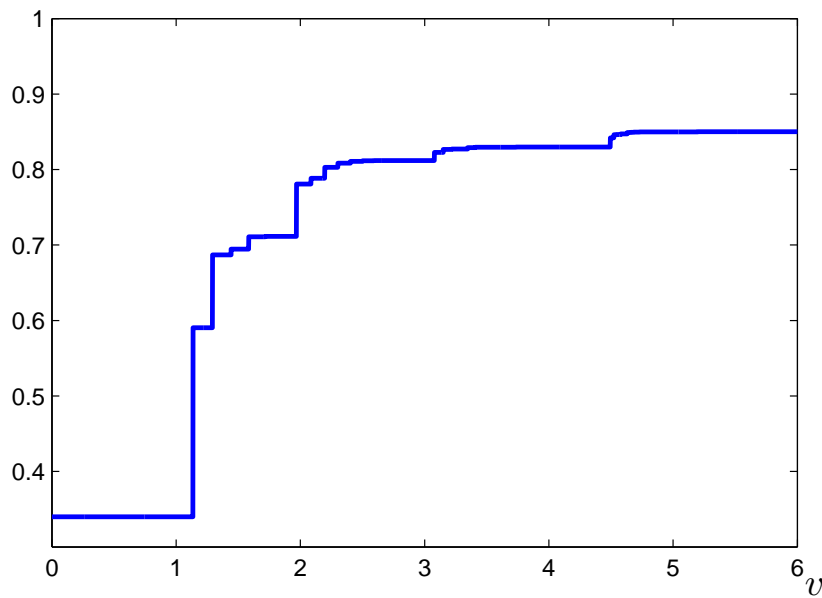


Figure 5: Volume of Trade as a Function of shock to Default Value.

This reasoning suggests that fragility under the second notion does not depend on the particular equilibrium that we have selected. In both the positive and negative reputational equilibria, the reputations of high-quality banks rise over time and tend to cluster together eventually. This clustering tends to make them react in the same way to fluctuations in the default value of the underlying loans. We conjecture that any continuous selection procedure will produce periods of high volumes of new issuances followed by sudden collapses.

We have analyzed the effect of other aggregate shocks in our model. In particular, we allowed the comparative advantage cost, \bar{c} , to be subject to aggregate shocks. In that version of the model, we found that banks with a wide variety of reputations tend to have cutoffs

that are very close to each other. That model displays fragility under our second notion because small fluctuations in holding costs around a critical value induce large changes in actions by banks with a wide variety of reputations. (Details are available upon request.)

7 Policy Exercises

In this section, we use our model to evaluate the effects of various policies intended to remedy problems of credit markets – policies that have been proposed since the 2007 collapse of secondary loan markets in the United States. We focus on the effects of policies in which the government would purchase asset-backed securities at prices above existing market value, such as the Public-Private Partnership plan, as well as on policies that decreased the costs of holding loans to maturity, including changes in the Federal Funds target rate, the Term Asset-Backed Securities Loan Facility (TALF), and increased FDIC insurance.

These policies were motivated by perceived inefficiencies in secondary loan markets. For example, the Treasury Department asserts, in its Fact Sheet dated March 23, 2009, releasing details of a proposed Public-Private Investment Program for Legacy Assets,

Secondary markets have become highly illiquid, and are trading at prices below where they would be in normally functioning markets. ([Treasury Department \(2009\)](#))

Similarly, the Federal Reserve Bank of New York asserts, in a White Paper dated March 3, 2009, making the case for the Term Asset-Backed Securities Loan Facility (TALF),

Nontraditional investors such as hedge funds, which may otherwise be willing to invest in these securities, have been unable to obtain funding from banks and dealers because of a general reluctance to lend. (TALF White Paper 2009)

Note that in our model sudden collapses are associated with increased inefficiency so that our model is consistent with policy makers concerns that the market had become more

inefficient. In this sense, our model is an appropriate starting point for analyzing policies intended to remedy inefficiencies.

We first consider policies in which the government attempts to purchase so-called toxic assets at above-market values. Consider the following government policy in the limiting version of the perturbed game as $\sigma \rightarrow 0$. The government offers to buy the asset at some price p in the first period.

Suppose first that $p \leq \hat{p}(\mu_1; v_1)$. We claim that the unique equilibrium without government is also the unique equilibrium with this government policy. To see this claim, note that the equilibrium in the second period is the same with and without the government policy so that the reputational gains are the same with and without the government policy. Consider the first period and a realization of first-period return $v_1 < v_1^*$. In the game without the government, the HH bank found it optimal not to sell at a price $\hat{p}(\mu_1; v_1)$. Since the reputational gains are the same with and without the government policy, in the game with the government, it is also optimal for the HH not to sell at this price. A similar argument implies that the equilibrium strategy of the HH bank is unchanged for $v_1 > v_1^*$. Thus, this government policy has no effect on the equilibrium strategy of the HH bank. Of course, under this policy, the government ends up buying the asset from low-quality banks. The only effect of this policy is to make transfers to low-quality banks.

Suppose next that the price set by the government, p , is sufficiently larger than $\hat{p}(\mu_1; v_1)$. Then, the HH bank will find it optimal to sell and will enjoy the reputational gain associated with a policy of selling. In this sense, if the government offers a sufficiently high price, it can ensure that reputational incentives work to overcome adverse selection problems. Note, however, that this policy necessarily implies that the government must earn negative profits.

Consider now a policy that reduces interest rates in period 1 and leaves period 2 interest rates unchanged. We begin the analysis with the unperturbed game. Such a policy increases the static payoff in period 1 from holding loans which worsens the static incentives for the HH bank to sell its loan. Specifically, this policy raises both the threshold $\underline{\mu}$ below which banks

find it optimal to hold in the positive reputational equilibrium and the threshold $\bar{\mu}$ below which banks find it optimal to hold their loans in the negative reputational equilibrium. Thus, this policy serves only to aggravate the lemons problem in secondary loans markets.

Consider next a policy under which the government commits to reducing period 2 interest rates but leaves period 1 interest rates unchanged. Obviously, this policy increases incentives for banks to hold their loans in period 2 and thereby increases the threshold below which banks hold their loans, μ_2^* . In this sense, it makes period 2 allocations less efficient. We will show that this policy reduces the region of multiplicity in period 1 and in this sense can improve period 1 allocations. To show the reduction in the region of multiplicity, consider the reputational gain in the positive reputational equilibrium evaluated at $\underline{\mu}$:

$$\beta (\bar{\pi} V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi}) V_2(\mu_{s0}) - V_2(\mu_h)).$$

Using (5), it is straightforward to see that an arbitrarily small reduction in interest rates of dr in period 2 reduces $V_2(\mu_{s\bar{v}})$ by $\alpha q dr$ since $\mu_{s\bar{v}} > \mu_2^*$. Moreover, since μ_{s0} and μ_h are strictly less than μ_2^* , $V_2(\mu_{s0})$ and $V_2(\mu_h)$ fall by $q dr$. As a result, the reputational gain falls by $\beta \bar{\pi} (1 - \alpha) q dr$. This decline in reputational gain induces an increase in the threshold $\underline{\mu}$. Similarly, we can show that the policy induces a fall in the threshold $\bar{\mu}$. Thus, the region of multiplicity shrinks and in this sense can improve period 1 allocations. Interestingly, such a policy is time inconsistent because the government has a strong incentive in period 2 not to make period 2 allocations less efficient.

An alternative policy that has not been proposed is to consider forced asset sales in which the government randomly forces banks to sell their loans. Such a policy in our model would mitigate the lemons problem in secondary loan markets by generating a pool of loans in secondary markets consistent with the ex ante mix of loan types. Although this is a standard intervention directed at increasing the price and volume of trade in markets that suffer from adverse selection, in our model such an intervention comes at the cost of misallocating loans to those without comparative advantage. Specifically, some banks with low costs of holding

loans will be forced to sell to the marketplace.

It is straightforward to show that a policy under which the government commits to purchase assets in period 2 at prices that are contingent on the realization of the signals can eliminate the multiplicity of equilibria and support the positive reputational equilibrium. Although such a policy would be desirable, the feasibility of such a policy can be analyzed only by developing a model in which private agents cannot commit but the government can.

8 Conclusion

This paper is an attempt to make three contributions: a theoretical contribution to the literature on reputation, a substantive contribution to the literature on the behavior of financial markets during crises, and a contribution to analyses of proposed and actual policies during the recent crisis. In terms of the theoretical contribution, we have combined insights from the literature that emphasizes the positive aspects of reputational incentives (see [Mailath and Samuelson \(2001\)](#)) with the literature that emphasizes the negative aspects of reputational incentives (see [Ely and Välimäki \(2003\)](#)) to show that multiplicity of equilibria naturally arise in reputation models like ours. We have also shown how techniques from the coordination games literature can be adapted to develop a refinement method that produces a unique equilibrium. In terms of the literature on the behavior of financial markets during crises, we have argued that sudden collapses in secondary loan market activity are particularly likely when the collateral value of the underlying loan declines. In terms of policy, we have argued that a wide variety of proposed policy responses would not have averted either the sudden collapse or the associated inefficiency. An important avenue for future work is to analyze policies that might in fact remedy the inefficiencies.

Another important avenue for future work is to introduce loan origination as a choice for banks in the model so that the model can be used to analyze the effects of sudden collapses on investment and other macroeconomic aggregates.

Appendix A Proofs

Proof of Lemma 1. First, consider the set $A = \{v_1; \hat{a}_1(v_1) > \hat{a}'_1(v_1)\}$. Then

$$\int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) - \int \hat{a}'_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) = \int_A dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) \geq 0$$

with equality only if A is measure zero. Given the Bayesian updating formulas, this inequality implies that for any \underline{v}_1 ,

$$\mu_{sg}(\underline{v}_1; \hat{a}_1) \geq \mu_{sg}(\underline{v}_1; \hat{a}'_1), \mu_{sd}(\underline{v}_1; \hat{a}_1) \geq \mu_{sd}(\underline{v}_1; \hat{a}'_1), \mu_h(\underline{v}_1; \hat{a}_1) \leq \mu_h(\underline{v}_1; \hat{a}'_1)$$

with strict inequalities only if A is zero measure. Therefore, for each v_1 , the integrand in (19) is higher for \hat{a}_1 and therefore $\Delta(v_1; \hat{a}_1) \geq \Delta(v_1; \hat{a}'_1)$ with equality only if A is measure zero.

Second, if \hat{a}_1 is a switching strategy with switching point k , from (18) it is straightforward to see that $\mu_{sg}(\underline{v}_1; \hat{a}_1), \mu_{sd}(\underline{v}_1; \hat{a}_1)$ are strictly increasing and $\mu_h(\underline{v}_1; \hat{a}_1)$ is strictly decreasing in \underline{v}_1 . Thus the integrand in (19) is increasing in \underline{v}_1 . Since we have assumed that $H(\hat{v}_1|v_1)$ is decreasing in v_1 , from first-order stochastic dominance, it follows that $\Delta(v_1; \hat{a}_1)$ is strictly increasing.

Finally, to show boundedness, we first show that for all μ_2 , $V_2(\mu_2)$ is well defined and continuous. Since μ_2 lies in a compact set, it follows that $V_2(\mu_2)$ is bounded. To show continuity, note that when $v_2 \geq (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2) = \hat{p}(\mu_2; v_2) - q$ and if $v_2 < (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2) = \bar{\pi}\bar{v} + (1 - \bar{\pi})v_2 - q(1 + r) - \bar{c}$. Therefore,

$$\begin{aligned} V_2(\mu_2) &= \int_{-\infty}^{\infty} \left[\int_{\mu^{*-1}(\mu_2)}^{\infty} \{\hat{p}(\mu_2; v_2) - q\} dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right) \right. \\ &\quad \left. + \int_{-\infty}^{\mu^{*-1}(\mu_2)} \{\bar{\pi}\bar{v} + (1 - \bar{\pi})v_2 - q(1 + r) - \bar{c}\} dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right) \right] dF(\underline{v}_2) \\ &= \{\hat{p}(\mu_2; \underline{v}_2) - q\} \\ &\quad + \int_{-\infty}^{\infty} \int_{-\infty}^{\mu^{*-1}(\mu_2)} \{(1 - \mu_2)(\bar{\pi} - \underline{\pi})(\bar{v} - v_2) - qr - \bar{c}\} dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right) dF(\underline{v}_2). \end{aligned} \tag{25}$$

Using our assumption that the random variable v_2 has a finite mean with respect to G

in (25), it follows that $V_2(\mu_2)$ is bounded. Continuity follows by inspection of (25) noting that so that G and F are continuous functions. Thus, there exist bounds $\underline{\Delta} \leq \bar{\Delta}$ such that for any v_1, \hat{a}_1

$$\underline{\Delta} \leq \bar{\pi}V_2(\mu_{s\bar{v}}(\underline{v}_1; \hat{a}_1)) + (1 - \bar{\pi})V_2(\mu_{s0}(\underline{v}_1; \hat{a}_1)) - V_2(\mu_h(\underline{v}_1; \hat{a}_1)) \leq \bar{\Delta}.$$

Q.E.D.

Proof of Lemma 2. We show that the best response function $b(k)$ is continuous and strictly increasing. Note that $b(k)$ satisfies the following:

$$\hat{p}(\mu_1; b(k)) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} + (1 - \bar{\pi})b(k) - q(1 + r) - \bar{c} \quad (26)$$

Since $\Delta(b; d_k)$ is continuous in b and k , it is obvious that $b(k)$ is continuous. An increase in k causes the function $\Delta(b; d_k)$ to decrease by Lemma 1. Since $\hat{p}(\mu_1; b) - (1 - \bar{\pi})b$ is increasing in b , from (26), $b(k)$ must be an increasing function of k .

Next, we show that the fixed point of $b(k)$ is unique. To see this, note that any fixed point of $b(k)$, v_1^* must satisfy

$$\hat{p}(\mu_1; v_1^*) - q + \Delta(v_1^*; d_{v_1^*}) = \bar{\pi}\bar{v} + (1 - \pi)v_1^* - q(1 + r) - \bar{c}.$$

Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following:

$$\begin{aligned} \Delta(v_1^*; d_{v_1^*}) = & \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi}V_2 \left(\hat{\mu}_{sg} \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) + (1 - \bar{\pi})V_2 \left(\hat{\mu}_{sd} \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right. \\ & \left. - V_2 \left(\hat{\mu}_h \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right\} dG \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \end{aligned}$$

Let $l = 1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Then the above integral becomes

$$\Delta(v_1^*; d_{v_1^*}) = \beta \int_0^1 [\bar{\pi}V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi})V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))] dl$$

and v_1^* must satisfy

$$-q + \beta \int_0^1 [\bar{\pi} V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi}) V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))] dl = \bar{\pi} \bar{v} + (1 - \underline{\pi}) v_1^* - \hat{p}(\mu_1; v_1^*) - q(1+r) - \bar{c}.$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^* . Since the right side ranges from plus infinity to minus infinity, there exist a unique v_1^* that satisfies the above equation. Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following:

$$\begin{aligned} \Delta(v_1^*; d_{v_1^*}) = & \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi} V_2 \left(\hat{\mu}_{sg} \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) + (1 - \bar{\pi}) V_2 \left(\hat{\mu}_{sd} \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right. \\ & \left. - V_2 \left(\hat{\mu}_h \left(1 - G \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right\} dG \left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \end{aligned}$$

and v_1^* must satisfy

$$-q + \beta \int_0^1 [\bar{\pi} V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi}) V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))] dl = \bar{\pi} \bar{v} + (1 - \underline{\pi}) v_1^* - \hat{p}(\mu_1; v_1^*) - q(1+r) - \bar{c}.$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^* . Since the right side ranges from plus infinity to minus infinity, there exists a unique v_1^* that satisfies the above equation.

Finally, we conclude by showing that when $k > v_1^*$, $b(k) < k$ and when $k < v_1^*$, $b(k) > k$. Suppose $k < v_1^*$ and $b(k) \leq k$. Since $\lim_{k \rightarrow -\infty} b(k) = \hat{v}^0 > -\infty$. Then by continuity of $b(\cdot)$, there must exist $k \in (-\infty, k]$ such that $b(\hat{k}) = \hat{k}$, contradicting part 2. Similarly, we can show that for all $k > v_1^*$, $b(k) < k$. *Q.E.D.*

Proof of Theorem 1. We prove Proposition 1 by mapping our environment into that described in Morris and Shin (2003) and show that their requirements for existence of a unique equilibrium in the limit are satisfied.

Given a value function $V_2(\mu_2)$, consider an equilibrium strategy profile in the first period $(a_1(\cdot), \hat{a}_1(\cdot), p_1(\cdot))$. In a game with full information about shocks to returns, when agents

in period 2 believe that the HH bank sells with probability l in the first period 1, the HH bank's differential gain from selling is given by

$$\hat{\pi}(v_1, l) = \hat{p}(\mu_1; v_1) + qr + \bar{c} - \bar{\pi}\bar{v} - (1 - \bar{\pi})v_1 + \beta [\bar{\pi}V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi})V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))].$$

Then, in the game with private information, $l = \int \hat{a}_1(v_1)dH(v_1|\underline{v}_1)$ is a random variable. We then show that $\hat{\pi}$ satisfies the conditions A1–A3, A4*, A5, and A6 in [Morris and Shin \(2003\)](#). We then can apply Theorem 2.2 in [Morris and Shin \(2003\)](#), and that completes the proof of our Proposition. It is easy to see that $\hat{\mu}_{sg}(l)$ and $\hat{\mu}_{sd}(l)$ are increasing in l and $\hat{\mu}_h(l)$ is decreasing in l . Since $V_2(\mu_2)$ is nondecreasing in μ_2 , $\hat{\pi}(v_1, l)$ is nondecreasing in l – condition A1. Obviously $\hat{\pi}(v_1, l)$ is increasing in v_1 – condition A2. Since $\hat{\pi}(v_1, l)$ is separable in v_1 and l , and $\hat{\pi}(v_1, l)$ is linearly increasing in v_1 , there must exist a unique v_1^* such that $\int \hat{\pi}(v_1^*, l)dl = 0$ – condition A3. Since $V_2(\mu_2)$ is a continuous function over a compact set $[0, 1]$, $\beta [\bar{\pi}V_2(\hat{\mu}_{sg}(l)) + (1 - \bar{\pi})V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))]$ is bounded above and below by $\underline{\Delta}$ and $\bar{\Delta}$, respectively. Now let \tilde{v}_1 and \hat{v}_1 be defined by

$$\begin{aligned} 0 &= -\hat{p}(\mu_1; \tilde{v}_1) - qr + \bar{\pi}\bar{v} + (1 - \bar{\pi})\tilde{v}_1 - \bar{c} - \bar{\Delta} - \varepsilon, \\ 0 &= -\hat{p}(\mu_1; \hat{v}_1) - qr + \bar{\pi}\bar{v} + (1 - \bar{\pi})\hat{v}_1 - \bar{c} - \underline{\Delta} + \varepsilon. \end{aligned}$$

Then, if $v_1 \leq \tilde{v}_1$, $\hat{\pi}(v_1, l) \leq -\varepsilon$ for all $l \in [0, 1]$. Moreover, if $v_1 \geq \hat{v}_1$, $\hat{\pi}(v_1, l) \geq -\varepsilon$ for all $l \in [0, 1]$ – condition A4*. Continuity of V_2 implies that $\hat{\pi}(v_1, l)$ is a continuous function of v_1 and l . Therefore, $\int_0^1 g(l)\hat{\pi}(v_1, l)dl$ is a continuous function of $g(\cdot)$ and v_1 – condition A5. Moreover, by definition of $F(\cdot)$ and $G(\cdot)$, noisy signal v_1 has a finite expectation, $E[v_1] \in R$ – condition A6. Therefore, we can rewrite Proposition 2.2 in [Morris and Shin \(2003\)](#) for our environment as follows:

Proposition Let v_1^* satisfy $\int \hat{\pi}(v_1^*, l)dl = 0$. For any $\delta > 0$, there exists a $\bar{\sigma} > 0$ such that for all $\sigma \leq \bar{\sigma}$, if strategy a_1 survives iterated elimination of dominated strategies, then $a_1(v_1) = 1$ for all $v_1 \geq v_1^* + \delta$ and $a_1(v_1) = 0$ for all $v_1 \leq v_1^* - \delta$.

Q.E.D.

Proof of Proposition 7. We proceed by induction. As described in Proposition 1, the game has a unique equilibrium in period T . The equilibrium strategy in the last period is a cutoff strategy with cutoff $v_T^*(\mu_T)$ given by

$$v_T^*(\mu_T) = \bar{v} - \frac{qr + \bar{c}}{(1 - \mu_T)(\bar{\pi} - \underline{\pi})}.$$

Using the equilibrium strategy, we define the last period's ex-ante value function, $V_T(\mu_T)$ according to

$$\begin{aligned} V_T(\mu_T) &= (1 - \alpha) \int_{-\infty}^{v_T^*(\mu_T)} \{\bar{\pi}\bar{v} + (1 - \underline{\pi})\underline{v}_t - q(1 + r) - \bar{c}\} dF(\underline{v}_t) \\ &\quad + (1 - \alpha) \int_{v_T^*(\mu_T)}^{\infty} \{\hat{p}(\mu_T; \underline{v}_t) - q\} dF(\underline{v}_t). \end{aligned}$$

From Theorem 1, as σ_{T-1} converges to zero, the set of equilibrium strategies in period $T - 1$ converges to a cutoff strategy with cutoff $v_{T-1}^*(\mu_{T-1})$ given by

$$v_{T-1}^*(\mu_{T-1}) = \bar{v} - \frac{qr + \bar{c} + \beta \int_0^1 [\bar{\pi}V_T(\hat{\mu}_{sg}(l; \mu_{T-1})) + (1 - \bar{\pi})V_T(\hat{\mu}_{sg}(l; \mu_{T-1})) - V_T(\hat{\mu}_h(l; \mu_{T-1}))] dl}{(1 - \mu_{T-1})(\bar{\pi} - \underline{\pi})}$$

Notice that for σ_{T-1} small and given the above cutoff strategy, the value function at period $T - 1$, $V_{T-1}(\mu_{T-1}; \sigma_{T-1})$ is given by

$$\begin{aligned} V_{T-1}(\mu_{T-1}; \sigma_{T-1}) &= (1 - \alpha) \int_{\underline{v}_t} \int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}} \{\bar{\pi}\bar{v} + (1 - \underline{\pi})\underline{v}_t - q(1 + r) - \bar{c} \\ &\quad + \beta V_T \left(\hat{\mu}_h \left(1 - G \left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}} \right) \right) \right) \} dG(\varepsilon_{T-1}) dF(\underline{v}_t) + \\ &\quad (1 - \alpha) \int_{\underline{v}_t} \int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}} \{\hat{p}(\mu_{T-1}; \underline{v}_t) - q \\ &\quad + \beta \bar{\pi} V_T \left(\hat{\mu}_{sg} \left(1 - G \left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}} \right) \right) \right) \\ &\quad + \beta (1 - \bar{\pi}) V_T \left(\hat{\mu}_{sb} \left(1 - G \left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}} \right) \right) \right) \} dG(\varepsilon_{T-1}) dF(\underline{v}_t) \\ &\quad + \alpha \int_{\underline{v}_t} \int_{-\infty}^{\infty} \{\bar{\pi}\bar{v} + (1 - \underline{\pi})\underline{v}_t - q(1 + r) - \bar{c} \\ &\quad + \beta V_T \left(\hat{\mu}_h \left(1 - G \left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}} \right) \right) \right) \} dG(\varepsilon_{T-1}) dF(\underline{v}_t), \end{aligned}$$

and hence, the above formula becomes the following as $\sigma_{T-1} \rightarrow 0$:

$$\begin{aligned}
V_{T-1}(\mu_{T-1}) &= (1 - \alpha) \int_{-\infty}^{v_{T-1}^*(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1 - \bar{\pi})\underline{v}_t - q(1 + r) - \bar{c} + \beta V_T(\hat{\mu}_h(0))\} dF(\underline{v}_t) \\
&+ (1 - \alpha) \int_{v_{T-1}^*(\mu_{T-1})}^{\infty} \{\hat{p}(\mu_{T-1}; \underline{v}_t) - q + \beta \bar{\pi} V_T(\hat{\mu}_{sg}(1)) + \beta(1 - \bar{\pi}) V_T(\hat{\mu}_{sd}(1))\} dF(\underline{v}_t) \\
&+ \alpha \int_{-\infty}^{v_{T-1}^*(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1 - \bar{\pi})\underline{v}_t - q(1 + r) - \underline{c} + \beta V_T(\hat{\mu}_h(0))\} dF(\underline{v}_t) \\
&+ \alpha \int_{v_{T-1}^*(\mu_{T-1})}^{\infty} \{\bar{\pi}\bar{v} + (1 - \bar{\pi})\underline{v}_t - q(1 + r) - \underline{c} + \beta V_T(\hat{\mu}_h(1))\} dF(\underline{v}_t)
\end{aligned} \tag{27}$$

Similarly, suppose for some period $t+1$ and any μ_{t+1} , the multi period model has a unique equilibrium with payoff for the HH bank given by $V_{t+1}(\mu_{t+1})$. If $V_{t+1}(\mu_{t+1})$ is increasing in μ_{t+1} , then the proof of Theorem 1 can be applied. As a result, as $\sigma_t \rightarrow 0$, the set of equilibrium strategies in period t converges to a cutoff strategy with cutoff $v_t^*(\mu_t)$ satisfying the properties defined in Proposition 7. In addition, this cutoff strategy can be used to construct the value function in period t , $V_t(\mu_t)$ in fashion similar to (27).

Q.E.D.

References

- ARORA, S., B. BOAZ, M. BRUNNERMEIER, AND R. GE (2009): “Computational Complexity and Information Asymmetry in Financial Products,” Working Paper, Princeton University, Department of Computer Science.
- ASHCRAFT, A. AND T. SCHUERMANN (2006): “Understanding the Securitization of Subprime Mortgage Credit,” *Foundations and Trends in Finance*, 2, 191–309.
- CARLSSON, H. AND E. VAN DAMME (1993): “Global Games and Equilibrium Selection,” *Econometrica*, 61, 989–1018.

- CARTER, S. AND R. SUTCH (2006): *Historical Statistics of the United States: Earliest Times to the Present*, Cambridge University Press, New York.
- DEMARZO, P. (2005): “The Pooling and Tranching of Securities: A Model of Informed Intermediation,” *Review of Financial Studies*, 18, 1–35.
- DEMARZO, P. AND D. DUFFIE (1999): “A Liquidity-Based Model of Security Design,” *Econometrica*, 67, 65–99.
- DEWATRIPONT, M. AND J. TIROLE (1994): *The Prudential Regulation of Banks*, MIT Press, Cambridge, MA.
- DIAMOND, D. (1989): “Reputation Acquisition in Debt Markets,” *Journal of Political Economy*, 97, 828–862.
- DOWNING, C., D. JAFFEE, AND N. WALLACE (2009): “Is the Market for Mortgage-Backed Securities a Market for Lemons?” *Review of Financial Studies*, 22, 2457–2494.
- DRUCKER, S. AND C. MAYER (2008): “Inside Information and Market Making in Secondary Mortgage Markets,” Working Paper, Renaissance Technologies LL3 and Duke University.
- EISFELDT, A. (2004): “Endogenous Liquidity in Asset Markets,” *The Journal of Finance*, 59, 1–30.
- ELUL, R. (2009): “Securitization and Mortgage Default: Reputation vs. Adverse Selection,” Working Paper, Federal Reserve Bank of Philadelphia.
- ELY, J., D. FUDENBERG, AND D. LEVINE (2008): “When Is Reputation Bad?” *Games and Economic Behavior*, 63, 498–526.
- ELY, J. AND J. VÄLIMÄKI (2003): “Bad Reputation,” *Quarterly Journal of Economics*, 118, 785–814.

- FRANKEL, D., S. MORRIS, AND A. PAUZNER (2003): “Equilibrium Selection in Global Games with Strategic Complementarities,” *Journal of Economic Theory*, 108, 1–44.
- GARLEANU, N. AND L. PEDERSEN (2004): “Adverse Selection and the Required Return,” *Review of Financial Studies*, 17, 643–665.
- GLOSTEN, L. AND P. MILGROM (1985): “Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders,” *Journal of Financial Economics*, 14, 71–100.
- GUERRIERI, V. AND R. SHIMER (2011): “Competitive Equilibrium in Asset Markets with Adverse Selection,” .
- GUERRIERI, V., R. SHIMER, AND R. WRIGHT (2010): “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 78, 1823–1862.
- HOLMSTROM, B. AND R. MYERSON (1983): “Efficient and Durable Decision Rules with Asymmetric Information,” *Econometrica*, 51, 1799–1819.
- IVASHINA, V. (2009): “Asymmetric Information Effects on Loan Spreads,” *Journal of Financial Economics*, 92, 300–319.
- IVASHINA, V. AND D. SCHARFSTEIN (2010): “Bank Lending During the Financial Crisis of 2008,” *Journal of Financial Economics*, 97, 319–338.
- KREPS, D. AND R. WILSON (1982): “Reputation and Imperfect Information,” *Journal of Economic Theory*, 27, 253–279.
- KURLAT, P. (2010): “Lemons, Market Shutdowns and Learning,” *Working Paper, Stanford University*.
- KYLE, A. (1985): “Continuous Auctions and Insider Trading,” *Econometrica*, 53, 1315–1335.

- MAILATH, G. AND L. SAMUELSON (2001): “Who Wants a Good Reputation?” *Review of Economic Studies*, 68, 415–441.
- MIAN, A. AND A. SUFI (2009): “The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis,” *Quarterly Journal of Economics*, 124, 1449–1496.
- MILGROM, P. AND J. ROBERTS (1982): “Predation, Reputation, and Entry Deterrence,” *Journal of Economic Theory*, 27, 280–312.
- MORRIS, S. AND H. SHIN (2003): “Global Games: Theory and Applications,” 1, 56–114, in *Advances in Economics and Econometrics: Theory and Applications*, Eighth world Congress, vol. 1, ed. M. Dewatripont, L.P. Hansen, and S.J. Turnovsky, Cambridge University Press.
- MYERS, S. AND N. MAJLUF (1984): “Corporate Financing and Investment Decisions When Firms Have Information that Investors Do Not Have,” *Journal of Financial Economics*, 13, 187–221.
- ORDOÑEZ, G. (2009): “Fragility of Reputation and Clustering in Risk-Taking,” Staff Report No. 431, Federal Reserve Bank of Minneapolis.
- PHILIPPON, T. AND V. SKRETA (2009): “Efficient Interventions in Markets with Adverse Selection,” Working Paper, New York University.
- TREASURY DEPARTMENT (2009): “Fact Sheet on Public-Private Investment Program for Legacy Assets,” http://www.treasury.gov/press-center/press-releases/Documents/ppip_fact_sheet.pdf.
- WHITE, E. N. (2009): “Lessons from the Great American Real Estate Boom and Bust of the 1920s,” Working Paper 15573, National Bureau of Economic Research.

Appendix B NOT FOR PUBLICATION

Proposition 8 *Suppose $\beta(1 - \alpha) \leq 1$ and $0 < \mu_2^* < 1$. Then, there exist $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that*

1. *if $\mu_1 \in [\underline{\mu}, \bar{\mu})$, the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan,*
2. *if $\mu_1 < \underline{\mu}$, the model has a unique equilibrium in which the HH bank holds its loan in period 1,*
3. *if $\mu_1 \geq \bar{\mu}$, the model has a unique equilibrium in which the HH bank sells its loan in period 1.*

Proof. We show that our economy has a positive reputational equilibrium. As an implication of Bayes Rule, if the HH bank sells its loan in the first period, the reciprocal of the posterior beliefs is a martingale. Formally, we have

$$\frac{\bar{\pi}}{\mu_{s\bar{v}}} + \frac{1 - \bar{\pi}}{\mu_{s0}} = \frac{1}{\mu_1} = \frac{1}{\mu_h}$$

Since $1/\mu$ is a convex function, it follows that

$$\bar{\pi}\mu_{s\bar{v}} + (1 - \bar{\pi})\mu_{s0} \geq \mu_1 = \mu_h. \quad (28)$$

Let the reputational gain be defined as

$$\Delta^g(\mu_1) = \beta(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h))$$

Recall from (5) that V_2 is a convex and increasing function, so that

$$\bar{\pi}V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi})V_2(\mu_{s0}) \geq V_2(\bar{\pi}\mu_{s\bar{v}} + (1 - \bar{\pi})\mu_{s0}).$$

This convexity together with (28) implies that $\Delta^g(\mu_1) \geq 0$.

Next we show that there is some critical value of μ_1 denoted $\mu_g < \mu_2^*$ such that for all μ_1 in the interval $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1 and $\Delta^g(\mu_1) = 0$

for $\mu_1 \leq \mu_g$. To obtain these results, define μ_g implicitly by

$$\mu_2^* = \frac{\mu_g \bar{\pi}}{\mu_g \bar{\pi} + (1 - \mu_g) \underline{\pi}}.$$

That is μ_g denotes that initial reputation level such that if the HH bank sells and receives a good signal, its reputation level would rise to μ_2^* . Since $\bar{\pi} > \underline{\pi}$, $\mu_g < \mu_2^*$. To see that for all $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1 , rewrite the reputational gain as

$$\Delta^g(\mu_1) = \beta (\bar{\pi}(V_2(\mu_{s\bar{v}}) - V_2(\mu_h)) + (1 - \bar{\pi})(V_2(\mu_{s0}) - V_2(\mu_h))).$$

Since $\mu_h = \mu_1$ and $\mu_{s0} < \mu_1$, from Proposition 1 it follows that for all $\mu_g < \mu_1 \leq \mu_1^*$, $V_2(\mu_{s0}) = V_2(\mu_h)$. Since $\mu_{s\bar{v}} > \mu_h = \mu_1$, it follows that $\Delta^g(\mu_1)$ is positive and since $\mu_{s\bar{v}}$ is strictly increasing in μ_1 it follows that $\Delta^g(\mu_1)$ is strictly increasing. To see that $\Delta^g(\mu_1) = 0$ for $\mu_1 \leq \mu_g$, note that $\mu_{s\bar{v}} \leq \mu_2^*$ so that $V_2(\mu_{s\bar{v}}) = V_2(\mu_h)$.

Next, rewrite (10) as

$$(\mu_1 \bar{\pi} + (1 - \mu_1) \underline{\pi}) \bar{v} - q + \Delta^g(\mu_1) \geq \bar{\pi} \bar{v} - q(1 + r) - \bar{c} \quad (29)$$

Consider $\mu_1 \leq \mu_2^*$. Since $\Delta^g(\mu_1)$ is a nondecreasing function of μ_1 in this range and $(\mu_1 \bar{\pi} + (1 - \mu_1) \underline{\pi}) \bar{v}$ is a strictly increasing function of μ_1 , it follows that the left side of (29) is strictly increasing in this range. Since $\Delta^g(\mu_1^*)$ is strictly positive, using (3) the left side of (29) is strictly greater than the right side of this inequality at μ_1^* . Since $\Delta^g(\mu_g) = 0$ and $\mu_g < \mu_2^*$, the left side is strictly less than the right side at μ_g . Thus, there is a unique value of μ at which (29) holds as an equality. For $\mu_1 > \mu_2^*$, $(\mu_1 \bar{\pi} + (1 - \mu_1) \underline{\pi}) \bar{v} - q > \bar{\pi} \bar{v} - q(1 + r) - \bar{c}$ and $\Delta^g(\mu_1) \geq 0$ so that (29) is satisfied. We have established that our model has an equilibrium in which all HH banks with reputation levels above $\mu_1 \geq \underline{\mu}$ sell.

To obtain the negative reputational equilibrium, define μ_b implicitly by

$$\mu_2^* = \frac{\mu_b}{\mu_b + (1 - \mu_b) \alpha}.$$

That is μ_b denotes that initial reputation level such that if the HH bank holds, its reputation

level would rise to μ_2^* . Clearly $\mu_b < \mu_2^*$.

Since $\mu_h = \mu_1/(\mu_1 + (1 - \mu_1)\alpha)$ is greater than μ_1 , it follows that $\Delta^b(\mu_1)$ is negative for $\mu_1 > \mu_b$. If $\mu_1 \in [\mu_b, \mu_2^*]$, selling has a static cost, i.e. $\hat{p}(\mu_2) - q \leq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$ as well as a loss from reputation, i.e. $\Delta^b(\mu_1) < 0$ so that the HH bank prefers to hold the asset. If $\mu_1 \in (\mu_2^*, 1]$, there are benefits from selling the asset, i.e. $\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$, while there is a loss from reputation $\Delta^b(\mu_1) < 0$. Our assumption that $\beta(1 - \alpha) \leq 1$ ensures that when $\mu_1 = 1$, the static benefit outweighs the loss from reputation, i.e. (12) is reversed at $\mu_1 = 1$. Moreover, Since $\mu_h = \mu_1/(\mu_1 + (1 - \mu_1)\alpha)$, it is easy to show that $(\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi})\bar{v} - q + \Delta^b(\mu_1)$ is a strictly convex function of μ_1 for $\mu_1 \in [\mu_2^*, 1]$. Since the value of this function is strictly less than $\bar{\pi}\bar{v} - q(1 + r) - \bar{c}$ at $\mu_1 = \mu_2^*$ and weakly higher when $\mu_1 = 1$, there exists a unique $\bar{\mu} \in (\mu_2^*, 1]$, at which (12) holds with equality. For $\mu_1 \leq \bar{\mu}$, (12) holds and for $\mu_1 > \bar{\mu}$ (12) is violated. Q.E.D.

Proposition 9 *Suppose $\beta(1 - \alpha) \leq 1$ and*

$$(\bar{\pi} - \underline{\pi})\bar{v} + qr + \max_{\mu_1 \in [0,1]} \Delta^g(\mu_1) < -\underline{c}. \quad (30)$$

Then the unique equilibrium of the static game described in Proposition 1 and the multiple equilibria of the dynamic game described in Proposition 2 are also equilibria of the associated games when all bank types behave strategically.

Proof. Consider the static game. It is sufficient to show that given the constructed equilibrium and specified strategies for all agents, there is no profitable deviation by any agent. Note that in the proof of Proposition 2 we show that $\Delta^g(\mu_1) \geq 0$ for all $\mu_1 \in [0, 1]$. Hence, (30) implies that

$$\mu_1 (\bar{\pi} - \underline{\pi})\bar{v} + qr < -\underline{c}$$

or

$$[\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q < \underline{\pi}\bar{v} - q(1 + r) - \underline{c} \quad (31)$$

Inequality (31) implies that facing break even prices the low cost type bank would like to

hold. Moreover a deviation by a buyer must attract these types of bank and (31) implies that buyers must offer a price higher than the actuarially fair price. Hence, there is no deviation by any buyer or a low cost bank type. Moreover, an LH bank wants to sell even at the lowest possible price, $\underline{\pi}\bar{v}$, since $\bar{c} > 0$. Thus there are no profitable deviation from the specified strategies in the static game.

Consider the positive equilibrium of the dynamic game. Given future beliefs, the value of selling to a low quality bank adjusted by the future reputational gain from holding is given by

$$[\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \beta [\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)]$$

where $\mu_{s\bar{v}} = \bar{\pi}\mu_1/(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})$ and $\mu_{s0}^g = (1 - \bar{\pi})\mu_1/((1 - \bar{\pi})\mu_1 + (1 - \underline{\pi})(1 - \mu_1))$. The value of selling to a high quality bank is given by

$$[\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \Delta^g(\mu_1)$$

From (30) and $\beta [\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)] = \Delta^g(\mu_1)$, we have

$$\begin{aligned} [\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \beta [\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)] &\leq \underline{\pi}\bar{v} - q(1 + r) - \underline{c} \\ [\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \Delta^g(\mu_1) &\leq \bar{\pi}\bar{v} - q(1 + r) - \underline{c} \end{aligned}$$

Hence, there is no profitable deviation by the low cost types. As for the LH type bank, note that in the positive equilibrium

$$[\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \beta [\bar{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \bar{\pi})V_2(\mu_{s0}^g) - V_2(\mu)] \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c} \quad (32)$$

We use the above inequality to show that the LH type bank does not have a profitable deviation. There are two possible cases: Case 1. $\bar{c} + qr \geq (\bar{\pi} - \underline{\pi})\bar{v}$. In this case, $\mu_2^* = 0$ and $V_2(\mu)$ is a constant function. Therefore, $\Delta^g(\mu_1) = 0$ for all μ_1 and $\beta [\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)] = 0$. In this case, we are back to the static game and as we have shown before, the LH bank finds it optimal to sell always. Case 2. $\bar{c} + qr <$

$(\bar{\pi} - \underline{\pi})\bar{v} < \bar{v}$. In this case, we have

$$\begin{aligned}\beta [V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})] &\leq \beta(1 - \alpha) \{[\mu_{s\bar{v}}\bar{\pi} + (1 - \mu_{s\bar{v}})\underline{\pi}]\bar{v} - q - \bar{\pi}\bar{v} + q(1 + r) + \bar{c}\} \\ &= \beta(1 - \alpha) \{-(1 - \mu_{s\bar{v}})(\bar{\pi} - \underline{\pi})\bar{v} + qr + \bar{c}\}\end{aligned}$$

The last expression is increasing in μ_1 and therefore maximized at $\mu_1 = 1$. Hence, we must have

$$\beta [V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})] \leq \beta(1 - \alpha)(qr + \bar{c}) < \bar{v}$$

Therefore,

$$-\beta(\bar{\pi} - \underline{\pi}) [V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})] > -\bar{v}(\bar{\pi} - \underline{\pi})$$

Adding this inequality to (32), we get

$$[\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi}]\bar{v} - q + \beta [\underline{\pi}V_2(\mu_{s\bar{v}}^g) + (1 - \underline{\pi})V_2(\mu_{s0}^g) - V_2(\mu)] \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$$

which implies that the LH type bank does not have a profitable deviation in the constructed equilibrium.

As for the negative equilibrium, it is clear that a bank with low cost does not want to sell its loan, since selling only punishes the bank. Therefore, it is sufficient to show that the LH bank wants to sell its loan. That is, we need to show that for all $\mu_1 \in [0, \bar{\mu}]$, we have

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] \geq \underline{\pi}\bar{v} - q(1 + r) - \bar{c} \quad (33)$$

where $\mu_h^b = \mu_1/(\mu_1 + (1 - \mu_1)\alpha)$. To do so, we first show that this inequality is satisfied at $\mu_1 = \bar{\mu}$. Now, since $\Delta^b(\mu_1) = \beta[V_2(0) - V_2(\mu_h^b)]$ is decreasing, this implies that (33) holds for all $\mu_1 \in [0, \bar{\mu}]$. By definition, $\bar{\mu}$ satisfies

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] = \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$$

Obviously, this equality leads to the above inequality. Therefore, we have shown that LH bank still finds it optimal to sell in the negative equilibrium. Q.E.D.

Proof of Proposition 3. We shall prove that when

$$\mu_2^* < \frac{\beta\bar{\pi} - \frac{\pi}{\bar{\pi}\alpha - \pi}}{1 + \beta\bar{\pi}(1 - \alpha)}$$

then if $\mu_1 = \underline{\mu}$, we must have $\mu_h^n < \mu_2^*$. Note that from (13),

$$\begin{aligned} \mu_h^n &= \frac{\underline{\mu}}{\underline{\mu} + (1 - \underline{\mu})\alpha} < \mu_2^* \\ \Leftrightarrow \underline{\mu} &< \mu_2^* [\underline{\mu} + (1 - \underline{\mu})\alpha] \\ \Leftrightarrow \underline{\mu} (1 - \mu_2^*(1 - \alpha)) &< \mu_2^* \alpha \\ \Leftrightarrow \underline{\mu} &< \frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha} \end{aligned} \quad (34)$$

Hence, we must show that the above inequality holds. Notice that from (10), $\underline{\mu}$ is defined by

$$\hat{p}(\underline{\mu}) + \beta [\bar{\pi}V(\mu_{s\bar{v}}) + (1 - \bar{\pi})V(\mu_{s0})] = \bar{\pi}\bar{v} - \bar{c} - qr + \beta V(\mu_h).$$

Since $\hat{p}(\mu_2^*) = \bar{\pi}\bar{v} - \bar{c} - qr$ and $V(\mu_h) = V(\mu_{s0}) = V(\mu_2^*)$, the above equality can be written as

$$\hat{p}(\underline{\mu}) + \beta\bar{\pi} [V(\mu_{s\bar{v}}) - V(\mu_2^*)] = \hat{p}(\mu_2^*).$$

Moreover, since low cost types always hold their assets, we must have

$$V(\mu_{s\bar{v}}) - V(\mu_2^*) = (1 - \alpha) [\hat{p}(\mu_{s\bar{v}}) - \hat{p}(\mu_2^*)].$$

Therefore, (10) becomes

$$\hat{p}(\underline{\mu}) + \beta\bar{\pi}(1 - \alpha) [\hat{p}(\mu_{s\bar{v}}) - \hat{p}(\mu_2^*)] = \hat{p}(\mu_2^*),$$

Using the fact that, $\hat{p}(\cdot)$ is a linear function and definition of $\mu_{s\bar{v}}$ from (11),

$$\underline{\mu} + \beta\bar{\pi}(1 - \alpha) \left[\frac{\underline{\mu}}{\underline{\mu} + (1 - \underline{\mu})\frac{\pi}{\bar{\pi}}} - \mu_2^* \right] = \mu_2^*$$

Given that the right hand side of the above equation is increasing in $\underline{\mu}$, (34) is equivalent to the following inequality

$$\frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha} + \beta\bar{\pi}(1 - \alpha) \left[\frac{\frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha}}{\frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha} + (1 - \frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha})\frac{\pi}{\bar{\pi}}} - \mu_2^* \right] > \mu_2^*$$

The above inequality can be further simplified in the following steps:

$$\begin{aligned}
& \frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha} + \beta \bar{\pi} (1 - \alpha) \left[\frac{\mu_2^* \alpha}{\mu_2^* \alpha + (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}} - \mu_2^* \right] > \mu_2^* \\
\Leftrightarrow & \beta \bar{\pi} (1 - \alpha) \mu_2^* \frac{\alpha(1 - \mu_2^*) - (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}}{\mu_2^* \alpha + (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}} > \mu_2^* - \frac{\mu_2^* \alpha}{1 - \mu_2^* + \mu_2^* \alpha} \\
\Leftrightarrow & \beta \bar{\pi} (1 - \alpha) \mu_2^* \frac{\alpha(1 - \mu_2^*) - (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}}{\mu_2^* \alpha + (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}} > \mu_2^* \frac{1 - \mu_2^* - \alpha(1 - \mu_2^*)}{1 - \mu_2^* + \mu_2^* \alpha}.
\end{aligned}$$

Since $0 < \mu_2^* < 1$, we can divide both sides of the above inequality by $\mu_2^*(1 - \mu_2^*)$ and we have

$$\begin{aligned}
& \beta \bar{\pi} (1 - \alpha) \frac{\alpha - \frac{\pi}{\bar{\pi}}}{\mu_2^* \alpha + (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}} > \frac{1 - \alpha}{1 - \mu_2^* + \mu_2^* \alpha} \\
\Leftrightarrow & \beta \bar{\pi} \frac{\alpha - \frac{\pi}{\bar{\pi}}}{\mu_2^* \alpha + (1 - \mu_2^*) \frac{\pi}{\bar{\pi}}} > \frac{1}{1 - \mu_2^* + \mu_2^* \alpha} \\
\Leftrightarrow & \beta \bar{\pi} \left(\alpha - \frac{\pi}{\bar{\pi}} \right) (1 - \mu_2^* (1 - \alpha)) > \mu_2^* \left(\alpha - \frac{\pi}{\bar{\pi}} \right) + \frac{\pi}{\bar{\pi}} \\
\Leftrightarrow & \beta \bar{\pi} \left(\alpha - \frac{\pi}{\bar{\pi}} \right) - \frac{\pi}{\bar{\pi}} > \mu_2^* \left(\alpha - \frac{\pi}{\bar{\pi}} \right) [1 + \beta \bar{\pi} (1 - \alpha)]
\end{aligned}$$

The above inequality is equivalent to

$$\frac{\beta \bar{\pi} - \frac{\pi}{\bar{\pi} \alpha - \pi}}{1 + \beta \bar{\pi} (1 - \alpha)} > \mu_2^*$$

and this completes the proof. *Q.E.D.*